Unit 06

BASIC STATISTICS

Frequency Distribution

A frequency distribution is a tabular arrangement for classifying data into different groups and the number of observation falling in each group corresponds to the respective group. In fact a frequency distribution is a method to summarize data.

Grouped Data:

The data presented in the form of frequency distribution is called grouped data.

Types of Frequency Distribution

On the basis of types of variable or data, there are two types of frequency distribution. These are

(a) Discrete Frequency Distribution

(b) Continuous Frequency Distribution

(a) Discrete Frequency Table

Following steps are involved in making of a discrete frequency distribution:

(i) Find the minimum and maximum observation in the data and write the values of the variable in the variable column from minimum to the maximum.

(ii) Record the observations by using tally marks. (Vertical bar '|')

(iii) Count the tally marks write down the frequency in the frequency column.

Example 1: Five coins are tossed 20 times and the number of head recorded at each toss are give below:

3, 4, 2, 3, 3, 5, 2, 2, 2, 1, 1, 2, 1, 4, 2, 2, 3,3,4,2.

Make frequency distribution of the number of heads observed.

Solution: Let X = Number of Heads. The frequency distribution is give below:

Frequency Distribution of number of heads		
X	**Tally Marks**	**Frequency**
1	|||	3
2	~~||||~~ |||	8
3	~~||||~~	5
4	|||	3
5	|	1
Total		**20**

(b) Continuous frequency distribution

The making of continuous frequency distribution involves the following steps:

(i) Find the Range where Range = $X_{max} - X_{min}$ (the difference between maximum and minimum observations).

(ii) Decide about the number of groups (denote it by k) into which the data is to be classified (usually an integer between 5 and 20). Usually it depends upon the range. The larger the range the more the number of groups.

(iii) Determine the size of class (denote by h) by using the formula:

$$h = \frac{\text{Range}}{k}$$

Note: The rule of approximation is relaxed in determining h. For example, h = 7.1 or h = 7.9 may be taken as 8.

(iv) Start writing the classes or groups of the frequency distribution usually starting from the minimum observation and keeping in view the size of a class.

(v) Record the observations from the data by using tally marks.

(vi) Count the number of tally marks and record them in the frequency column for each class.

Example 2: The following are the marks obtained by 40 students in mathematics of class X. Make a frequency distribution with a class interval of size 10.

51, 55, 32, 41, 22, 30, 35, 53, 30, 60, 59, 15, 7, 18 , 40, 49, 40, 25, 14, 18, 19, 2, 43, 22, 39, 26, 34, 19, 10, 17, 47, 38, 13, 30, 34, 54, 10, 21, 51, 52.

Solution: Let X = Marks of a student.

From the above data we have $X_{min} = 2$, $X_{max} = 60$. It is given that h = 10. We can either start from 2 or the nearest smallest integer 0 for our convenience. There are two way to make frequency distribution.

(a) We may write the actual observations falling in the respective groups. This is given as follows:

Groups	Observations	F
0–9	2,7	2
10–19	10,10,13,14,15,17,18,18,19,19	10
20–29	21,22.22.25.26	5
30–39	30,30,30,32,34,34,35,38,39	9
40–49	40,40,41,43,47,49	6
50–59	51,51,52,53,54,55,59	7
60–69	60	1
Total		**40**

(b) Use tally marks for recording each observation in the respective group. This is given in the following table:

Classes/Groups	Tally Marks	Frequency
0–9	||	2
10–19	||||̸ ||||̸	10
20–29	||||̸	5
30–39	||||̸ ||||	9
40–49	||||̸ |	6
50–59	||||̸ ||	7
60–69	|	1
Total		**40**

Note: The Tally bar method is usually adopted to construct a frequency distribution.

Concepts involved in a Continuous frequency table.

The following terms are frequently used in a continuous frequency distribution.

(a) Class Limits: The minimum and the maximum values defined for a class or group are called class limits. The minimum value is called the lower class limit and the maximum value is called the upper class limit of that class. For example in the group (5–10), 5 is lower class limit and 10 is called upper class limit.

(b) Class Boundaries:

The real class limits of a class are called class boundaries. A class boundary is obtained by adding two successive class limits and dividing the sum by 2. The value so obtained is taken as upper class boundary for the previous class and lower class boundary for the next class.

(c) Midpoint or Class Mark:

For a given class the average of that class obtained by dividing the sum of upper and lower class limits by 2, is called the midpoint or class mark of that class.

For example the midpoint of a class (6–10) is 8. i.e.

$$\text{Class Marks } x = \frac{6+10}{2} = \frac{16}{2} = 8$$

(d) **Cumulative Frequency:** The total of frequency up to an upper class limit or boundary is called the cumulative frequency.

Example 3: Compute class boundaries, class marks and cumulative frequency for data of example 2.

Solution:

Class Limits	Class Boundaries	Midpoint/ Class Marks	f	C.F
0–9	–0.5–9.5	4.5	2	2
10–19	9.5–19.5	14.5	10	2+10=12
20–29	19.5–29.5	24.5	5	12+5=17
30–39	29.5–39.5	34.5	9	17+9=26
40–49	39.5–49.5	44.5	6	26+6=32
50–59	49.5–59.5	54.5	7	32+7=39
60–69	59.5–69.5	64.5	1	39+1=40
Total			40	

Construction of Histograms:

Histogram

A Histogram is a graph of adjacent rectangles constructed on XY–plane. It is a graph of frequency distribution. In practice both discrete and continuous frequency distribution are represented by means of histogram. However there is a little difference in the construction procedure. We explain this with the help of examples.

Equal Intervals Histogram:

Example 1: Make a histogram of the following distribution of the number of heads when 5 coins were tossed.

X (number of heads)	Frequency
0	1
1	3
2	8
3	5
4	3
5	1

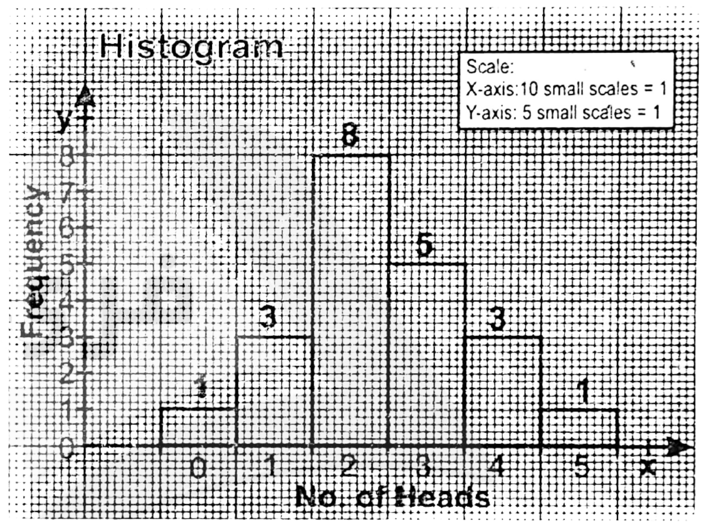
Solution: We proceed as follows:

Step 1: Mark the value of variable X along x–axis using a suitable interval.

