EXERCISE 1.3

Q.1 $2x^4 - 11x^2 + 5 = 0$

Solution: $2x^4 - 11x^2 + 5 = 0$ .... (i)

Let $x^2 = y$ .... (ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2x^4 - 11x^2 + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(y-5)(2y-1) = 0$

$y - 5 = 0$ or $2y - 1 = 0$

$y = 0 + 5$ or $2y = 1$

$y = 5$ or $y = \frac{1}{2}$

From equation (ii) Put $y = x^2$

$x^2 = 5$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{5}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Q.2 $2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ....(i)

Let $x^2 = y$ ....(ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ or $2y - 1 = 0$

$y = 4$ or $2y = 1$

$\Rightarrow$ $y = 4$ or $y = \frac{1}{2}$

From equation (ii) put $y = x^2$

$x^2 = 4$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm 2$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

Q.3 $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution: $5x^{1/2} = 7x^{1/4} - 2$ ....(i)

Let $x^{\frac{1}{4}} = y$ ....(ii)

Taking square

$\left(x^{\frac{1}{4}}\right)^2 = y^2$

$x^{\frac{1}{2}} = y^2$

Put $x^{\frac{1}{4}} = y$ and $x^{\frac{1}{2}} = y^2$ in equation (i)

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5y^2 = 7y - 2$

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(y-1)(5y-2) = 0$

$y - 1 = 0$ or $5y - 2 = 0$

$y = 1$ or $5y = 2$

$y = 1 \quad$ or $\quad y = \frac{2}{5}$

From equation (ii) put $y = x^{\frac{1}{4}}$

$x^{\frac{1}{4}} = 1 \quad$ or $\quad x^{\frac{1}{4}} = \frac{2}{5}$

$\left(x^{\frac{1}{4}}\right)^4 = (1)^4 \quad$ or $\quad \left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$

$x = 1 \quad$ or $\quad x = \frac{2^4}{5^4}$

$x = \frac{16}{625}$

Solution set is $\left\{\frac{16}{625}, 1\right\}$

Q.4 $\quad \mathbf{x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}}$

Solution: $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$ ....(i)

Let $\quad x^{\frac{1}{3}} = y$ ....(ii)

Taking square

$\left(x^{\frac{1}{3}}\right)^2 = y^2$

$x^{\frac{2}{3}} = y^2$

Put $\quad x^{\frac{1}{3}} = y$ and $x^{\frac{2}{3}} = y^2$ in equation (i)

$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

$y^2 + 54 = 15y$

$y^2 - 15y + 54 = 0$

$y^2 - 9y - 6y + 54 = 0$

$y(y-9) - 6(y-9) = 0$

$(y-9)(y-6) = 0$

$y - 9 = 0 \quad$ or $\quad y - 6 = 0$

$y = 9 \quad$ or $\quad y = 6$

From equation (ii) put $y = x^{\frac{1}{3}}$

$x^{\frac{1}{3}} = 9 \quad$ or $\quad x^{\frac{1}{3}} = 6$

$\left(x^{\frac{1}{3}}\right)^3 = (9)^3 \quad$ or $\quad \left(x^{\frac{1}{3}}\right)^3 = (6)^3$

$x = 9^3 \quad$ or $\quad x = 6^3$

$x = 729 \quad$ or $\quad x = 216$

Solution set is $\{216, 729\}$

Q.5 $\quad \mathbf{3x^{-2} + 5 = 8x^{-1}}$

Solution: $3x^{-2} + 5 = 8x^{-1}$ ....(i)

Let $\quad x^{-1} = y$ ....(ii)

$(x^{-1})^2 = y^2$

$x^{-2} = y^2$

Put $\quad x^{-1} = y$ and $x^{-2} = y^2$ in equation (i)

