EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i) $(x+7)(x-3)=-7$

Solution: $(x+7)(x-3)=-7$

$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The standard form of quadratic equation is:

$$x^2+4x-14=0$$

(ii) $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

Solution: $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

$$\frac{7(x^2+4)-3x}{21}=1$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The standard form of quadratic equation is:

$$7x^2-3x+7=0$$

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

Solution: $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+1+2x=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$0=6x^2+6x-2x^2-2x-1$$
$$0=4x^2+4x-1$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The standard form of quadratic equation is:

$$4x^2+4x-1=0$$

(iv) $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

Solution: $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

$$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$$
$$x^2+4x-[x^2+2^2-2(x)(2)]+4x^2-8x=0$$
$$\cancel{x^2}+4x-\cancel{x^2}-4+4x+4x^2-8x=0$$
$$4x^2+4x+4x-8x-4=0$$
$$4x^2+\cancel{8x}-\cancel{8x}-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$\therefore \quad x^2-1=0 \quad (\because 4\neq 0)$$

So, $x^2-1=0$ is Pure Quadratic Equation

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

Solution: $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$
$$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$$
$$x^2+3x-(x^2-1x-20)=x^2+4x$$
$$\cancel{x^2}+3x-\cancel{x^2}+x+20=x^2+4x$$
$$3x+x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$\Rightarrow \quad x^2+\cancel{4x}-\cancel{4x}-20=0$$
$$x^2-20=0$$
$$x^2+0x-20=0$$

As, $b=0$

So, $x^2-20=0$ is Pure Quadratic Equation

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution: $7(x+2a)^2+3a^2=5a(7x+23a)$

$$7(x^2+4a^2+4ax)+3a^2=35ax+115a^2$$
$$7x^2+28a^2+28ax+3a^2-35ax-115a^2=0$$
$$7x^2-35ax+28ax+28a^2+3a^2-115a^2=0$$
$$7x^2-7ax-84a^2=0$$

Dividing each term of the equation by 7 we get

$$\frac{7x^2}{7}-\frac{7ax}{7}=\frac{\overset{12}{84}a^2}{7}$$
$$x^2-ax=12a^2$$
$$(x)^2-2(x)\left(\frac{a}{2}\right)=12a^2$$

Adding $\left(\frac{a}{2}\right)^2$ an both sides

$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a\pm 7a}{2}$$

$\Rightarrow \quad x=\frac{a+7a}{2}$ or $x=\frac{a-7a}{2}$

$x=\frac{8a}{2}$ or $x=\frac{-6a}{2}$

$x=4a$ or $x=-3a$

Solution set is $\{-3a,\ 4a\}$

Quadratic Formula

Derivation of Quadratic Formula By Using Completing Square Method

The quadratic equation in standard form is

$$ax^2+bx+c=0,\quad a\neq 0$$

Dividing each term of the equation by 'a' we get.

$$\frac{ax^2}{a}+\frac{b}{a}x+\frac{c}{a}=0$$

Shifting constant term $\frac{c}{a}$ to the right, we have

$$x^2+\frac{b}{a}x=\frac{-c}{a}$$
$$x^2+\frac{2}{2}(x)\left(\frac{b}{a}\right)=-\frac{c}{a}$$
$$(x)^2+2(x)\left(\frac{b}{2a}\right)=\frac{-c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides we obtain

$$(x)^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

or $\quad \left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

Taking square root of the both sides, we get

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$\Rightarrow \quad x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$

$\Rightarrow \quad x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

or $\quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $a\neq 0$ is known as "quadratic Formula"

Example 1: Solve the Quadratic equation $2 + 9x = 5x^2$ by using quadratic formula.

Solution: $2 + 9x = 5x^2$

The given equation in standard form can be written as

$$5x^2 - 9x - 2 = 0$$

Comparing with the standard quadratic equation $ax^2 + bx + c = 0$, we observe that

$$a = 5, \quad b = -9, \quad c = -2$$

Putting the values of a , b and c in quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \text{we have}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(-2)}}{2(5)}$$

Or $$x = \frac{9 \pm \sqrt{81+40}}{10} = \frac{9 \pm \sqrt{121}}{10} = \frac{9 \pm 11}{10}$$

Either $x = \frac{9+11}{10}$ or $x = \frac{9-11}{10}$

$x = \frac{20}{10} = 2$ or $x = \frac{-2}{10} = -\frac{1}{5}$

$2, \frac{-1}{5}$ are the roots of the given equation

Thus, the solution set is $\left\{\frac{-1}{5}, 2\right\}$

Example 2: Using quadratic formula, solve the equation $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$

Solution: $$\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$$

Simplifying and writing in the standard form

$$\frac{(2x+1)(x+4) - (x-2)(x+2)}{(x+2)(x+4)} = 0$$

$$(2x + 1)(x + 4) - (x - 2)(x + 2) = 0$$

$$2x^2 + 8x + x + 4 - (x^2 - 4) = 0$$

$$2x^2 + 9x + 4 - x^2 + 4 = 0$$

Or $$x^2 + 9x + 8 = 0$$

$$ax^2 + bx + c = 0$$

Here $a = 1, \quad b = 9, \quad c = 8$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we have}$$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{-9 \pm \sqrt{81-32}}{2} = \frac{-9 \pm \sqrt{49}}{2} = \frac{-9 \pm 7}{2}$$

$x = \frac{-9+7}{2}$ | $x = \frac{-9-7}{2}$

$x = \frac{-2}{2} = -1$ | $x = \frac{-16}{2} = -8$

$\therefore$ $-1, -8$, are the roots of the given equation. Thus, the solution set is $\{-8, -1\}$