Exercise 10.2

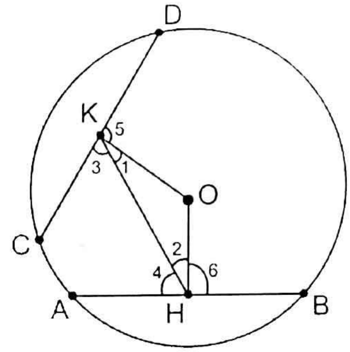
Q. 1 $\overline{AB}$ and $\overline{CD}$ are two equal chords in a circle with centre O. H and K are respectively the mid points of the chords. Prove that $\overline{HK}$ makes equal angles with $\overline{AB}$ and $\overline{CD}$.

Given:

A circle with centre 'O'. Two chords such that

$m\overline{AB} = m\overline{CD}$. H and K are mid points of chords AB and CD respectively.

H is joined with K



To Prove:

(i) $m\angle AHK = m\angle CKH$

(ii) $m\angle BHK = m\angle DKH$

Proof:

Statements	Reasons
In ΔHOK	
$m\overline{OH} = m\overline{OK}$	Two equal chords are equidistant from the center.
$\therefore$ $m\angle 1 = m\angle 2$ .... (i)	Angles opposite to the equal line segments
And $m\angle 5 = m\angle 6$ .... (ii)	Each 90°
$m\angle 1 + m\angle 5 = m\angle 2 + m\angle 6$	Adding (i) and (ii)
Thus, $m\angle DKH = m\angle BHK$	
or $m\angle BHK = m\angle DKH$ Proved	
$m\angle AHO = m\angle CKO$	Each 90°
$m\angle 2 + m\angle 4 = m\angle 1 + m\angle 3$	
But $m\angle 2 = m\angle 1$	Proved in (i)
$\cancel{m\angle 1} + m\angle 4 = \cancel{m\angle 1} + m\angle 3$	
$m\angle 4 = m\angle 3$	By cancellation property
$m\angle AHK = m\angle CKH$	

Q.2 The radius of a circle is 2.5 cm. $\overline{AB}$ and $\overline{CD}$ are two chords 3.9cm apart. If $m\overline{AB} = 1.4$ cm, then measure the other chord.

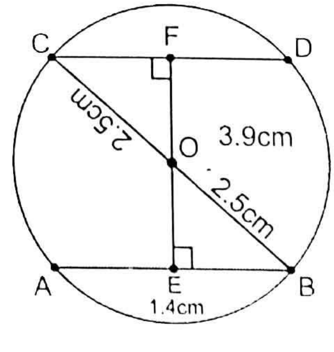
Given:

O is the centre of a circle.

(i) $m\overline{OB} = m\overline{OC} = 2.5$cm

(ii) $m\overline{AB} = 1.4$cm

(iii) $m\overline{EF} = 3.9$cm



To Find:

$m\overline{CD} = ?$

Construction:

Join O with B and C.

Calculations:

Steps	Reasons
In ΔOEB	Given
$m\overline{EB} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(1.4cm) = 0.7\ cm$ ....(i)	
$m\overline{OB} = 2.5cm$ ....(ii)	
$(m\overline{OB})^2 = (m\overline{OE})^2 + (m\overline{EB})^2$	By Pythagoras theorem in right angled ΔOEB
$(2.5cm)^2 = (m\overline{OE})^2 + (0.7cm)^2$	From (i) and (ii)
$\Rightarrow (m\overline{OE})^2 = (2.5cm)^2 - (0.7cm)^2$	
$(m\overline{OE})^2 = 6.25cm^2 - 0.49cm^2$	
$(m\overline{OE})^2 = 5.76cm^2$	
$\sqrt{(m\overline{OE})^2} = \sqrt{5.76cm^2}$	
$m\overline{OE} = 2.4cm$ .... (iii)	
Now	
$m\overline{OF} = m\overline{EF} - m\overline{OE}$	
$m\overline{OF} = 3.9cm - 2.4cm$	
$m\overline{OF} = 1.5cm$ .... (iv)	
In right angled triangle ΔOCF	
$(m\overline{OC})^2 = (m\overline{OF})^2 + (m\overline{CF})^2$	
$\Rightarrow (m\overline{CF})^2 = (2.5cm)^2 - (1.5cm)^2$	From (iv)
$(m\overline{CF})^2 = 6.25cm^2 - 2.25cm^2$	
$(m\overline{CF})^2 = 4cm^2$	
$\therefore \sqrt{(m\overline{CF})^2} = \sqrt{4cm^2}$	
$(m\overline{CF}) = 2\ cm$ .... (v)	
$\therefore (m\overline{CD}) = 2(m\overline{CF})$	$\because m\overline{CF} = \frac{1}{2} m\overline{CD}$
$m\overline{CD} = 2(2cm)$	From (v)
$m\overline{CD} = 4cm$	