$3x^{-2} + 5 = 8x^{-1}$

$3y^2 + 5 = 8y$

$3y^2 - 8y + 5 = 0$

$3y^2 - 5y - 3y + 5 = 0$

$y(3y-5) - 1(3y-5) = 0$

$(3y-5)(y-1) = 0$

$3y - 5 = 0 \quad$ or $\quad y - 1 = 0$

$3y = 5 \quad$ or $\quad y = 1$

$y = \frac{5}{3}$

From equation (ii) put $y = x^{-1}$

$x^{-1} = \frac{5}{3} \quad$ or $\quad x^{-1} = 1$

$\frac{1}{x} = \frac{5}{3} \quad$ or $\quad \frac{1}{x} = 1$

$x = \frac{3}{5} \quad$ or $\quad x = 1$

Solution set is $\left\{\frac{3}{5}, 1\right\}$

Q.6 $(2x^2+1)+\frac{3}{2x^2+1}=4$

Solution: $(2x^2+1)+\frac{3}{2x^2+1}=4$ ....(i)

Let $2x^2+1=y$ ....(ii)

Replacing $2x^2+1$ by "y" in equation (i), we get

$$y+\frac{3}{y}=4$$

Multiplying both sides by "y"

$$y^2+3=4y$$
$$y^2-4y+3=0$$
$$y^2-3y-1y+3=0$$
$$y(y-3)-1(y-3)=0$$
$$(y-3)(y-1)=0$$
$$y-3=0 \quad \text{or} \quad y-1=0$$
$$y=3 \quad \text{or} \quad y=1$$

From equation (ii) Put $y=2x^2+1$

$$2x^2+1=3 \quad \text{or} \quad 2x^2+1=1$$
$$2x^2=3-1 \quad \text{or} \quad 2x^2=1-1$$
$$2x^2=2 \quad \text{or} \quad 2x^2=0$$
$$x^2=\frac{2}{2} \quad \text{or} \quad x^2=\frac{0}{2}$$
$$x^2=1 \quad \text{or} \quad x^2=0$$
$$\sqrt{x^2}=\pm\sqrt{1} \quad \text{or} \quad \sqrt{x^2}=\pm\sqrt{0}$$
$$x=\pm1 \quad \text{or} \quad x=0$$

Solution set is $\{-1, 0, 1\}$

Q.7 $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$

Solution: $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$ ....(i)

Let $\frac{x}{x-3}=y$ ....(ii)

$$\frac{x-3}{x}=\frac{1}{y}$$

Equation (i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by "y"

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$y^2-2y-2y+4=0$$
$$y(y-2)-2(y-2)=0$$
$$(y-2)(y-2)=0$$
$$(y-2)^2=0$$
$$y-2=0$$
$$y=2$$

From equation (ii) Put $y=\frac{x}{x-3}$

$$\frac{x}{x-3}=2$$
$$x=2(x-3)$$
$$x=2x-6$$
$$6=2x-x$$
$$6=x \Rightarrow x=6$$

Solution set is $\{6\}$

Q.8 $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$

Solution: $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$ .... (i)

Let $\frac{4x+1}{4x-1}=y$ .... (ii)

$$\frac{4x-1}{4x+1}=\frac{1}{y}$$

Equation (i) becomes

$$y+\frac{1}{y}=2\frac{1}{6}$$
$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by "6y"

$$6y^2+6=13y$$
$$6y^2-13y+6=0$$

$$6y^2-9y-4y+6=0$$
$$3y(2y-3)-2(2y-3)=0$$
$$(2y-3)(3y-2)=0$$
$$2y-3=0 \quad \text{or} \quad 3y-2=0$$
$$2y=3 \quad \text{or} \quad 3y=2$$
$$y=\frac{3}{2} \quad \text{or} \quad y=\frac{2}{3}$$

From equation (ii) Put $y=\frac{4x+1}{4x-1}$

$$\frac{4x+1}{4x-1}=\frac{3}{2} \quad \text{or} \quad \frac{4x+1}{4x-1}=\frac{2}{3}$$
$$2(4x+1)=3(4x-1) \quad \text{or} \quad 3(4x+1)=2(4x-1)$$
$$8x+2=12x-3 \quad \text{or} \quad 12x+3=8x-2$$
$$2+3=12x-8x \quad \text{or} \quad 12x-8x=-2-3$$
$$4x=5 \quad \text{or} \quad 4x=-5$$
$$x=\frac{5}{4} \quad \text{or} \quad x=\frac{-5}{4}$$

