EXERCISE 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution: $2 - x^2 = 7x$

$1x^2 + 7x - 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1,\ b = 7,\ c = -2$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$\therefore \quad x = \frac{-7 \pm \sqrt{57}}{2}$$

Solution set is $\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$

Solution: $5x^2 + 8x + 1 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 5\ ,\ b = 8\ ,\ c = 1$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{\not{2}\left(-4 \pm \sqrt{11}\right)}{\not{10}\ 5}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution: $\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}\ x^2 + 1x - 4\sqrt{3} = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = \sqrt{3}\ ,\ b = 1\ ,\ c = -4\sqrt{3}$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$\Rightarrow$ $x = \frac{-1 - 7}{2\sqrt{3}}$ or $x = \frac{-1 + 7}{2\sqrt{3}}$

$x = \frac{-\not{8}\,^4}{\not{2}\sqrt{3}}$ or $x = \frac{\not{6}\,^3}{\not{2}\sqrt{3}}$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \frac{3}{\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \sqrt{3} \qquad \left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$$

Solution set is $\left\{\sqrt{3}, -\frac{4}{\sqrt{3}}\right\}$

(iv) $\mathbf{4x^2 - 14 = 3}$

Solution: $4x^2 - 14 = 3x$

$$4x^2 - 3x - 14 = 0$$

As $\quad ax^2 + bx + c = 0$

$\Rightarrow \quad a = 4, b = -3, c = -14$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

Solution set is $\left\{\frac{3 \pm \sqrt{233}}{8}\right\}$

(v) $\mathbf{6x^2 - 3 - 7x = 0}$

Solution: $\mathbf{6x^2 - 3 - 7x = 0}$

$$6x^2 - 7x - 3 = 0$$

As $\quad ax^2 + bx + c = 0$

$\Rightarrow \quad a = 6 \;,\; b = -7 \;,\; c = -3$

Using Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$$\Rightarrow \quad x = \frac{7 - 11}{12} \quad \text{or} \quad x = \frac{7 + 11}{12}$$

$$x = \frac{-4}{12} \quad \text{or} \quad x = \frac{18}{12}$$

$$x = \frac{-1}{3} \quad \text{or} \quad x = \frac{3}{2}$$

Solution set is $\left\{\frac{-1}{3}, \frac{3}{2}\right\}$

(vi) $\mathbf{3x^2 + 8x + 2 = 0}$

Solution: $3x^2 + 8x + 2 = 0$

As $\quad ax^2 + bx + c = 0$

$\Rightarrow \quad a = 3 \;,\; b = 8 \;,\; c = 2$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$$

$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$

$$x = \frac{\cancel{2}(-4 \pm \sqrt{10})}{\cancel{6}\,3}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii) $\dfrac{3}{x-6}-\dfrac{4}{x-5}=1$

Solution: $\dfrac{3}{x-6}-\dfrac{4}{x-5}=1$

$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$

$$3(x-5)-4(x-6)=(x-6)(x-5)$$

$$3x-15-4x+24=x^2-5x-6x+30$$

$$-1x+9=x^2-11x+30$$

$$x^2-11x+1x+30-9=0$$

$$1x^2-10x+21=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=1$, $b=-10$, $c=21$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm 4}{2}$$

$$x=\cancel{2}\frac{(5\pm 2)}{\cancel{2}}$$

$\Rightarrow$ $x=5+2$ or $x=5-2$

$x=7$ or $x=3$

Solution set is $\{3, 7\}$

(viii) $\dfrac{x+2}{x-1}-\dfrac{4-x}{2x}=2\dfrac{1}{3}$

Solution: $\dfrac{x+2}{x-1}-\dfrac{4-x}{2x}=2\dfrac{1}{3}$

$$\frac{(x+2)2x-(4-x)(x-1)}{(x-1)(2x)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x}=\frac{7}{3}$$

$$\frac{3x^2-x+4}{2x^2-2x}=\frac{7}{3}$$

$$3(3x^2-x+4)=7(2x^2-2x)$$

$$9x^2-3x+12=14x^2-14x$$

$\Rightarrow$ $14x^2-9x^2-14x+3x-12=0$

$$5x^2-11x-12=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=5$, $b=-11$, $c=-12$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121+240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm 19}{10}$$