Q.3. The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm then find the distance between the centres.

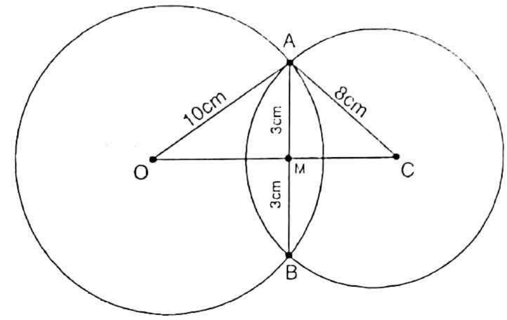
Solution:

Given: Two intersecting circles with centers O and C having radius $m\overline{OA} = 10cm$, radius $m\overline{AC} = 8cm$ respectively.

Length of common chord $m\overline{AB} = 6cm$

To Find:

Distance between the centers $m\overline{OC} = ?$



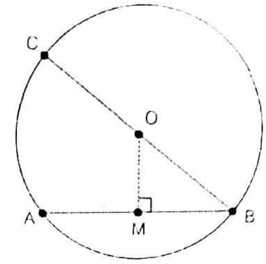
Construction:

Join the point A to the centers O and C. Joint O to C which meets the chord $\overline{AB}$ at its midpoint.

Calculations:

Steps	**Reasons**
$\overline{OM} \perp \overline{AB}$	
$m\overline{AM} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(6cm) = 3cm$ ....(i)	$\perp$ from the center to the chord bisect it.
By Pythagoras theorem	
$(m\overline{OM})^2 = (m\overline{OA})^2 - (m\overline{AM})^2$	
$= (10cm)^2 - (3cm)^2$	From given and from (i)
$= 100cm^2 - 9cm^2$	
$(m\overline{OM})^2 = 91\,cm^2$	
$\sqrt{(m\overline{OM})^2} = \sqrt{91cm^2}$	
$m\overline{OM} = 9.54cm$ ....(ii)	
In ΔAMC	
By Pythagoras theorem	
$(m\overline{MC})^2 = (m\overline{CA})^2 - (m\overline{AM})^2$	
$= (8cm)^2 - (3cm)^2$	$m\overline{CA} = 8cm$, $m\overline{AM} = 3cm$
$= 64cm^2 - 9cm^2$	
$(m\overline{MC})^2 = 55\ cm^2$	
$\sqrt{(m\overline{MC})^2} = \sqrt{55cm^2}$	
$m\overline{MC} = 7.42\,cm$ ....(iii)	
We know that the distance between the centers	
$(m\overline{OC}) = (m\overline{OM}) + (m\overline{MC})$	From (ii) and (iii)
$= 9.54mc + 7.42cm$	
$= 16.96cm$	
Distance between the centres is 16.96cm	

Q.4. Show that greatest chord in a circle is its diameter.



Given: O be the centre of the circle, $m\overline{BC}$ is central chord and $\overline{AB}$ be any chord of the circle

To prove: Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$

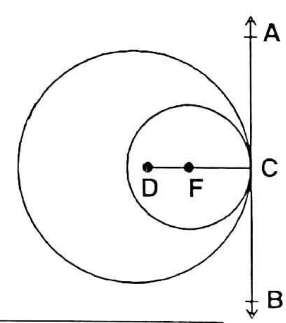
Construction: Draw $\overline{OM} \perp \overline{AB}$ to make right angled triangle OMB.