Solution set is $\left\{\pm\frac{5}{4}\right\}$

Q.9 $\quad \mathbf{\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}}$

Solution: $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ ....(i)

Let $\quad \frac{x-a}{x+a}=y$ ....(ii)

Or $\quad \frac{x+a}{x-a}=\frac{1}{y}$

The equation (i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$
$$\frac{y^2-1}{y}=\frac{7}{12}$$
$$12(y^2-1)=7y$$
$$12y^2-12=7y$$
$$12y^2-7y-12=0$$
$$12y^2-16y+9y-12=0$$
$$4y(3y-4)+3(3y-4)=0$$
$$(3y-4)(4y+3)=0$$
$$3y-4=0 \quad \text{or} \quad 4y+3=0$$
$$3y=4 \quad \text{or} \quad 4y=-3$$
$$y=\frac{4}{3} \quad \text{or} \quad y=\frac{-3}{4}$$

From equation (ii) Putting $y=\frac{x-a}{x+a}$, we get

$$\frac{x-a}{x+a}=\frac{4}{3} \quad \text{or} \quad \frac{x-a}{x+a}=\frac{-3}{4}$$
$$3(x-a)=4(x+a) \quad \text{or} \quad 4(x-a)=-3(x+a)$$
$$3x-3a=4x+4a \quad \text{or} \quad 4x-4a=-3x-3a$$
$$-3a-4a=4x-3x \quad \text{or} \quad 4x+3x=-3a+4a$$
$$-7a=x \quad \text{or} \quad 7x=a$$
$$\Rightarrow \quad x=-7a \quad \text{or} \quad x=\frac{a}{7}$$

The solution set is $\left\{-7a, \frac{a}{7}\right\}$

Q.10 $\quad \mathbf{x^4-2x^3-2x^2+2x+1=0}$

Solution: $x^4-2x^3-2x^2+2x+1=0$

Dividing both sides by "x^2"

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$$
$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2x+\frac{2}{x}-2=0$$
$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0 \quad \text{....(i)}$$

Let $\quad x-\frac{1}{x}=y$ ....(ii)

Taking square of both sides

$$\left(x-\frac{1}{x}\right)^2=y^2$$
$$x^2+\frac{1}{x^2}-2=y^2$$
$$x^2+\frac{1}{x^2}=y^2+2$$

Putting these values in equation (i)

$$y^2 + \cancel{2} - 2(y) - \cancel{2} = 0$$

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

$$y = 0 \quad \text{or} \quad y - 2 = 0$$

$$y = 0 \quad \text{or} \quad y = 2$$

From equation (ii) Put $y = x - \frac{1}{x}$

$$x - \frac{1}{x} = 0 \quad \text{or} \quad x - \frac{1}{x} = 2$$

$$\frac{x^2 - 1}{x} = 0 \quad \text{or} \quad \frac{x^2 - 1}{x} = 2$$

$$x^2 - 1 = 0 \quad \text{or} \quad x^2 - 1 = 2x$$

$$x^2 = 1 \quad \text{or} \quad x^2 - 2x - 1 = 0$$

$$\sqrt{x^2} = \pm\sqrt{1} \quad \text{or} \quad \boxed{1x^2 - 2x - 1 = 0}$$

$$\boxed{x = \pm 1}$$

Now Solving $1x^2 - 2x - 1 = 0$ by quadratic formula

$$a = 1 \;,\; b = -2 \;,\; c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{\cancel{2}\left(1 \pm \sqrt{2}\right)}{\cancel{2}}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

The solution set is $\left\{\pm 1,\ 1 \pm \sqrt{2}\right\}$

Q.11 $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Solution: $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Dividing each terms by x^2

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6\cancel{x^2}}{\cancel{x^2}} + \frac{x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \text{......(i)}$$

Let $\quad x + \frac{1}{x} = y \quad$ ......(ii)

Taking square of both sides

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Put $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in equation (i)