Step 2: Mark the frequency along y–axis using a suitable scale.

Step 3: At each interval make a rectangle of height corresponding to the respective frequency of values of the variable X.

The resulting Histogram is given below:



Example 2: Make a Histogram for the following distribution of marks.

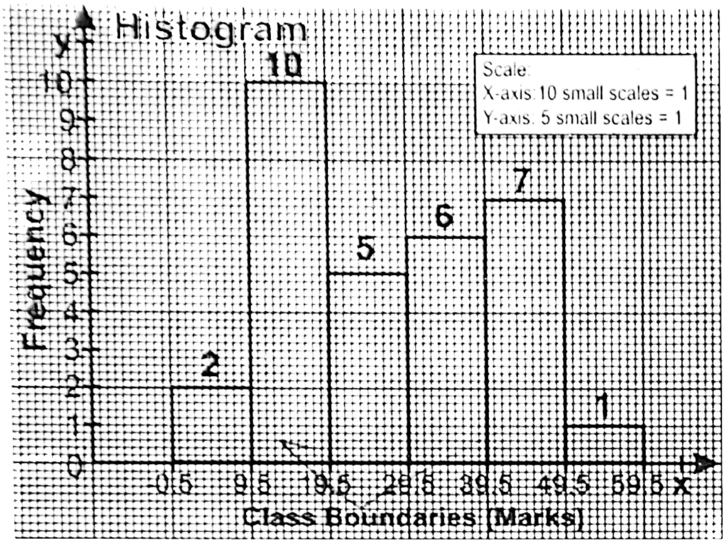
Class Boundaries	Frequency
–0.5–9.5	2
9.5–19–5	10
19.5–29.5	5
29.5–39.5	6
39.5–49.5	7
49.5–59.5	1

Solution: Since this is a continuous frequency distribution so we proceed as follows:

(a) Mark the class boundaries along x–axis using a suitable scale.

(b) Mark the frequency along y–axis using a suitable scale.

(c) At each class interval construct a rectangle of height corresponding to the frequency of that group.



Note: On graph above 0 and –0.5 are written on positive side of x–axis just to better understand the histogram.

Unequal Intervals Histogram:

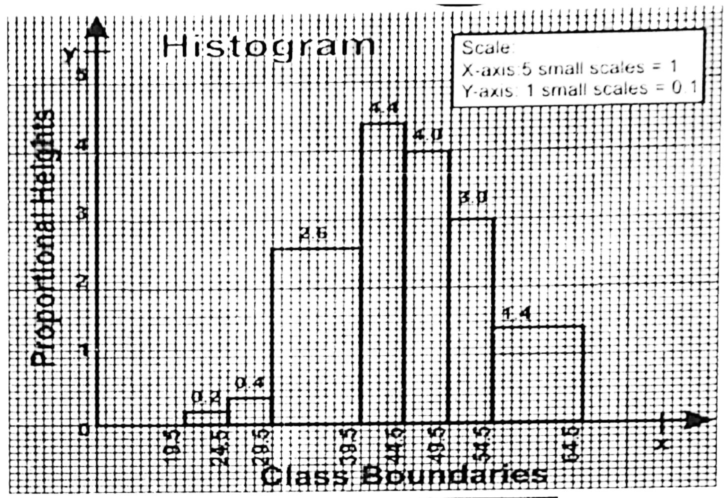
If the class intervals are un–equal, the frequency must be adjusted by dividing each class frequency on its class interval size. If the interval becomes double, then frequency is divided by 2, so that the area of the bar is in proportion to the areas of other bars etc.

Example 3: Draw a histogram illustrating the following data:

Ages	Number of Men
20–24	1
25–29	2
30–39	26
40–44	22
45–49	20
50–54	15
55–64	14

Solution: As the class intervals are unequal the height of each rectangle cannot be made equal to the frequency. Therefore we obtain proportional heights by dividing each frequency with class interval size. This is shown in the following table.

Ages	Class Boundaries	Class Interval (h)	f	Proportional Heights
20–24	19.5–24.5	5	1	1÷5= 0.20
25–29	24.5–29.5	5	2	2÷5=0.4
30–39	29.5–39.5	10	26	26÷10=2.6
40–44	39.5–44.5	5	22	22÷5=4.4
45–49	44.5–49.5	5	20	20÷5=4.0
50–54	49.5–54.5	5	15	15÷5=3.0
55–64	54.5–64.5	10	14	14÷10=1.4



Construction of Frequency Polygon

A Frequency Polygon is a many sided closed figure. Its construction is explained by the following example.

Example 1: For the following data make a Frequency Polygon.

Class Limits	Class Boundaries	Frequency
10–19	9.5–19.5	10
20–29	19.5–29.5	5
30–39	29.5–39.5	9
40–49	39.5–49.5	6
50–59	49.5–59.5	7
60–69	59.5–69.5	1

Solution: Step 1. Take two additional groups with the same class interval size. One before the very first group and the second after the very last group. Also calculate midpoints for these two groups. These groups will have frequency "Zero".