$\Rightarrow$ $x=\dfrac{11-19}{10}$ or $x=\dfrac{11+19}{10}$

$x=\dfrac{-8}{10}$ or $x=\dfrac{30}{10}$

$x=-\dfrac{4}{5}$ or $x=3$

Solution set is $\left\{3, \dfrac{-4}{5}\right\}$

(ix) $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

Solution: $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

$$\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$$

$$\frac{ax-a^2+bx-b^2}{x^2-ax-bx+ab}=2$$

$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$

$$ax-a^2+bx-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-2ax-2bx+2ab-ax+a^2-bx+b^2=0$$

$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-(3a+3b)x+(a^2+b^2+2ab)=0$$

$$2x^2-(3a+3b)x+(a+b)^2=0$$

As $Ax^2+Bx+C=0$

$\Rightarrow$ $A=2$, $B=-(3a+3b)$, $C=(a+b)^2$

Using Formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-[-(3a+3b)]\pm\sqrt{[-(3a+3b)]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{+3(a+b)\pm\sqrt{[-3(a+b)]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{\left[(-3)^2(a+b)\right]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$x=\dfrac{3(a+b)-(a+b)}{4}$ or $x=\dfrac{3(a+b)+(a+b)}{4}$

$x=\dfrac{2(a+b)}{4}$ or $x=\dfrac{4(a+b)}{4}$

$x=\dfrac{a+b}{2}$ or $x=a+b$

$x=\dfrac{1}{2}(a+b)$ or $x=a+b$

Solution set is $\left\{(a+b),\ \dfrac{1}{2}(a+b)\right\}$

(x) $-(\ell+m)-\ell x^2+(2\ell+m)\,x=0\ ,\ \ell\neq 0$

Solution: $-(\ell+m)-\ell x^2+(2\ell+m)\,x=0\ ,\ \ell\neq 0$

$\Rightarrow$ $(\ell+m)+\ell x^2-(2\ell+m)x=0$ **01(032)**

$\Rightarrow$ $\ell x^2-(2\ell+m)x+(\ell+m)=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=\ell$, $b=-(2\ell+m)$, $c=\ell+m$

Using Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-[-(2\ell+m)]\pm\sqrt{[-(2\ell+m)]^2-4(\ell)(\ell+m)}}{2(\ell)}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(-1)^2(2\ell+m)^2-4\ell(\ell+m)}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(2\ell)^2+(m)^2+2(2\ell)(m)-4\ell^2-4\ell m}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{\cancel{4\ell^2}+m^2+\cancel{4\ell m}-\cancel{4\ell^2}-\cancel{4\ell m}}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{m^2}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm m}{2\ell}$$

$x=\dfrac{2\ell+\cancel{m}-\cancel{m}}{2\ell}$ or $x=\dfrac{2\ell+m+m}{2\ell}$

$x=\dfrac{\cancel{2\ell}}{\cancel{2\ell}}$ or $x=\dfrac{2\ell+2m}{2\ell}$

$x=1$ or $x=\dfrac{\cancel{2}(\ell+m)}{\cancel{2}\ell}$

$$x=\frac{\ell+m}{\ell}$$

Solution set is $\left\{1,\ \dfrac{\ell+m}{\ell}\right\}$

Equations reducible to quadratic form

Type (i):

The equations of the type $\mathbf{ax^4+bx^2+c=0}$

Example 1:

Solve the equation $x^4-13x^2+36=0$

Solution: $x^4-13x^2+36=0$ .... (i)

Let $x^2=y$ .... (ii)