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0$$

$$2(y^2 - 2) + y - 6 = 0$$

$$2y^2 - 4 + y - 6 = 0$$

$$2y^2 + y - 10 = 0$$

$$2y^2 + 5y - 4y - 10 = 0$$

$$y(2y+5) - 2(2y+5) = 0$$

$$(2y+5)(y-2) = 0$$

$$2y + 5 = 0 \quad \text{or} \quad y - 2 = 0$$

$$2y = -5 \quad \text{or} \quad y = 2$$

$$y = \frac{-5}{2}$$

From equation (ii) Put $y = x + \frac{1}{x}$

$$x + \frac{1}{x} = \frac{-5}{2}$$
$$\frac{x^2+1}{x} = \frac{-5}{2}$$
$$2(x^2+1) = -5x$$
$$2x^2 + 2 = -5x$$
$$2x^2 + 5x + 2 = 0$$
$$2x^2 + 4x + 1x + 2 = 0$$
$$2x(x+2) + 1(x+2) = 0$$
$$(x+2)(2x+1) = 0$$
$$x + 2 = 0 \text{ or } 2x + 1 = 0$$
$$x = -2 \quad \text{or} \quad 2x = -1$$
$$\boxed{x = -2} \text{ or } \boxed{x = \frac{-1}{2}}$$

$$x + \frac{1}{x} = 2$$
$$\frac{x^2+1}{x} = 2$$
$$x^2 + 1 = 2x$$
$$x^2 - 2x + 1 = 0$$
$$x^2 - 1x - 1x + 1 = 0$$
$$x(x-1) - 1(x-1) = 0$$
$$(x-1)(x-1) = 0$$
$$(x-1)^2 = 0$$
$$\sqrt{(x-1)^2} = \sqrt{0}$$
$$x - 1 = 0$$
$$x = 1$$

Solution set is $\left\{1, -2, \frac{-1}{2}\right\}$

Q.12 $4.2^{2x+1} - 9.2^x + 1 = 0$

Solution: $4.2^{2x+1} - 9.2^x + 1 = 0$

$$4.2^{2x}2^1 - 9.2^x + 1 = 0$$
$$8(2^x)^2 - 9(2^x) + 1 = 0 \quad \ldots\ldots\ldots (i)$$

Let $2^x = y \quad \ldots\ldots\ldots (ii)$

The equation (i) becomes

$$8y^2 - 9y + 1 = 0$$
$$8y^2 - 8y - 1y + 1 = 0$$
$$8y(y-1) - 1(y-1) = 0$$
$$(y-1)(8y-1) = 0$$
$$y - 1 = 0 \quad \text{or} \quad 8y - 1 = 0$$
$$y = 1 \quad \text{or} \quad 8y = 1$$
$$\text{or} \quad y = \frac{1}{8}$$

From equation (ii) Put $y = 2^x$

$$2^x = 1 \quad \text{or} \quad 2^x = \frac{1}{8}$$
$$2^x = 2^0 \quad \text{or} \quad 2^x = \frac{1}{2^3}$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad 2^x = 2^{-3}$$
$$\Rightarrow \quad x = -3$$

Solution set is $\{-3, 0\}$

Q.13 $3^{2x+2} = 12.3^x - 3$

Solution: $3^{2x+2} = 12.3^x - 3$

$$3^{2x}\ 3^2 - 12.3^x + 3 = 0$$
$$9(3^x)^2 - 12.(3^x) + 3 = 0 \ldots\ldots\ldots (i)$$

Let $3^x = y \quad \ldots\ldots\ldots (ii)$

Put $3^x = y$ in equation (i)

$$9y^2 - 12y + 3 = 0$$
$$9y^2 - 9y - 3y + 3 = 0$$
$$9y(y-1) - 3(y-1) = 0$$
$$(y-1)(9y-3) = 0$$
$$y - 1 = 0 \quad \text{or} \quad 9y - 3 = 0$$
$$y = 1 \quad \text{or} \quad 9y = 3$$
$$\boxed{y = 1} \quad \text{or} \quad y = \frac{3}{9} \Rightarrow \boxed{y = \frac{1}{3}}$$

From equation (ii) Put $y = 3^x$

$$3^x = 1 \quad \text{or} \quad 3^x = \frac{1}{3}$$
$$3^x = 3^0 \quad \text{or} \quad 3^x = 3^{-1}$$
$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = -1$$