Class Limits	Class Boundaries	Frequency
0–9	**–0.5–9.5**	**0**
10–19	9.5–19.5	10
20–29	19.5–29.5	5
30–39	29.5–39.5	9
40–49	39.5–49.5	6
50–59	49.5–59.5	7
60–69	59.5–69.5	1
70–79	**69.5–79.5**	**0**

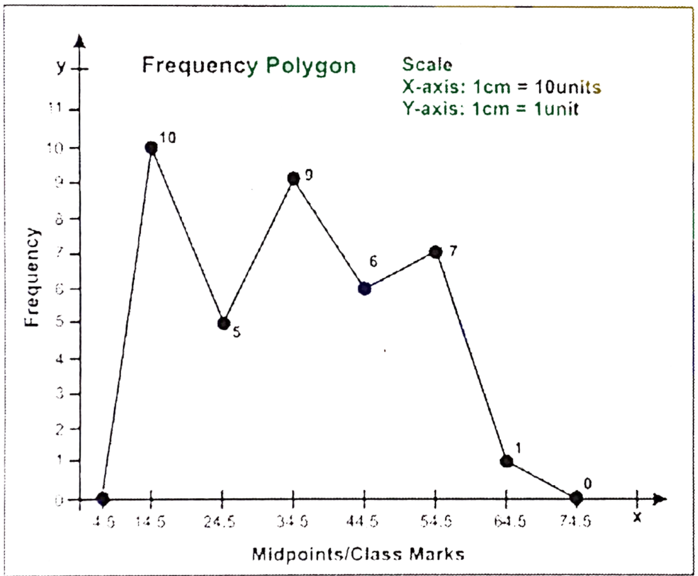
Step 2: Calculate class marks or midpoints for the given distribution.

Midpoints /Class Marks	Frequency
4.5	**0**
14.5	10
24.5	5
34.5	9
44.5	6
54.5	7
64.5	1
74.5	**0**

Step 3: Mark midpoints at x–axis and frequency along y–axis using appropriate scale.

Step 4: Plot a point against the frequency for each of the corresponding midpoint / class mark.

Step 5: Join all the points by means of line segment.



Cumulative Frequency Distribution:

(i) Construction of Cumulative Frequency Table

A table showing cumulative frequencies against upper class boundaries is called a cumulative frequency distribution. It is also called a less than cumulative frequency distribution.

Example 1: Construct a cumulative frequency distribution of for the following data.

Classes	20–24	25–29	30–34	35–39	40–44	45–49	50–54
F	1	2	26	22	20	15	14

Solution: The cumulative frequency distribution is constructed below.

Class Boundaries	f	CF	Class Boundaries	CF
14.5–19.5	0	0	Less than 19.5	0
19.5–24.5	1	0+1=1	Less than 24.5	1
24.5–29.5	2	1+2=3	Less than 29.5	3
29.5–34.5	26	3+26=29	Less than 34.5	29
34.5–39.5	22	29+22=51	Less than 39.5	51
39.5–44.5	20	51+20=71	Less than 44.5	71
44.5–49.5	15	71+15=86	Less than 49.5	86
49.5–54.5	14	86+14=100	Less than 54.5	100

(ii) Drawing of cumulative Frequency Polygon or Ogive

A cumulative frequency polygon or ogive is a graph of less than cumulative frequency distribution. It involves the following steps.

Step 1: Mark the class boundaries on x–axis and frequency (cumulative) on y–axis.

Step 2: Plot the points for the given frequencies corresponding to the upper class boundaries.

Step 3: Join the points by means of line segments.

Step 4: Drop perpendicular from the last point to x–axis to make a closed figure.

Example 2: Construct a cumulative frequency polygon for the given data.

Class Limits	Frequency
4–6	2
7–9	4
10–12	8
13–15	3

Solution: First we add one group before the first group. Then we make the class boundaries and also calculate the cumulative frequencies.

Class Limits	Class Boundaries	F	CF
1–3	**0.5–3.5**	**0**	**0**
4–6	3.5–6.5	2	0+2=2
7–9	6.5–9.5	4	2+4=6
10–12	9.5–12.5	8	6+8=14
13–15	12.5–15.5	3	14+3=17

Classes	Tally Marks	f	Mid points (x)	Class Boundaries
20–24	𝍸 I	6	22	19.5–24.5
25–29	𝍸 𝍸	10	27	24.5–29.5
30–34	𝍸 𝍸 II	12	32	29.5–34.5
35–39	𝍸 IIII	9	34	34.5–39.5
40–44	III	3	39	39.5–44.5
	$\sum f = 40$			

Less than Cumulative Frequency Distribution

C.B	f	C.F	Class Boundaries	F
14.5–19.5	0	0	Less than 19.5	0
19.5–24.5	6	0+6=6	Less than 24.5	6
24.5–29.5	10	6+10=16	Less than 29.5	16
29.5–34.5	12	16+12=28	Less than 34.5	28
34.5–39.5	9	28+9=37	Less than 39.5	37
39.5–44.5	3	37+3=40	Less than 44.5	40

Q. 3 From the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of Rs. **100, 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240** 06(003)
(**Hints:** Make classes 450—549, 550—649,..).

Solution: Let Salaries of teachers are X.

X: 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240

Minimum value = $X_{min} = 450$

Maximum value = $X_{max} = 1240$

Number of observations = n = 30

Size of class intervals = Rs.100

Classes	Tally Marks	Frequency (f)
450–549	II	2
550–649	II	2
650–749	IIII	4
750–849	𝍸	5
850–949	III	3
950–1049	IIII	4
1050–1149	𝍸	5
1150–1249	𝍸	5
		Σf = 30

Q.4 The following data shows the daily load shedding duration in hours in 30 localities of a certain city. Make a frequency distribution of the load shedding duration taking 2 hours as class intervals size and answer the following questions.
6, 12, 5, 7, 8, 3, 6, 7, 10, 2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14,12.

(a) Find the most frequent load shedding hours
(b) Find least load shedding intervals
(Hint: Make classes 2—3,4—5,6—7,.....)

Solution:
Let load shedding hours are X.

X: 6, 12, 5, 7, 8, 3, 6, 7, 10, 2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14, 12

Minimum value = $X_{min} = 2$

Maximum value = $X_{max} = 14$

Classes	Tally marks	Frequency (f)
2–3	II	2
4–5	I	1
6–7	𝍸 IIII	9
8–9	𝍸	5
10–11	𝍸 I	6
12–13	𝍸	5
14–15	III	3
Total		**Σf = 31**

(a) The most frequent load shedding hours are (6–7)

(b) The least load shedding interval is (4–5)

Classes	Tally Marks	f	Mid points (x)	Class Boundaries
20–24	𝍸 I	6	22	19.5–24.5
25–29	𝍸 𝍸	10	27	24.5–29.5
30–34	𝍸 𝍸 II	12	32	29.5–34.5
35–39	𝍸 IIII	9	34	34.5–39.5
40–44	III	3	39	39.5–44.5
	$\sum f = 40$			

Less than Cumulative Frequency Distribution

C.B	f	C.F	Class Boundaries	F
14.5–19.5	0	0	Less than 19.5	0
19.5–24.5	6	0+6=6	Less than 24.5	6
24.5–29.5	10	6+10=16	Less than 29.5	16
29.5–34.5	12	16+12=28	Less than 34.5	28
34.5–39.5	9	28+9=37	Less than 39.5	37
39.5–44.5	3	37+3=40	Less than 44 5	40

Q. 3 From the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of Rs. 100, 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240 06(003)

(Hints: Make classes 450—549, 550–649,..).

