EXERCISE 6.3

Q.1 What do you understand by Dispersion?

Dispersion: Statistically, dispersion means the spread or scatterness of observations in a data set.

Q.2 How do you define measures of dispersion?

Measures of Dispersion:

The measures or techniques that are used to determine the degree or extent of variation in a data set are called Measure of Dispersion.

For example Range, Variance and Standard Deviation etc.

Q.3 Define range, standard deviation and variance?

Definitions:

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula.

Range = $X_{max} - X_{min} = X_m - X_o$ Where,

$X_{max} = X_m$: the maximum, observation.

$X_{min} = X_o$: the minimum observation.

Variance:

Variance is defined as the mean of the squared deviation of X_i,(i=1,2,3,......., n) observations from their arithmetic mean.

Variance from Ungrouped Data

- **Definitional Formula**

$$\text{Variance of x} = \text{var}(X) = S^2 = \frac{\sum (x - \bar{x})^2}{n}$$

- **Computational formula:**

$$\text{Var}(x) = S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

Variance from Grouped Data

- **Definitional Formula**

$$\text{Var}(x) = S^2 = \frac{\sum f(x - \bar{x})^2}{\sum f}$$

- **Computational Formula**

$$\text{Var}(x) = S^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$$

Standard Deviation:

Standard deviation is defined as the positive square root of mean of the squared deviations of X_i, (i= 1,2,3….., n) observations from their arithmetic mean.

Standard Deviation from Un-Grouped Data

- **Definitional Formula**

$$S.D(X) = S = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

- **Computational Formula**

$$S.D(x) = S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

Standard Deviation from Grouped Data

- **Definitional Formula**

$$S.D(x) = S = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

- **Computational Formula**

$$S.D(x) = S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Q.4 The salaries of five teachers in Rupees are as follows. 11500, 12400, 15000, 14500, 14800

Find range and standard deviation

Solution

Maximum value = X_{max} = Rs. 15000

Minimum value = X_{min} = Rs. 11500

Range (R)= ?

Range $= X_{max} - X_{min}$

R = 15000–11500

Range = Rs. 3500

Standard Deviation:

Let X = 11500, 12400, 15000, 14500, 14800

We know that $\bar{X} = \frac{\sum X}{n} = \frac{68200}{5} = 13640$

X	$x-\bar{x}$	$(x-\bar{x})^2$
11500	11500 – 13640 = –2140	4579600
12400	12400 – 13640 = –1240	1537600
15000	15000 – 13640 = 1360	1849600
14500	14500 – 13640 = 860	739600
14800	14800 – 13640 = 1160	1345600
$\sum x = 68200$	$\sum (x-\bar{x})^2 = 10052000$	

Standard Deviation:

$$S = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

$$S = \sqrt{\frac{10052000}{5}}$$

$$S = \sqrt{2010400}$$

$$S = 1417.88575$$

Q. 5

a. Find the standard deviation "S" of each set of numbers.

(i) 12, 6, 7, 3, 15, 10, 18, 5

(ii) 9, 3, 8, 8, 9, 8, 9, 18

b. Calculate variance for the data:

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

(i) Solution:

X = 12, 6, 7, 3, 15, 10, 18, 5

Number of observations = n = 8

X	X^2
3	9
5	25
6	36
7	49
10	100
12	144
15	225
18	324
$\sum x = 76$	$\sum x^2 = 912$

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$S = \sqrt{\frac{912}{8} - \left(\frac{76}{8}\right)^2}$$

$$S = \sqrt{114 - 90.25}$$

$$S = \sqrt{23.75}$$

$$S = 4.87$$

(ii) Solution:

X = 9, 3, 8, 8, 9, 8, 9, 18

Number of observations = n = 8

X	X^2
3	9
8	64
8	64
8	64
9	81
9	81
9	81
18	324
$\sum x = 72$	$\sum x^2 = 768$

We know that,

$$S = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$S = \sqrt{\frac{768}{8} - \left(\frac{72}{8}\right)^2}$$

$$S = \sqrt{96 - 81}$$

$$S = \sqrt{15} \quad \Rightarrow \quad S = 3.87$$

(b). Calculate variance for the data

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Solution:

Number of Observations = n = 10

X	X^2
10	100
8	64
9	81
7	49
5	25
12	144
8	64
6	36
8	64
2	4
$\sum x = 75$	$\sum x^2 = 631$

We know that,

$$S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$S^2 = \frac{631}{10} - \left(\frac{75}{10}\right)^2$$

$$S^2 = 63.1 - (7.5)^2$$

$$S^2 = 63.1 - 56.25 \Rightarrow S^2 = 6.85$$

Q. 6. The lengths of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	20–22	23–25	26–28	29–31	32–34
f	3	6	12	9	2

Solution:

Length	(f)	Class Marks (x)	fx	x^2	fx^2
20–22	3	21	63	441	1323
23–25	6	24	144	576	3456
26–28	12	27	324	729	8748
29–31	9	30	270	900	8100
32–34	2	33	66	1089	2178
	$\sum f = 32$		$\sum fx = 867$		$\sum fx^2 = 23805$

(i) Arithmetic Mean

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{867}{32} = 27.093$$

(ii) Standard Deviation

$$S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$S = \sqrt{\frac{23805}{32} - \left(\frac{867}{32}\right)^2}$$

$$S = \sqrt{743.90 - (27.093)^2}$$

$$S = \sqrt{743.90 - 734.03065}$$

$$S = \sqrt{9.86935}$$

$$S = 3.1415$$