The solution set is $\{0, -1\}$

Q.14 $2^x + 64.2^{-x} - 20 = 0$

Solution: $2^x + 64.2^{-x} - 20 = 0$

$$2^x + \frac{64}{2^x} - 20 = 0 \quad \ldots\ldots\ldots\ldots\ldots (i)$$

Let $2^x = y \quad \ldots\ldots\ldots\ldots\ldots (ii)$

Put $2^x = y$ in equation (i)

$$y + \frac{64}{y} - 20 = 0$$

Multiply both sides by "y"

$$y^2 + 64 - 20y = 0$$
$$y^2 - 20y + 64 = 0$$
$$y^2 - 16y - 4y + 64 = 0$$
$$y(y-16) - 4(y-16) = 0$$
$$(y-16)\ (y-4) = 0$$
$$y - 16 = 0 \quad \text{or} \quad y - 4 = 0$$
$$y = 16 \quad \text{or} \quad y = 4$$

From equation (ii) put $y = 2^x$

$$2^x = 16 \quad \text{or} \quad 2^x = 4$$
$$2^x = 2^4 \quad \text{or} \quad 2^x = 2^2$$
$$\Rightarrow \quad x = 4 \quad \text{or} \quad x = 2$$

The solution set is $\{2, 4\}$

Q.15 $(x+1)(x+3)(x-5)(x-7)=192$

Solution: $(x+1)(x+3)(x-5)(x-7)=192$

$$\because \quad a+b=c+d$$
$$1-5=3-7$$
$$-4=-4$$

$$(x+1)(x-5)(x+3)(x-7)=192$$
$$(x^2-5x+1x-5)(x^2-7x+3x-21)=192$$
$$(x^2-4x-5)(x^2-4x-21)=192 \quad ....(i)$$

Let $\quad x^2-4x=y \quad ....(ii)$

The equation (i) becomes

$$(y-5)(y-21)=192$$
$$y^2-21y-5y+105=192$$
$$y^2-26y+105-192=0$$
$$y^2-26y-87=0$$
$$y^2-29y+3y-87=0$$
$$y(y-29)+3(y-29)=0$$
$$(y-29)(y+3)=0$$
$$y-29=0 \quad \text{or} \quad y+3=0$$
$$y=29 \quad \text{or} \quad y=-3$$

From equation (ii) Put $y=x^2-4x$

$$x^2-4x=29 \quad \text{or} \quad x^2-4x=-3$$
$$x^2-4x-29=0 \quad \text{or} \quad x^2-4x+3=0$$
$$\boxed{x^2-4x-29=0} \quad \text{or} \quad \boxed{x^2-4x+3=0}$$

First we solve $x^2-4x-29=0$ by quadratic formula

$$a=1 \quad , \quad b=-4 \quad , \quad c=-29$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+116}}{2}$$
$$x=\frac{4\pm\sqrt{132}}{2}$$
$$x=\frac{4\pm\sqrt{4\times33}}{2}$$
$$x=\frac{4\pm2\sqrt{33}}{2}$$
$$x=\frac{\cancel{2}(2\pm\sqrt{33})}{\cancel{2}}$$
$$\boxed{x=2\pm\sqrt{33}}$$

Now we solve $x^2-4x+3=0$ by factorization

$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-3)(x-1)=0$$
$$x-3=0 \quad \text{or} \quad x-1=0$$
$$\boxed{x=3} \quad \text{or} \quad \boxed{x=1}$$

Solution set is $\{1,\ 3,\ 2\pm\sqrt{33}\}$

Q.16 $(x-1)(x-2)(x-8)(x+5)+360=0$

Solution: $(x-1)(x-2)(x-8)(x+5)+360=0$

$$[(x-1)(x-2)][(x-8)(x+5)]+360=0$$
$$[x^2-2x-1x+2][x^2+5x-8x-40]+360=0$$
$$(x^2-3x+2)(x^2-3x-40)+360=0 \quad ....(i)$$