Solution: Let Salaries of teachers are X.

X: 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240

Minimum value = X_{min} = 450

Maximum value = X_{max}= 1240

Number of observations = n = 30

Size of class intervals = Rs.100

Classes	Tally Marks	Frequency (f)
450–549	II	2
550–649	II	2
650–749	IIII	4
750–849	𝍸	5
850–949	III	3
950–1049	IIII	4
1050–1149	𝍸	5
1150–1249	𝍸	5
		Σf = 30

Q.4 The following data shows the daily load shedding duration in hours in 30 localities of a certain city. Make a frequency distribution of the load shedding duration taking 2 hours as class intervals size and answer the following questions.

6, 12, 5, 7, 8, 3, 6, 7, 10, 2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14,12.

(a) Find the most frequent load shedding hours
(b) Find least load shedding intervals
(Hint: Make classes 2–3,4–5,6–7,.....)
Solution:
Let load shedding hours are X.

X: 6, 12, 5, 7, 8, 3, 6, 7, 10, 2, 14, 11, 12, 8, 6, 8, 9, 7, 11, 6, 9, 12, 13, 10, 14, 7, 6, 10, 11, 14, 12

Minimum value = X_{min} = 2

Maximum value = X_{max}= 14

Classes	Tally marks	Frequency (f)
2–3	II	2
4–5	I	1
6–7	𝍸 IIII	9
8–9	𝍸	5
10–11	𝍸 I	6
12–13	𝍸	5
14–15	III	3
Total		**Σf = 31**

(a) The most frequent load shedding hours are (6–7)

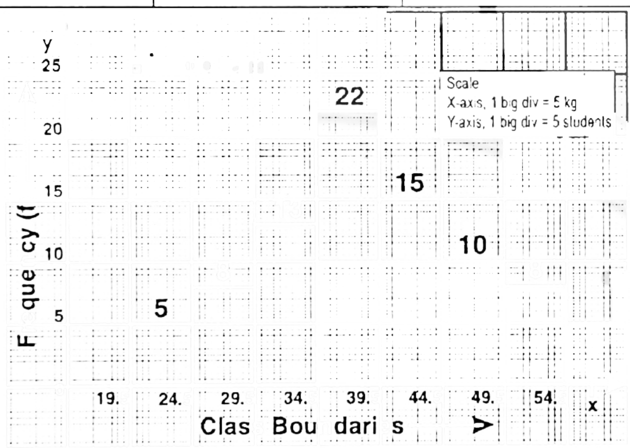
(b) The least load shedding interval is (4–5)

Q.5 Construct a Histogram and frequency Polygon for the following data showing weights of students in Kg.

Weights	Frequency / No. of students
20–24	5
25–29	8
30–34	13
35–39	22
40–44	15
45–49	10
50–54	8

(i) Solution: Histogram

Weights	Frequency (f)	Class Boundaries
20–24	5	19.5–24.5
25–29	8	24.5–29.5
30–34	13	29.5–34.5
35–39	22	34.5–39.5
40–44	15	39.5–44.5
45–49	10	44.5–49.5
50–54	8	49.5–54.5

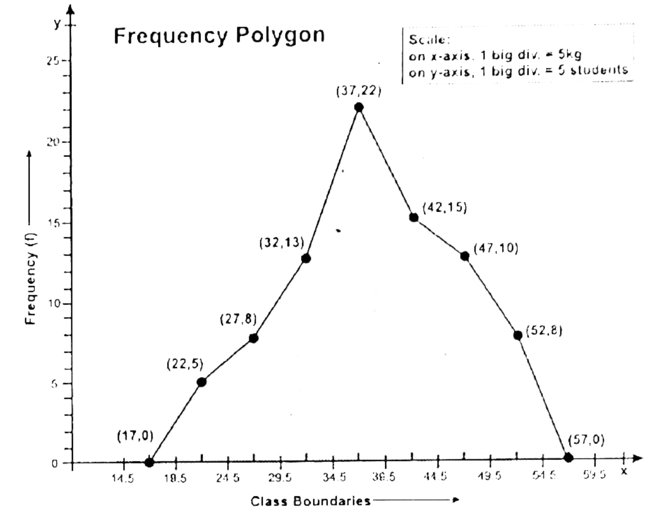


(ii) Frequency Polygon:

Weights (kg)	(f)	Class Boundaries	Midpoint
15–19	0	14.5–19.5	17
20–24	5	19.5–24.5	22
25–29	8	24.5–29.5	27
30–34	13	29.5–34.5	32
35–39	22	34.5–39.5	37
40–44	15	39.5–44.5	42
45–49	10	44.5–49.5	47
50–54	8	49.5–54.5	52
55–59	0	54.5–59.5	57

Note: Two additional groups with same size of class interval are taken. One before the very first group and the second after the very last group. These two groups will have frequency "0".

Frequency Polygon:



Average or Central Value

The representative value of the variables around which the majority of the observations tends to concentrate shows the tendency or behavior of the distribution. This representative value is called average or the central value.

Measures of Central Tendency:

The measures or techniques that are used to determine the central value are called measure of central tendency. Some measures of central tendency are discussed here.

(1) Arithmetic Mean:

Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. We denote Arithmetic mean by $\overline{X}$. In symbols we define arithmetic mean of n observations as

$$\overline{X} = \frac{\sum X}{n} = \frac{\text{Sum of all values of observations}}{\text{No. of observations}}$$

Computation of Arithmetic Mean:

There are two types of data, ungrouped and grouped. We, therefore, have different methods to determine Mean for the two types of data. These are explained with the help of examples.

