EXERCISE 1.4

Solve the following equations:

Q.1 $2x+5=\sqrt{7x+16}$

Solution: $2x+5=\sqrt{7x+16}$ ....(i)

Squaring both sides

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$

$$(2x)^2+(5)^2+2(2x)(5)=7x+16$$

$$4x^2+25+20x=7x+16$$

$$4x^2+20x-7x+25-16=0$$

$$4x^2+13x+9=0$$

$$4x^2+4x+9x+9=0$$

$$4x(x+1)+9(x+1)=0$$

$$(x+1)(4x+9)=0$$

$x+1=0$ or $4x+9=0$

$x=-1$ or $4x=-9$

$x=-1$ or $x=\dfrac{-9}{4}$

Checking

Putting $x=-1$ in the equation (i) we have

$$2x+5=\sqrt{7x+16}$$

$$2(-1)+5=\sqrt{7(-1)+16}$$

$$-2+5=\sqrt{-7+16}$$

$$3=\sqrt{9}$$

$3=3$ which is true

Putting $x=\dfrac{-9}{4}$ in the Equation (i) we have

$$2x+5=\sqrt{7x+16}$$

$$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$$

$$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$$

$$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$$

$\dfrac{2}{4}=\sqrt{\dfrac{1}{4}}$ $\Rightarrow \dfrac{1}{2}=\dfrac{1}{2}$ which is true

So, the solution set is $\left\{-1,\dfrac{-9}{4}\right\}$

Q.2 $\sqrt{x+3}=3x-1$

Solution: $\sqrt{x+3}=3x-1$ ......(i)

Squaring both sides

$$\left(\sqrt{x+3}\right)^2=(3x-1)^2$$

$$x+3=(3x)^2+(1)^2-2(3x)(1)$$

$$x+3=9x^2+1-6x$$

$$9x^2-6x-x+1-3=0$$

$$9x^2-7x-2=0$$

$$9x^2-9x+2x-2=0$$

$$9x(x-1)+2(x-1)=0$$

$$(x-1)(9x+2)=0$$

$x-1=0$ or $9x+2=0$

$x=1$ or $9x=-2$

or $x=\dfrac{-2}{9}$

Checking:

Putting $x=1$ in the equation (i) we have

$$\sqrt{x+3}=3x-1$$

$$\sqrt{1+3}=3(1)-1$$

$$\sqrt{4}=3-1$$

$2=2$ which is true

Putting $x=\dfrac{-2}{9}$ in the equation (i) we have

$$\sqrt{x+3}=3x-1$$

$$\sqrt{\frac{-2}{9}+3}=3\left(\frac{-2}{9}\right)-1$$

$$\sqrt{\frac{-2+27}{9}}=\frac{-6}{9}-1$$

$$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$$

$$\frac{5}{3}=\frac{-\cancel{15}^{\,5}}{\cancel{9}_{\,3}}$$

$\dfrac{5}{3}=\dfrac{-5}{3}$ which is not true

As $\dfrac{-2}{9}$ is an extraneous root

So, the solution set is $\{1\}$

Q.3 $4x = \sqrt{13x+14} - 3$

Solution: $4x = \sqrt{13x+14} - 3$ ....(i)

$4x + 3 = \sqrt{13x+14}$

Squaring both sides

$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$

$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$

$16x^2 + 9 + 24x = 13x + 14$

$16x^2 + 9 + 24x - 13x - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(x+1)(16x-5) = 0$

$x + 1 = 0$ or $16x - 5 = 0$

$x = -1$ or $16x = 5$

$\boxed{x = -1}$ or $\boxed{x = \frac{5}{16}}$

Checking

Putting $x = -1$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4(-1) = \sqrt{13(-1)+14} - 3$

$-4 = \sqrt{-13+14} - 3$

$-4 = \sqrt{1} - 3$

$-4 = 1 - 3$

$-4 \neq -2$

So -1 is an extraneous root

Putting $x = \frac{5}{16}$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3$

$\frac{20}{16} = \sqrt{\frac{65}{16} + 14} - 3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$ which is true

So, the solution set is $\left\{\frac{5}{16}\right\}$

Q.4 $\sqrt{3x+100} - x = 4$

Solution: $\sqrt{3x+100} - x = 4$........(i)