Then $x^4=y^2$

Equation (i) becomes

$$y^2-13y+36=0$$

$$y^2-9y-4y+36=0$$

$$y(y-9)-4(y-9)=0$$

$$(y-9)(y-4)=0$$

Either $y-9=0$ or $y-4=0$,

$y=9$ or $y=4$

From equation (ii) Put $y=x^2$

$x^2=9$ or $x^2=4$

$\sqrt{x^2}=\pm\sqrt{9}$ or $\sqrt{x^2}=\pm\sqrt{4}$

$\Rightarrow$ $x=\pm3$ or $x=\pm2$

$\therefore$ The solution set is $\{\pm2,\pm3\}$

Type (ii):

The equations of the type $\mathbf{ap(x)+\frac{b}{p(x)}=c}$

Example 2:

Solve the equation $2(2x-1)+\frac{3}{2x-1}=5$

Solution:

$2(2x-1)+\frac{3}{2x-1}=5$ ....(i)

Let $2x-1=y$ ....(ii)

Then the equation (i) becomes

$$2y+\frac{3}{y}=5$$

or $2y^2+3=5y$

$$2y^2-5y+3=0$$

Using quadratic formula

$$ay^2+by+c=0$$

$\Rightarrow$ $a=2$, $b=-5$, $c=3$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$y=\frac{-(-5)\pm\sqrt{(-5)^2-4\times2\times3}}{2\times2}$$

$$y=\frac{5\pm\sqrt{25-24}}{4}=\frac{5\pm\sqrt{1}}{5}=\frac{5\pm1}{4}$$

We have

$y=\frac{5+1}{4}$ or $y=\frac{5-1}{4}$

$y=\frac{6}{4}=\frac{3}{2}$ or $y=\frac{4}{4}=1$

From equation (ii) putting $(y=2x-1)$

$2x-1=\frac{3}{2}$ or $2x-1=1$

$2x=\frac{3}{2}+1$ or $2x=1+1=2$

$2x=\frac{5}{2}$ or $x=\frac{2}{2}$

$x=\frac{5}{4}$ or $x=1$

Thus, the solution set is $\left\{1,\frac{5}{4}\right\}$

Type (iii): Reciprocal equations of the type:

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$$

or $ax^4+bx^3+cx^2+bx+a=0$

Reciprocal Equation

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Replacing x by $\frac{1}{x}$ in

$ax^4-bx^3+cx^2-bx+a=0$, we have

$$a\left(\frac{1}{x}\right)^4 - b\left(\frac{1}{x}\right)^3 + c\left(\frac{1}{x}\right)^2 - b\left(\frac{1}{x}\right) + a = 0$$

$$\left(\frac{a}{x^4}\right) - \left(\frac{b}{x^3}\right) + \left(\frac{c}{x^2}\right) - \left(\frac{b}{x}\right) + a = 0$$

Multiplying both side by x^4, we get

$a - bx + cx^2 - bx^3 + ax^4 = 0$.

By arranging these terms we get the same equation .

Thus $ax^4 - bx^3 + cx^2 - bx + a = 0$ is a reciprocal equation.

Example 3:

Solve the equation

$$\mathbf{2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0}$$

Solution:

$$2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$$

Dividing each term by x^2.

$$\frac{2x^4}{x^2} - \frac{5x^3}{x^2} - \frac{14x^2}{x^2} - \frac{5x}{x^2} + \frac{2}{x^2} = 0$$

$$2x^2 - 5x - 14 - \frac{5}{x} + \frac{2}{x^2} = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 14 = 0 \quad \ldots\ldots \text{(i)}$$

Let $x + \frac{1}{x} = y$, $\quad \ldots\ldots$ (ii)

Taking square both side

$$x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So equation (i) becomes

$$2(y^2 - 2) - 5y - 14 = 0$$

$$2y^2 - 4 - 5y - 14 = 0$$

$$2y^2 - 5y - 18 = 0$$

$$2y^2 - 9y + 4y - 18 = 0$$

$$y(2y - 9) + 2(2y - 9) = 0$$

$$\Rightarrow \quad (2y - 9)(y + 2) = 0$$

Either $2y - 9 = 0$ or $y + z = 0$

From equation (ii) Putting $y = x + \frac{1}{x}$, we have

$$2\left(x + \frac{1}{x}\right) - 9 = 0 \quad \text{or} \quad x + \frac{1}{x} + 2 = 0$$