Let $\quad x^2-3x=y \quad ......(ii)$

Put it in equation (i)

$$(y+2)(y-40)+360=0$$
$$y^2-40y+2y-80+360=0$$
$$y^2-38y+280=0$$
$$y^2-28y-10y+280=0$$
$$y(y-28)-10(y-28)=0$$
$$(y-28)(y-10)=0$$
$$y-28=0 \quad \text{or} \quad y-10=0$$
$$y=28 \quad \text{or} \quad y=10$$

From equation (ii) Put $y=x^2-3x$

$x^2-3x=28$	$x^2-3x=10$
$x^2-3x-28=0$	$x^2-3x-10=0$
$x^2-7x+4x-28=0$	$x^2-5x+2x-10=0$
$x(x-7)+4(x-7)=0$	$x(x-5)+2(x-5)=0$
$(x-7)(x+4)=0$	$(x-5)(x+2)=0$
$x-7=0$ or $x+4=0$	$x-5=0$ or $x+2=0$
$x=7$ or $x=-4$	$x=5$ or $x=-2$

The solution set is

$$\{5,-2,7,-4\}$$

Radical Equation

An equation involving expression of the variable under radical sign is called radical equation. For example $\sqrt{x+1}=2$

(i) The equation of the type $\sqrt{ax+b}=cx+d$

Example 1:

Solve the equation $\sqrt{3x+7}=2x+3$

Solution: $\sqrt{3x+7}=2x+3$ .... (i)

Squaring both sides of the equation (i) , we get

$$\left(\sqrt{3x+7}\right)^2=(2x+3)^2$$

$$3x+7=(2x)^2+2(2x)(3)+(3)^2$$

$$3x+7=4x^2+12x+9$$

$$0=4x^2+12x+9-3x-7$$

$$\Rightarrow \quad 4x^2+9x+2=0$$

Applying quadratic formula,

$$a=4\ ,\ b=9\ ,\ c=2$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-9\pm\sqrt{(9)^2-4\times4\times2}}{2\times4}$$

$$x=\frac{-9\pm\sqrt{81-32}}{8}=\frac{-9\pm\sqrt{49}}{8}=\frac{-9\pm7}{8}$$

$$x=\frac{-9+7}{8} \quad \text{or} \quad x=\frac{-9-7}{8}$$

$$x=\frac{-2}{8} \quad \text{or} \quad x=\frac{-16}{8}$$

$$x=\frac{-1}{4} \quad \text{or} \quad x=-2$$

Checking: Putting $x=-\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right)+7}=2\left(-\frac{1}{4}\right)+3$$

$$\Rightarrow \sqrt{\frac{-3+28}{4}}=-\frac{1}{2}+3$$

$$\Rightarrow \sqrt{\frac{25}{4}}=\frac{-1+6}{2}$$

$$\frac{5}{2}=\frac{5}{2} \quad \text{which is true.}$$

Putting $x=-2$ in the equation (i) , we get

$$\sqrt{3(-2)+7}=2(-2)+3$$

$$\Rightarrow \quad \sqrt{1}=-1$$

$$1\neq-1$$

On checking, we find that $x=-2$ does not satisfy the equation (i) , so it is an extraneous root. Thus the solution set is $\left\{-\frac{1}{4}\right\}$

(ii) The equation of the type $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

Example 2: **Solve the equation**

$$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$$

Solution: $\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$ ....(i)

Squaring both sides of the equation (i), we have

$$\left(\sqrt{x+3}+\sqrt{x+6}\right)^2=\left(\sqrt{x+11}\right)^2$$

$$\left(\sqrt{x+3}\right)^2+\left(\sqrt{x+6}\right)^2+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$x+3+x+6+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$2x+9+2\sqrt{(x+3)(x+6)}=x+11$$

$$2\sqrt{x^2+6x+3x+18}=x+11-2x-9$$

$$\text{or} \quad 2\sqrt{x^2+9x+18}=-x+2$$

Again taking square of both sides we get

$$(2)^2\left(\sqrt{x^2+9x+18}\right)^2=(-x)^2+(2)^2+2(-x)(2)$$

$$4(x^2+9x+18)=x^2+4-4x$$

$$(4x^2+36x+72)=x^2-4x+4$$

$$4x^2+36x+72-x^2+4x-4=0$$

$$3x^2+40x+68=0$$

Applying quadratic formula

$$a=3\ ,\ b=40\ ,\ c=68$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-40\pm\sqrt{(40)^2-4(3)(68)}}{2(3)}$$