Ungrouped Data:

For ungrouped data we use three approaches to find mean. These are as follows.

a. **Direct method**

$$\bar{X} = \frac{\sum X}{n} = \frac{\text{Sum of all values of observations}}{\text{No. of observations}}$$

b. **Indirect method**

i. **Short Cut method**

$$\bar{x} = A + \frac{\sum D}{n}$$

- $D = X - A$
- X = Observations
- A = Assumed mean from observation

ii. **Coding Method**

$$\bar{x} = A + \frac{\sum U}{n} \times h$$

- $U = \frac{x - A}{h}$
- X = Observations
- A = Assumed mean from observation
- h = common difference or common divisor

Example 1: The marks of seven students in Mathematics are as follows. Calculate the Arithmetic Mean and interpret the results.

Students	1	2	3	4	5	6	7
Marks	45	60	74	58	65	63	49

Solution: Let X = marks of a students.

$$\bar{x} = \frac{\sum x}{n} = \frac{x_1 + x_2 + x_3 + \dots + x_7}{7}$$

or $$\bar{x} = \frac{45 + 60 + 74 + 58 + 65 + 63 + 49}{7}$$

$$\bar{x} = \frac{414}{7} = 59.14 \text{ Marks}$$

Example 2: The salaries of five teaches are as follows. Find the mean salary using direct and indirect methods and compare the results.

11500, 12400, 15000, 14500, 14800

Solution:

a. **Mean by Direct Method**

$$\bar{x} = \frac{\sum_{i=1}^{5} x_i}{5} = \frac{11500 + 12400 + 15000 + 14500 + 14800}{5}$$

$$= \frac{74000}{5} = Rs.\ 13640$$

Mean by Indirect Method (Short Cut, Coding)

We assume A = 13000, $D_i = (x_i - 13000)$

h=100 and $u_i = \frac{(x_i - A)}{100}$, the computations are shown in the following table.

X	$D_i = (x_i - 13000)$	$u_i = \frac{(x_i - A)}{100}$
11500	–1500	–15
12400	–600	–6
15000	2000	20
14500	1500	15
14800	1800	18
$\sum x_i = 74000$	$\sum D_i = 3200$	$\sum u_i = 32$

(i) Using Short method

$$\bar{x} = A + \frac{\sum D}{n}$$

$$\bar{x} = 13000 + \frac{3200}{5} = 13000 + 640 = Rs.\ 13640$$

(ii) Using Coding Method

$$\bar{x} = A + \frac{\sum U}{n} \times h$$

$$\bar{x} = 13000 + \frac{32}{5} \times 100$$

$$\bar{x} = 13000 + \frac{3200}{5} = 13000 + 640 = Rs.13640$$

Group Data:

A data in the form of frequency distribution is called grouped data. To find the arithmetic mean from the group data we define formulae under direct and indirect methods as given below.

(a) Using Direct method

$$\bar{x} = \frac{\sum fx}{\sum f}$$

(b) Using Indirect method

(i) Short Cut method

$$\bar{x} = A + \frac{\sum fD}{\sum f}$$

- $D = X - A$
- X = Midpoint of a class
- A = Assumed mean

(ii) **Coding method**

$$\overline{X} = A + \frac{\sum fU}{\sum f} \times h$$

$$U = \frac{X - A}{h}$$

- X = Midpoint of a class
- A = Assumed mean
- h = Size of Class Interval

Example 3. Find the arithmetic mean using direct method for the following frequency distribution.

(Number of Heads) X	Frequency
1	3
2	8
3	5
4	3
5	1

Solution: we compute mean as follows.

(Number of Heads) X	Frequency (f)	fx
1	3	.
2	8	16
3	5	15
4	3	12
5	1	5
Total	$\sum f = 20$	$\sum fx = 51$

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{51}{20} = 2.55 \text{ or 3 heads}$$

(since the variable is discrete)

Example 4: For the following data showing weights of toffee boxes gms. Determine the mean weight of boxes.

Classes / Groups	Frequency
0 – 9	2
10 – 19	10
20 – 29	5
30 – 39	9
40 – 49	6
50 – 59	7
60 – 69	1
Total	40

Solution: First we calculate midpoints for each class and then find arithmetic mean.

Classes / Groups	Frequency (f)	Midpoints (x)	fx
0 – 9	2	4.5	9
10 – 19	10	14.5	145
20 – 29	5	24.5	122.5
30 – 39	9	34.5	310.5
40 – 49	6	44.5	267
50 – 59	7	54.5	381.5
60 – 69	1	64.5	64.5
Total	$\sum f = 40$		$\sum fx = 1300$

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{1300}{40} = 32.5 \ gm$$

Example 5: Find arithmetic mean using indirect formulae taking x = 34.5 as the provisional mean in example 4.

Solution: We use the following formulae.

(i) $\overline{X} = A + \frac{\sum fD}{\sum f}$ (Short Cut Formula)

(ii) $\overline{X} = A + \frac{\sum fU}{\sum f} \times h$ (Coding Formula)

Given A = 34.5. We may take h = 10 and make the following calculations.

Classes / Groups	f	Mid Points	D= X–34.5	U = (X–A)/ 10	fD	fU
0–9	2	4.5	–30	–3	–60	–6
10–19	10	14.5	–20	–2	–200	–20
20–29	5	24.5	–10	–1	–50	–5
30–39	9	34.5	0	0	0	0
40–49	6	44.5	10	1	60	6
50–59	7	54.5	20	2	140	14
60–69	1	64.5	30	3	30	3
Total	**$\Sigma f = 40$**		**1300**		**$\Sigma fD = -80$**	**$\Sigma fU = -8$**

Substituting the totals in the above formulae we get.

(i) $\bar{X}=A+\frac{\sum fD}{\sum f}$ $\Rightarrow$ $\bar{X}=34.5+\left(\frac{-80}{40}\right)=34.5-2=32.5$ gm

(ii) $\bar{X}=A+\frac{\sum fU}{\sum f}\times h$

$\bar{X}=34.5+\left(\frac{-8}{40}\right)\times 10$ $\Rightarrow$ $\bar{X}=34.5+\left(\frac{-80}{40}\right)=34.5-2=32.5$ gm

(2) Median:

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts. $\tilde{x}$ is used to represent median. We determine Median by using the following formulae.

Median From Ungrouped Data

Case1. When the number of observations (n) is odd.

$$\tilde{x}=\left(\frac{n+1}{2}\right)^{th} \text{ term in arranged data}$$

Case2. When the number of observations (n) is even.

$$\tilde{x}=\frac{1}{2}\left[\frac{n}{2}\text{th term}+\frac{n+2}{2}\text{th term}\right]$$

Example 1: On 5 term tests in mathematics, a student has made marks of 82,93,86,92 and 79. Find the median for the marks.