$$2x + \frac{2}{x} - 9 = 0 \quad \text{or} \quad x^2 + 1 + 2x = 0$$

$$\boxed{2x^2 - 9x + 2 = 0} \quad \text{or} \quad \boxed{x^2 + 2x + 1 = 0}$$

First we solve $2x^2 - 9x + 2 = 0$ by quadratic formula, we get

$a = 2$, $b = -9$, $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times 2}}{2 \times 2}$$

$$x = \frac{9 \pm \sqrt{81 - 16}}{4}$$

$$x = \frac{9 \pm \sqrt{65}}{4}$$

Now we solve $x^2 + 2x + 1 = 0$ by quadratic formula

$a = 1$, $b = 2$, $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$x = \frac{-2 \pm \sqrt{4 - 4}}{2} \quad \text{or} \quad x = \frac{-2 \pm 0}{2}$$

$$x = -1, -1$$

Thus, the solution set is $\left\{-1, \frac{9 \pm \sqrt{65}}{4}\right\}$

Type (iv): Exponential equations:

In exponential equations, variable occurs in exponent. For example $2^x = 8$

Example 4:

Solve the equation $5^{1+x} + 5^{1-x} = 26$

Solution: $5^{1+x} + 5^{1-x} = 26$

$5^1.5^x + 5^1.5^{-x} = 26$

or $5.5^x + \frac{5}{5^x} - 26 = 0$ ....(i)

Let $5^x = y$. ....(ii)

Then equation (i) becomes

$5y + \frac{5}{y} - 26 = 0$

$5y^2 + 5 - 26y = 0$

$5y^2 - 26y + 5 = 0$

$5y^2 - 25y - y + 5 = 0$

$5y(y-5) - 1(y-5) = 0$

$(y-5)(5y-1) = 0$

Either $y - 5 = 0$ or $5y - 1 = 0,$

$y = 5$ or $5y = 1$

or $y = \frac{1}{5} = 5^{-1}$

From equation (ii) Put $y = 5^x$

$5^x = 5^1$ or $5^x = 5^{-1}$

$\Rightarrow x = 1$ or $\Rightarrow x = -1$

$\therefore$ The solution set is $\{\pm 1\}$.

Type (v): The equations of the type:

$(x+a)(x+b)(x+c)(x+d) = k$,

where $a + b = c + d$

Example 5:

Solve the equation

$(x-1)(x+2)(x+8)(x+5) = 19$

Solution:

$(x-1)(x+2)(x+8)(x+5) = 19$

Or $[(x-1)(x+8)][(x+2)(x+5)] - 19 = 0$

$(\because -1+8 = 2+5)$

$(x^2+7x-8)(x^2+7x+10) - 19 = 0$ .... (i)

Let $x^2 + 7x = y$ .... (ii)

Then equation (i) becomes

$(y-8)(y+10) - 19 = 0$

$y^2 + 2y - 80 - 19 = 0$

$y^2 + 2y - 99 = 0$

$y^2 + 11y - 9y - 99 = 0$

$y(y+11) - 9(y+11) = 0$

$(y+11)(y-9) = 0$

Either $y + 11 = 0$ or $y - 9 = 0$

From equation (ii) Put $y = x^2 + 7x$

$\boxed{x^2+7x+11=0}$ or $\boxed{x^2+7x-9=0}$

First we solve $x^2 + 7x + 11 = 0$ by quadratic formula

$a = 1$, $b = 7$, $c = 11$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(11)}}{2(1)}$

Or $= \frac{-7 \pm \sqrt{49-44}}{2} = \frac{-7 \pm \sqrt{5}}{2}$

Now we solve $x^2 + 7x - 9 = 0$

$a = 1$, $b = 7$, $c = -9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-9)}}{2(1)}$

or $x = \frac{-7 \pm \sqrt{49+36}}{2} = \frac{-7 \pm \sqrt{85}}{2}$

$\therefore$ The solution set is $\left\{\frac{-7 \pm \sqrt{5}}{2}, \frac{-7 \pm \sqrt{85}}{2}\right\}$