$\sqrt{3x+100} = 4 + x$

Squaring both sides

$\left(\sqrt{3x+100}\right)^2 = (4+x)^2$

$3x + 100 = (4)^2 + (x)^2 + 2(4)(x)$

$3x + 100 = 16 + x^2 + 8x$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x+12) - 7(x+12) = 0$

$(x+12)(x-7) = 0$

$x + 12 = 0$ or $x - 7 = 0$

$x = -12$ or $x = 7$

Checking

Putting $x = -12$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$

$\sqrt{-36+100} + 12 = 4$

$\sqrt{64} + 12 = 4$

$8 + 12 = 4$

$20 \neq 4$

So -12 is an extraneous root

Putting $x = 7$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(7)+100} - 7 = 4$

$\sqrt{21+100} - 7 = 4$

$\sqrt{121} - 7 = 4$

$11 - 7 = 4$

$4 = 4$ which is true

So, the solution set is $\{7\}$

Q.5 $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution: $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ ...(i)

Squaring on both sides

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$x+5+x+21+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$2x+26+2\sqrt{x^2+21x+5x+105}=x+60$$

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$

$$2\sqrt{x^2+26x+105}=-x+34$$

$$2\sqrt{x^2+26x+105}=34-x$$

Again squaring both sides

$$\left(2\sqrt{x^2+26x+105}\right)^2=(34-x)^2$$

$$4(x^2+26x+105)=(34)^2+x^2-2(34)(x)$$

$$4x^2+104x+420=1156+x^2-68x$$

$$4x^2+104x+420-1156-x^2+68x=0$$

$$3x^2+172x-736=0$$

$$a=3\ ,\ b=172\ ,\ c=-736$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$x=\frac{-172\pm\sqrt{38416}}{6}$$

$$x=\frac{-172\pm196}{6}$$

$$x=\frac{-172-196}{6}\text{ or }x=\frac{-172+196}{6}$$

$$x=\frac{-368}{6}\quad\text{or}\quad x=\frac{24}{6}$$

$$x=\frac{-184}{3}\quad\text{or}\quad x=4$$

Checking:

Putting $x=4$ in equation (i)

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

$$3+5=8$$

$$8=8 \quad \text{which is true}$$

Putting $x=\dfrac{-184}{3}$

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$$\sqrt{\frac{-184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$$

$$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$$

$$\sqrt{\frac{-1\times169}{3}}+\sqrt{\frac{-1\times121}{3}}=\sqrt{\frac{-1\times4}{3}}$$

$$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{13i+11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$

$$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}} \quad \text{Which is not true}$$

As $x=\dfrac{-184}{3}$ is extraneous root

so solution set is $\{4\}$

Q.6 $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Solution: $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$ ......(i)

Squaring both sides

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$\left(\sqrt{x+1}\right)^2+\left(\sqrt{x-2}\right)^2+2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right)=x+6$$

$$x+1+x-2+2\sqrt{(x+1)(x-2)}=x+6$$

$$2x-1+2\sqrt{x^2-2x+1x-2}=x+6$$

$$2\sqrt{x^2-x-2}=x+6-2x+1$$

$$2\sqrt{x^2-x-2}=7-x$$

Again squaring both sides

$$\left(2\sqrt{x^2-x-2}\right)^2=(7-x)^2$$

$$4(x^2-x-2)=(7)^2+(x)^2-2(7)(x)$$

$$4x^2-4x-8=49+x^2-14x$$

$$4x^2-x^2-4x+14x-8-49=0$$

$$3x^2+10x-57=0$$

$$3x^2+19x-9x-57=0$$

$$x(3x+19)-3(3x+19)=0$$

$$(3x+19)(x-3)=0$$

$3x+19=0$ or $x-3=0$

$3x=-19$ or $x=3$

$$x=\frac{-19}{3}$$

Checking

Putting $x=3$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$

$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

$$2+1=3$$

$3=3$ which is true

Putting $x=\frac{-19}{3}$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

$$\sqrt{\frac{-19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{\frac{-19}{3}+6}$$

$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$

$$\sqrt{\frac{-1\times16}{3}}+\sqrt{\frac{-1\times25}{3}}=\sqrt{\frac{-1\times1}{3}}$$

$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{4i+5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$$