Solution:

Arranged Data: 79, 82, 86, 92, 93

Since number of observation is odd i.e., n =5

$$\tilde{x}=\left(\frac{n+1}{2}\right)^{th} \text{ term in arranged data}$$

$$\tilde{x}=\left[\left(\frac{5+1}{2}\right)^{th}\text{ term}\right] \text{ in arranged data}$$

$$\tilde{x}=\frac{6}{2}\,th\ term \text{ in arranged data}$$

$$\tilde{x}=3^{rd} \text{ term in arranged data}$$

$$\tilde{x}=86$$

Example 2: The sugar contents for a random sample of 6 packs of juices of a certain brand are found to be 2.3, 2.7,2.5,2.9,3.1 and 1.9 milligram. Find the median.

Solution:

Arranged Data: 1.9, 2.3, 2.5, 2.7, 2.9, 3.1

As number of observations are even i.e., n = 6

So, $\tilde{x}=\frac{1}{2}\left[\frac{n}{2}\text{th term}+\frac{n+2}{2}\text{th term}\right]$

$$\tilde{x}=\frac{1}{2}\left(\frac{6}{2}\text{th term}+\frac{6+2}{2}\text{th term}\right)$$

$$\tilde{x}=\frac{1}{2}\left(3^{rd}\text{ term}+4^{th}\text{ term}\right)$$

$$\tilde{x}=\frac{2.5+2.7}{2}=\frac{5.2}{2}=2.6 \text{ milligram}$$

Grouped Data (Discrete)

The following steps are involved in determining median for grouped data (discrete).

- Make cumulative frequency column.
- Determine the median observation using cumulative frequency i.e., the class containing $\left(\frac{n}{2}\right)^{th}$ observation.

Example 3: Find median for the following frequency distribution.

(Number of heads) x	Frequency
1	3
2	8
3	5
4	3
5	1

Solution: we first make cumulative frequency column as given below:

X	Frequency	Cumulative frequency
1	3	3
2	8	3 + 8 = 11
3	5	11 + 5 =16
4	3	16 + 3 =19
5	1	19 + 1 = 20

Median = the class containing $\left(\frac{n}{2}\right)^{th}$ observation

Median = the class containing $\left[\frac{20}{2}\right]^{th}$ observation

Median = the class containing $(10)^{th}$ observation.
Median = 2

Grouped Data (Continuous)

The following steps are involved in determining median for grouped data (continuous)

i. Determine class boundaries
ii. Make cumulative frequency column
iii. Determine the median class using cumulative frequency, i.e., the class containing $\left[\frac{n}{2}\right]^{th}$ observation.
iv. Use the formula $\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

Where
l = Lower class boundary of the median class.
h = class interval size of the median class.
f = frequency of the median class.
c = cumulative frequency of the class preceding the median class.

Example 4: The following data is the time taken by 40 students to solve a problem. Find the median time taken by the students.

138	164	150	132	144	125	149	157
146	158	140	147	136	148	152	144
168	126	138	176	163	119	154	165
146	173	142	147	135	153	140	135
161	145	135	142	150	156	145	128

Solution:

Class Intervals	Tally Marks	f	Class Boundaries	CF
118–126	𝍷𝍷𝍷	3	117.5–126.5	3
127–135	𝍸	5	126.5–135.5	8
136–144	𝍸 𝍷𝍷𝍷𝍷	9	135.5–144.5	17
145–153	𝍸 𝍸 𝍷𝍷	12	144.5–153.5	29
154–162	𝍸	5	153.5–162.5	34
163–171	𝍷𝍷𝍷𝍷	4	162.5–171.5	38
172–180	𝍷𝍷	2	171.5–180.5	40
Total		$\Sigma f = 40$	–	–

Now

Median class = class containing $\left(\frac{n}{2}\right)^{th}$ observation

Median class = class containing $\left[\frac{40}{2}\right]^{th}$ observation

Median class = class containing 20^{th} observation
Median class = 144.5–153.5
Where
l = Lower class boundary of median class = 144.5
h = class interval size of the median class = 9
f = frequency of the median class = 12
c = cumulative frequency of the class preceding the median class =17

So, $\tilde{x} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$\tilde{x} = 14.5 + \frac{9}{12}\left(\frac{40}{2} - 17\right)$

$\tilde{x} = 144.5 + \frac{9}{12}(20 - 17)$

$\tilde{x} = 144.5 + \frac{9 \times 3}{12} = 144.5 + \frac{27}{12}$

$\tilde{x} = 144.5 + 2.25 = 146.75 \Rightarrow \tilde{x} = 146.8$

(b) Mode:

Mode is defined as the most frequent occurring observation in the data. It is the observation that occurs maximum number of times in given data. The following formula is used to determine Mode.

(i) Ungrouped Data and Discrete Grouped Data:
Mode = The Most frequent observation

(ii) Grouped Data (Continuous)

The following steps are involved in determining mode for grouped data.

- Find modal group that has the maximum frequency
- Use the formula

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

Where
l = lower class boundary of the modal class.
h = class interval size of the modal class

f_m = frequency of the modal class

f_1 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class.

Example 1: Find the modal size of shoe for the following data.

4, 4.5 ,5, 6, 6, 6, 7, 7.5, 7.5, 8, 8, 8, 6, 5, 6.5, 7

Solution:

We note that most occurring observation in the given data is 6. So, Mode = 6

Example 2: Find Mode for the frequency distribution

(number of Heads) x	Frequency
1	3
2	8
3	5
4	3
5	1

Solution:

Since for x = 2 the frequency is maximum, means 2 heads appear the maximum number of times i.e. 8. So, Mode = 2

Example 3: For the following data showing weights of toffee boxes in gm. Determine the modal weight of boxes.

Classes / Groups	Frequency
0 – 9	2
10 – 19	10
20 – 29	5
30 – 39	9
40 – 49	6
50 – 59	7
60 – 69	1

Solution: Since the data is continuous so we proceed as follows.

