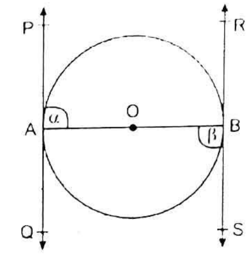
Exercise 10.1

Q.1 Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel.



Given: A circle with centre O, has $\overline{AB}$ as diameter. $\overleftrightarrow{PAQ}$ is tangent at point A.

$\overleftrightarrow{RBS}$ is tangent at point B

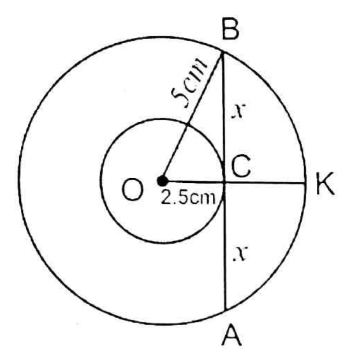
To Prove: $\overleftrightarrow{PAQ} \parallel \overleftrightarrow{RBS}$

Proof:

Statements	Reasons
$\overleftrightarrow{PAQ} \perp \overline{OA}$	Tangent is ⊥ at the outer end of radial segment.
$\Rightarrow$ $\overleftrightarrow{PAQ} \perp \overline{AB}$	
$\therefore$ $m\angle\alpha = 90^\circ$ ……… (i)	
$\overleftrightarrow{RBS} \perp \overline{OB}$	
$\Rightarrow$ $\overleftrightarrow{RBS} \perp \overline{AB}$	
$\therefore$ $m\angle\beta = 90^\circ$ ……… (ii)	
Thus, $m\angle\alpha = m\angle\beta$	
Therefore $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$	If alternate Angles are equal in measurement, then lines are parallel.

Q.2 The diameters of two concentric circles are 10cm and 5cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

Solution: Let $\overline{AB}$ be any chord of the outer circle that touches the inner circle.



Diameter of outer circle = 10 cm

Radius of outer circle = $m\overline{OB} = \dfrac{10cm}{2} = 5cm$

Diameter of inner circle = 5 cm

Radius of inner circle = $m\overline{OC} = \dfrac{5cm}{2} = 2.5cm$

ΔOCB is right angled triangle with right angle at C. $\left(\because \overline{OC} \perp \overline{AB}\right)$

By Pythagoras theorem

$$\left(m\overline{OB}\right)^2 = \left(m\overline{BC}\right)^2 + \left(m\overline{OC}\right)^2$$

$$(5\text{ cm})^2 = (x)^2 + (2.5cm)^2$$

$\Rightarrow$ $x^2 = (5cm)^2 - (2.5cm)^2$

$x^2 = 25cm^2 - 6.25\text{ cm}^2$

$x^2 = 18.75cm^2$

Taking Square root of both sides.

$$\sqrt{x^2} = \sqrt{18.75cm^2}$$

$$x = \sqrt{18.75}cm$$

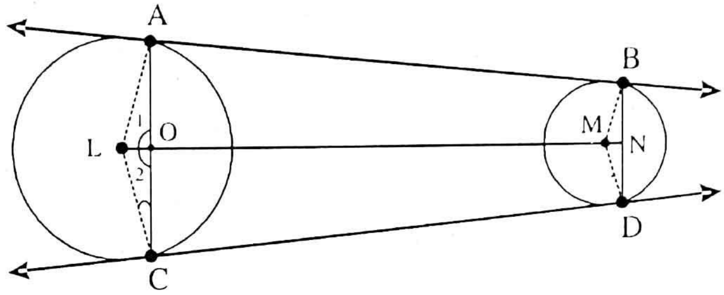
Length of Chord = $m\overline{AB} = 2x$

$m\overline{AB} = 2(\sqrt{18.75}cm)$

$m\overline{AB} = 8.66cm$ $\Rightarrow$ $\boxed{m\overline{AB} \simeq 8.7cm}$

Q.3 $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are the common tangents drawn to the pair of circles. If A and C are the points of tangency of 1^{st} circle where B and D are the points of tangency of 2^{nd} circle, then prove that $\overline{AC} \parallel \overline{BD}$.

Given: Two circles with centre L and M. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are their common tangents. A is joined with C and B is joined with D.



To prove:

$\overline{AC} \parallel \overline{BD}$

Construction:

Join L to A and C. Join M to B and D. Join L to M and produce it to meet the $\overline{BD}$ at N.

Proof:

Statements	Reasons
In $\Delta AOL \leftrightarrow \Delta COL$	
$\overline{AL} \cong \overline{CL}$	Radii of the same circle
$\angle A \cong \angle C$	angles opposite to congruent sides
$\overline{LO} \cong \overline{LO}$	common side.
$\therefore$ $\Delta AOL \cong \Delta COL$	S.A.S ≅ S.A.S
$m\angle 1 = m\angle 2$ ....(i)	Corresponding angles of congruent triangle.
$m\angle 1 + m\angle 2 = 180^{\circ}$ ....(ii)	O is the point on line segment $\overline{AC}$.
$\Rightarrow$ $m\angle 1 = m\angle 2 = 90^{\circ}$	
$\overline{LO} \perp \overline{AO}$	
or $\overline{LO} \perp \overline{AC}$	
or $\overline{AC} \perp \overline{LOMN}$ ....(iii)	
Similarly in the circle with centre M, it can be proved that	
$\overline{BD} \perp \overline{MN}$	
or $\overline{BD} \perp \overline{LOMN}$ ....(iv)	
Both $\overline{AC}$ and $\overline{BD}$ are $\perp$ to the same line segment	
$\therefore$ $\overline{AC} \parallel \overline{BD}$	Two line segments making same angle with a line are parallel to each other.