$\frac{9i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$ which is not true

As $x=\frac{-19}{3}$ is an extraneous root

So the solution set is $\{3\}$

Q.7 $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$

Solution: $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$ ......(i)

Squaring both sides

$$\left(\sqrt{11-x}+\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$

$$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2+2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$$

$$11-x+6-x+2\sqrt{(11-x)(6-x)}=27-x$$

$$17-2x+2\sqrt{66-11x-6x+x^2}=27-x$$

$$2\sqrt{66-17x+x^2}=27-x-17+2x$$

$$2\sqrt{66-17x+x^2}=10+x$$

Again squaring both sides

$$\left(2\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$

$$4(66-17x+x^2)=(10)^2+(x)^2+2(10)(x)$$

$$264-68x+4x^2=100+x^2+20x$$

$$4x^2-x^2-68x-20x+264-100=0$$

$$3x^2-88x+164=0$$

By applying quadratic formula

$a=3$, $b=-88$, $c=164$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$

$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$

$$x=\frac{88\pm\sqrt{5776}}{6}$$

$x=\frac{88-76}{6}$ or $x=\frac{88+76}{6}$

$x=\frac{12}{6}$ or $x=\frac{164}{6}$

$x=2$ or $x=\frac{82}{3}$

Checking

Putting $x = 2$ in the equation (i)

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$$

$$\sqrt{11-2}+\sqrt{6-2}=\sqrt{27-2}$$

$$\sqrt{9}+\sqrt{4}=\sqrt{25}$$

$$3+2=5$$

$$5=5 \qquad \text{which is true}$$

Putting $x=\frac{82}{3}$ in the equation (i)

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$$

$$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$

$$\sqrt{\frac{33-82}{3}}+\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$$

$$\sqrt{\frac{-1\times 49}{3}}+\sqrt{\frac{-1\times 64}{3}}=\sqrt{\frac{-1\times 1}{3}}$$

$$\frac{7i}{\sqrt{3}}+\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{7i+8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$$\frac{15i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \text{which is not true}$$

As $\frac{82}{3}$ is an extraneous root

So, the solution set is $\{2\}$

Q.8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution: $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ ....(i)

Squaring both sides

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$

$$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$$

$$4a+\cancel{x}+a-\cancel{x}-2\sqrt{(4a+x)(a-x)}=a$$

$$5a-2\sqrt{4a^2-4ax+ax-x^2}=a$$

$$-2\sqrt{4a^2-3ax-x^2}=a-5a$$

$$-2\sqrt{4a^2-3ax-x^2}=-4a$$

$$\sqrt{4a^2-3ax-x^2}=\frac{-\cancel{4}^{2}a}{-\cancel{2}}$$

$$\sqrt{4a^2-3ax-x^2}=2a$$

Again squaring both sides

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4\cancel{a^2}-4\cancel{a^2}=3ax+x^2$$

$$\Rightarrow \quad 3ax+x^2=0$$

$$x(3a+x)=0$$

$$x=0 \quad \text{or} \quad 3a+x=0$$

$$x=0 \quad \text{or} \quad x=-3a$$

Checking:

Putting $x=0$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$

$$\sqrt{4a}-\sqrt{a}=\sqrt{a}$$

$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$

$$\sqrt{a}=\sqrt{a} \qquad \text{which is true}$$

Putting $x=-3a$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$

$$\sqrt{a}-\sqrt{4a}=\sqrt{a}$$

$$\sqrt{a}-2\sqrt{a}=\sqrt{a}$$

$$-\sqrt{a}=\sqrt{a} \qquad \text{which is not true}$$

As $-3a$ is extraneous root

So, the solution set is $\{0\}$

Q.9 $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution: $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ ......(i)

Let $x^2+x=y$ ....(ii)

Put it in equation (i)

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=(1)^2$$

$$\left(\sqrt{y+1}\right)^2+\left(\sqrt{y-1}\right)^2-2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right)=1$$

$$y+\cancel{1}+y-\cancel{1}-2\sqrt{(y+1)(y-1)}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$$\Rightarrow \quad 2y-1=2\sqrt{y^2-1}$$