(a) Determine the class boundaries first

(b) Find the class with maximum frequency

Classes / Groups	Class Boundaries	Frequency
0 – 9	–0.5 – 9.5	2
10 – 19	9.5 – 19.5	10
20 – 29	19.5 – 29.5	5
30 – 39	29.5 – 39.5	9
40 – 49	39.5 – 49.5	6
50 – 59	49.5 – 59.5	7
60 – 69	59.5 – 69.5	1
Total		**40**

From the above table we see that the group (9.5 – 19.5) has maximum frequency (i.e.10) so, Modal Group = (9.5 – 19.5)

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$f_m = 10,\ l = 9.5,\ h = 10,\ f_1 = 2 \text{ and } f_2 = 5$$

$$\text{Mode} = 9.5 + \frac{10-2}{2(10)-2-5} \times 10$$

$$\text{Mode} = 9.5 + \frac{8}{20-7} \times 10$$

$$\text{Mode} = 9.5 + \frac{80}{13} = 9.5 + 6.134 = 15.654$$

(4) Geometric Mean:

Geometric mean of a variable x is the n^{th} positive root of the product of the $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$ observation. In symbols we write, (Basic Formula)

$$G.M = (X_1 . X_2 . X_3 . \ldots\ldots . X_n)^{1/n}$$

The Geometric mean can also be calculated by using logarithm.

For Ungrouped data

$$G.M = Anti\log\left(\frac{\sum \log X}{n}\right)$$

For Grouped data

$$G.M = Anti\log\left(\frac{\sum f \log X}{\sum f}\right)$$

Example 1: Find the geometric mean of the observation 2, 4, 8 using (i) basic formula and (ii) using logarithmic formula.

Solution: (i) Using basic formula

$$G.M = (x_1 . x_2 . x_3)^{\frac{1}{3}}$$

$$G.M = (2 \times 4 \times 8)^{1/3}$$

$$G.M = (64)^{1/3} = 4$$

(ii) Using logarithmic formula.

X	Log X
2	0.3010
4	0.6021
8	0.9031
Total	$\sum \log X = 1.8062$

$$G.M = Anti\log\left(\frac{\sum \log X}{n}\right)$$

$$G.M = Anti\log\left(\frac{1.8062}{3}\right)$$

G.M = Antilog (0.6021)

G.M = 4.00003 = 4

Example 2: Find the geometric mean for the following data.

Marks in Percentage	Frequency / (No of students)
33–40	28
41–50	31
51–60	12
61–70	9
71–75	5

Solution: We proceed as follows:

Marks in %	f	X	log X	f log X
33–40	28	36.5	1.562293	43.7442
41–50	31	45.5	1.658011	51.39835
51–60	12	55.5	1.744293	20.93152
61–70	9	65.5	1.816241	16.34617
71–75	5	73	1.863323	9.316614
$\sum f = 85$			$\sum f \log X = 141.7369$	

$$G.M = Anti\log\left(\frac{\sum f \log X}{\sum f}\right)$$

$$G.M = Anti\log\left(\frac{141.7369}{85}\right)$$

G.M = Antilog (1.66749) = 46.50

(5) Harmonic Mean:

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \ldots\ldots, x_n$ observations. In symbols,

For Ungrouped data.

$$H.M = \frac{n}{\sum \frac{1}{X}}$$

For Grouped data.

$$H.M = \frac{n}{\sum \frac{f}{X}}$$

Example 1: For the following data find the Harmonic Mean.

X	12	5	8	4

Solution:

X	1/X
12	0.0833
5	0.2
8	0.125
4	0.25
Total	$\sum \frac{1}{X} = 0.6583$

$$H.M = \frac{n}{\sum \frac{1}{X}} = \frac{4}{0.6583} = 6.076$$

Example 2: Find the Harmonic mean for the following data.

Classes	o. of Students
33–40	28
41–50	31
51–60	12
61–70	9
71–75	5

Solution:

Classes	f	X	f/X
33–40	28	36.5	0.767123
41–50	31	45.5	0.681319
51–60	12	55.5	0.216216
61–70	9	65.5	0.137405
71–75	5	73	1.870556
Total	$\sum f = 85$	$\sum \frac{f}{X} = 1.870556$	

$$H.M = \frac{n}{\sum \frac{f}{X}}$$

$$H.M = \frac{85}{1.8706} = 45.441$$

Properties of Arithmetic Mean:

(a) Mean of a variable with similar observations say constant k is the constant k itself.

(b) Mean is affected by change in origin.
(c) Mean is affected by change in scale.
(d) Sum of the deviations of the variable x from its mean is always zero.

Example 1: Find the mean of observations:

34, 34, 34, 34, 34, 34

Solution: Since the variable say X here is taking same observations so by property (i) $\overline{X} = 34$

Verification:

$$\overline{X} = \frac{\sum x}{n} = \frac{34+34+34+34+34+34}{6}$$

$$\overline{X} = \frac{204}{6} \Rightarrow \overline{X} = 34$$

Example 2: A variable X takes the following values 4, 5, 8, 6, 2. Find the mean of X. Also find the mean when (a) 5 is added to each observation (b) 10 is multiplied with each observation (c) Prove that sum of the deviation from mean is zero.

Solution: Given the values of X.

X: 4, 5, 8, 6, 2

We may introduce two new variable Y and Z under (a) and (b) respectively. So we are given that (a) Y = X+5 (b) Z = 10X. The following table shows the desired results.

X	Y= X+5	Z = 10X	$X-\overline{X}$
4	9	40	4–5= –1
5	10	50	5–5=0
8	13	80	8–5=3
6	11	60	6–5=1
2	7	20	2–5= –3
Σx =25	Σy =50	Σz = 250	$\sum(X-\overline{X})=0$

From the above table we get.

$$\overline{X} = \frac{\sum x}{n}, \quad \overline{X} = \frac{25}{5} = 5$$

$$\overline{Y} = \frac{\sum y}{n}, \quad \overline{Y} = \frac{50}{5} = 10$$

$$\overline{Z} = \frac{\sum z}{n}, \quad \overline{Z} = \frac{250}{5} = 50$$

We note that

a. $\overline{Y} = 10 = 5 + 5 = \overline{X} + 5$

b. $\overline{Z} = 50 = 10(5) = 10\overline{X}$

Which shows that mean is affected by change in origin and scale.

c. From the last column of the table we note that $\sum(X-\overline{X}) = 0$, the sum of the deviations from mean is zero.

The Weighted Arithmetic Mean

The relative importance of a number is called its weight. When numbers $x_1, x_2, \ldots x_n$ are not equally important, we associate them with certain weights, $w_1, w_2, w_3, \ldots w_n$ depending on the importance or significance.

$$\overline{x}_w = \frac{w_1x_1 + w_2x_2 + \ldots + w_nx_n}{w_1 + w_2 + \ldots + w_n} = \frac{\sum wx}{\sum w}$$

Is called the weighed arithmetic mean.