$$x=\frac{-40\pm\sqrt{1600-816}}{6}$$

$$x=\frac{-40\pm\sqrt{784}}{6}=\frac{-40\pm28}{6}$$

We have

$x = \frac{-40+28}{6}$

$x = \frac{-12}{6}$

$x = -2$

$x = \frac{-40-28}{6}$

$x = \frac{-68}{6}$

$x = \frac{-34}{3}$

Checking:

Putting $x = -2$ in (i)

$\sqrt{-2+3} + \sqrt{-2+6} = \sqrt{-2+11}$

$\sqrt{1} + \sqrt{4} = \sqrt{9}$

$1 + 2 = 3$

$3 = 3$ which is true

So one root is 3

Now Putting $x = \frac{-34}{3}$ in the equation (i), we have

$$\sqrt{\frac{-34}{3}+3} + \sqrt{\frac{-34}{3}+6} = \sqrt{\frac{-34}{3}+11}$$

or

$$\sqrt{\frac{-34+9}{3}} + \sqrt{\frac{-34+18}{3}} = \sqrt{\frac{-34+33}{3}}$$

$$\Rightarrow \sqrt{\frac{25}{3}\times(-1)} + \sqrt{\frac{16}{3}\times(-1)} = \sqrt{\frac{1}{3}\times(-1)}$$

$$\Rightarrow \frac{5}{\sqrt{3}}i + \frac{4}{\sqrt{3}}i = \frac{1}{\sqrt{3}}i \qquad \left(\because i = \sqrt{-1}\right)$$

$\frac{9}{\sqrt{3}}i = \frac{1}{\sqrt{3}}i$ which is not true.

As $\frac{-34}{3}$ is an extraneous root,

So, the solution set is $\{-2\}$.

(iii) The equation of the type $\sqrt{x^2+px+m} + \sqrt{x^2+px+n} = q$

Example 3: **Solve the equation**

$$\sqrt{x^2-3x+36} - \sqrt{x^2-3x+9} = 3$$

Solution: $\sqrt{x^2-3x+36} - \sqrt{x^2-3x+9} = 3$ ...(i)

Let $x^2 - 3x = y$ ....(ii)

Put it in equation (i)

Then $\sqrt{y+36} - \sqrt{y+9} = 3$

Squaring both sides

$$\left(\sqrt{y+36} - \sqrt{y+9}\right)^2 = (3)^2$$

$$\left(\sqrt{y+36}\right)^2 + \left(\sqrt{y+9}\right)^2 - 2\left(\sqrt{y+36}\right)\left(\sqrt{y+9}\right) = 9$$

$$y+36+y+9-2\sqrt{(y+36)(y+9)} = 9$$

$$2y+45-2\sqrt{(y^2+36y+9y+324)} = 9$$

$$-2\sqrt{(y^2+45y+324)} = 9-2y-45$$

$$-2\sqrt{y^2+45y+324} = -2y-36$$

$$\cancel{-2}\sqrt{y^2+45y+324} = \cancel{-2}(y+18)$$

Again squaring both sides

$$\left(\sqrt{y^2+45y+324}\right)^2 = (y+18)^2$$

$$y^2+45y+324 = (y)^2 + 2(y)(18) + (18)^2$$

$$\cancel{y^2} + 45y + \cancel{324} = \cancel{y^2} + 36y + \cancel{324}$$

$45y - 36y = 0$

$9y = 0$

$\Rightarrow$ $y = 0$

From equation (ii) put $y = x^2 - 3x$ so

$x^2 - 3x = 0$

$\Rightarrow$ $x(x-3) = 0$

Either $x = 0$ or $x - 3 = 0 \Rightarrow x = 3$

$\therefore$ $x = 0, 3$ are the roots of the equation.

Thus, the solution set is $\{0, 3\}$