Again squaring both sides

$$(2y-1)^2=\left(2\sqrt{y^2-1}\right)^2$$

$$(2y)^2+(1)^2-2(2y)(1)=4\left(y^2-1\right)$$

$$4y^2+1-4y=4y^2-4$$

$$4\cancel{y^2}+1-4y=4\cancel{y^2}-4$$

$$1-4y=-4$$

$$1+4=4y$$

$$5=4y$$

$$\Rightarrow \quad 4y=5$$

From equation (ii) Put $y=x^2+x$

$$4y=5$$

$$4(x^2+x)=5$$

$$4x^2+4x=5$$

$$4x^2+4x-5=0$$

Solving by quadratic formula

$$a=4 \quad , \quad b=4 \quad , \quad c=-5$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times 6}}{8}$$

$$x=\frac{-4\pm 4\sqrt{6}}{8}$$

$$x=\frac{\cancel{4}\left(-1\pm\sqrt{6}\right)}{\cancel{8}\,2}$$

$$x=\frac{-1\pm\sqrt{6}}{2}$$

The solution set is $\left\{\dfrac{-1\pm\sqrt{6}}{2}\right\}$

Q.10 $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution: $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ ....(i)

Let $x^2+3x=y$ ....(ii)

Put it in equation (i)

$$\sqrt{y+8}+\sqrt{y+2}=3$$

Squaring on both sides

$$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$$

$$\left(\sqrt{y+8}\right)^2+\left(\sqrt{y+2}\right)^2+2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right)=9$$

$$y+8+y+2+2\sqrt{y^2+2y+8y+16}=9$$

$$2y+10+2\sqrt{y^2+10y+16}=9$$

$$2\sqrt{y^2+10y+16}=9-2y-10$$

$$2\sqrt{y^2+10y+16}=-2y-1$$

$$2\sqrt{y^2+10y+16}=-(2y+1)$$

Again squaring on both sides

$$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$$

$$4(y^2+10y+16)=(2y+1)^2$$

$$4y^2+40y+64=(2y)^2+(1)^2+2(2y)(1)$$

$$4y^2+40y+64=4y^2+1+4y$$

$$40y+64=1+4y$$

$$40y-4y=1-64$$

$$36y=-63$$

From equation (ii) Put $y=x^2+3x$

$$36(x^2+3x)=-63$$

$$36x^2+108x+63=0$$

Solving by quadratic formula

$$a=36 \quad , \quad b=108 \quad , \quad c=63$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$=\frac{-108\pm\sqrt{11664-9072}}{72}$$

$$=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{-108\pm\sqrt{4\times4\times9\times9\times2}}{72}$$

$$x=\frac{-108\pm\sqrt{4^2\times9^2\times2}}{72}$$

$$x=\frac{-108\pm4\times9\sqrt{2}}{72}$$

$$x=\frac{-108\pm36\sqrt{2}}{72}$$

$$x=\frac{\cancel{36}\left(-3\pm\sqrt{2}\right)}{\cancel{72}_2}$$

$$x=\frac{-3\pm\sqrt{2}}{2}$$

So, the solution set is $\left\{\dfrac{-3\pm\sqrt{2}}{2}\right\}$

Q.11 $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution: $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$ ....(i)

Let $x^2+3x=y$ ....(ii)

Put it in equation (i)

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring both sides

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=5^2$$

$$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right)=25$$

$$y+9+y+4+2\sqrt{(y+9)(y+4)}=25$$

$$2y+13+2\sqrt{y^2+4y+9y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=12-2y$$

$$\cancel{2}\sqrt{y^2+13y+36}=\cancel{2}(6-y)$$

$$\sqrt{y^2+13y+36}=6-y$$

Again taking squaring both sides

$$\left(\sqrt{y^2+13y+36}\right)^2=(6-y)^2$$

$$y^2+13y+36=(6)^2+(y)^2-2(6)(y)$$

$$\cancel{y^2}+13y+\cancel{36}=\cancel{36}+\cancel{y^2}-12y$$

$$13y=-12y$$

$$13y+12y=0$$

$$25y=0$$

$$y=\frac{0}{25}$$

$$y=0$$

From equation (ii) Put $x^2+3x=y$

$$x^2+3x=0$$

$$x(x+3)=0$$

$x=0$ or $x+3=0$

$x=-3$

So, the solution set is $\{-3, 0\}$