Example 1: The following table gives the monthly earnings and the number of workers in a factory, compute the weighted average.

No. of employees	Monthly earnings. Rs.
4	800
22	45
20	100
30	30
80	35
300	15

Solution:

Number of employee are treated as a weight (*w*) and monthly earnings as variable (x).

No. of employees (w)	Monthly earnings in Rs. (x)	(xw)
4	800	3200
22	45	990
20	100	2000
30	30	900
80	35	2800
300	15	4500
$\sum w = 456$	——	$\sum xw = 14390$

$$\overline{x}_w = \frac{\sum xw}{\sum w} = \frac{14390}{456} = 31.5$$

Moving Average: Moving averages are defined as the successive averages (arithmetic means) which are computed for a sequence of days/months/years at a time. If we want to find 3–days moving average, we find the average of first 3–days, then dropping the first day and add the succeeding day to this group. Place the average of each 3–days against the mid of 3–days. This process continues until all the days, beginning from first to the last, are exhausted.

Example 2: Calculate three days moving average for the following record of attendance.

Week	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	24	55	28	45	51	54	60

Solution

Days	Attendance	3–days moving	
		Total	Average
Sun.	24	–	–
Mon.	55	107	107/3 = 35.67
Tue.	28	128	128/3 = 42.67
Wed.	45	124	124/3 = 41.33
Thu.	51	150	150/3 = 50
Fri.	54	165	165/3 = 55
Sat.	60	–	–

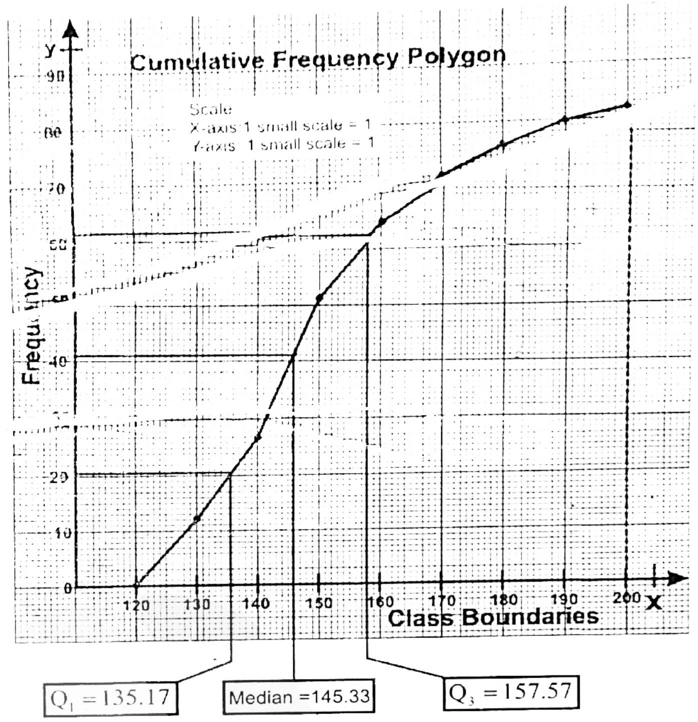
By adding the first three values we get 107 which is placed at the center of these values i.e., Monday and then dropping the first observation i.e., 24 and adding the next 3 values, we get 128 and placed at the middle of these three values and so on. For average values, divide 3 days moving total by "3" which shows in last column of the table.

Graphical Location of Median, Quartiles and Mode:

Example 1: For the following distribution locate Median and Quartiles on graph.

Class Boundaries	Cumulative Frequency
Less than 120	0
Less than 130	12
Less than 140	27
Less than 150	51
Less than 160	64
Less than 170	71
Less than 180	76
Less than 190	80
Less than 200	82

Cumulative Frequency Polygon:



Finding

Q_1:

a. Find $(\frac{n}{4})^{th}$ observation which is $\frac{82}{4} = 20.5$

b. On the graph locate 20.5 on y–axis and draw a horizontal line segment parallel to x–axis touching the polygon.

c. Draw a vertical line segment from this point touching x–axis.

d. Read the value of first quartile at the point where the line segment meets x–axis which is 135.17.

Finding

Q_2 or Median

a. Find $2(\frac{n}{4})^{th}$ observation which is $2(\frac{82}{4}) = 41$

b. On the graph locate 41 on y–axis and draw a horizontal line segment parallel to x–axis touching the polygon.

c. Draw a vertical line segment from this point touching x–axis.

d. Read the value of Median at the point where line segment meets x–axis which is 145.33.

Finding Q_3

a. Find the $3(\frac{n}{4})^{th}$ observation which is $3(\frac{82}{4}) = 61.5$.

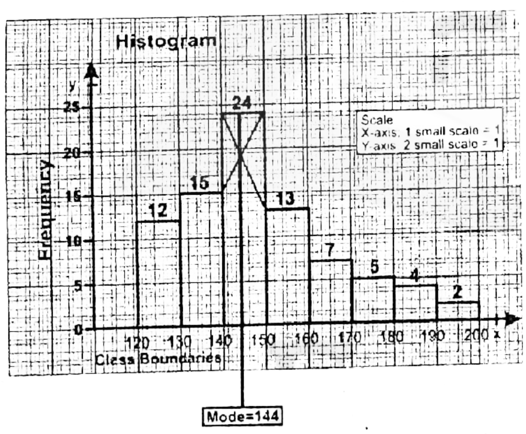
b. On the graph locate 61.5 on y–axis and draw a horizontal line segment parallel to x–axis touching the polygon.

c. Draw a vertical line segment from the point touching x–axis.

d. Read the value of Median at the point where the line segment meets x–axis which is 157.57.

Example 2: For the following distribution locate Mode on graph.

Salaries in Rupees	No. of Teachers
120–130	12
130–140	15
140–150	24
150–160	13
160–170	7
170–180	5
180–190	4
190–200	2



Steps:

- Determine the rectangle having the highest peak indicating the modal class.
- Draw a line segment from the top left corner of the rectangle to the top left corner of the succeeding rectangle.
- Draw another line segment from the top right corner of the rectangle to the top right corner of the preceding rectangle.
- Drop perpendicular form the top of the rectangle to the x–axis passing through the point of intersection of the two line segments.
- Read the value at the point where the perpendicular meets the x–axis. This is the Mode of the data which is 144.