Proof:

Statements	Reasons
In right angled triangle OMB $(m\overline{OB})^2 = (m\overline{OM})^2 + (m\overline{MB})^2$	By Pythagoras theorem
It means $m\overline{OB} > m\overline{MB}$ $\therefore \quad 2(m\overline{OB}) > 2(m\overline{MB})$	The length of hypotenuse is greater than the length of other two sides.
As $2(m\overline{OB})$ is length of the central chord and $2(m\overline{MB})$ is length of the chord $\overline{AB}$ thus, Central chord $m\overline{CB}$ > Any chord $m\overline{AB}$.	
It means central chord of the circle i.e. diameter is greater than any other chord of the circle, which proved that the greatest chord in a circle is its diameter.	

THEOREM 4(B)

If two circles touch each other internally, then the point of contact lies on the line segment through their centres and distance between their centres is equal to the difference of their radii.



Given: Two circles with centres D and F touch each other internally at point C.

So that $\overline{CD}$ and $\overline{CF}$ are the radii of two circles.

To Prove: Point C lies on the join of centres D and F extended, and $m\overline{DF} = m\overline{DC} - m\overline{CF}$

Construction: Draw $\overleftrightarrow{ACB}$ as the common tangent to the pair of circles at C.

Proof:

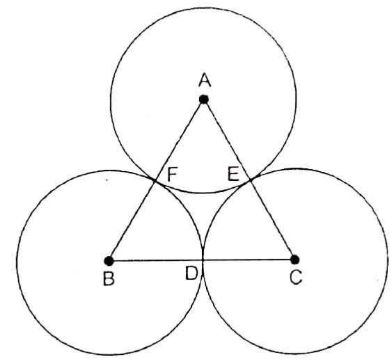
Statements	Reasons
Both circles touch internally at C whereas $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CD}$ is the radial segment of the first circle. $\therefore m\angle ACD = 90^o$ ……………..(i)	Radial segment $\overline{CD} \perp$ the tangent line $\overline{AB}$
Similarly $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CF}$ is the radial segment of the second circle. $\therefore m\angle ACF = 90^o$ ………….. (ii)	Radi al segment $\overline{CF} \perp$ the tangent line $\overline{AB}$.
$\Rightarrow \quad m\angle ACD = m\angle ACF = 90^o$ Where $\angle ACD$ and $\angle ACF$ coincide each other with point F between D and C. Hence $m\overline{DC} = m\overline{DF} + m\overline{FC}$ i.e., $\quad m\overline{DC} - m\overline{FC} = m\overline{DF}$ or $\quad m\overline{DF} = m\overline{DC} - m\overline{FC}$	Using (i) and (ii)

Example 1:

Three circles touch in pairs externally. Prove that the perimeter of a triangle formed by joining centres is equal to the sum of their diameters.

Given:

Three circles have centres A, B and C their radii are r_1, r_2 and r_3 respectively. They touch in pairs externally at D, E and F. So that ΔABC is formed by joining the centres of these circles.



To Prove:

Perimeter of $\Delta ABC = 2r_1 + 2r_2 + 2r_3$

$= d_1 + d_2 + d_3$

= Sum of the diameters of these circles.

Proof:

Statements	**Reasons**
Three circles with centres A, B and C touch in pairs externally at the points, D, E and F.	Given
$\therefore \quad m\overline{AB} = m\overline{AF} + m\overline{FB}$ ....(i)	
$m\overline{BC} = m\overline{BD} + m\overline{DC}$ ....(ii)	
And $m\overline{CA} = m\overline{CE} + m\overline{EA}$ ....(iii)	
$m\overline{AB} + m\overline{BC} + m\overline{CA} = m\overline{AF} + m\overline{FB} + m\overline{BD} + m\overline{DC} + m\overline{CE} + m\overline{EA}$	Adding (i), (ii) and (iii)
$P = (m\overline{AF} + m\overline{EA}) + (m\overline{FB} + m\overline{BD}) + (m\overline{CD} + m\overline{CE})$	Sum of three sides of a triangle is equal to its perimeter (P).
Perimeter of $\Delta ABC = 2r_1 + 2r_2 + 2r_3$ $= d_1 + d_2 + d_3$	
Perimeter of ΔABC = Sum of diameters of the circles.	$d_1 = 2r_1$, $d_2 = 2r_2$ and $d_3 = 2r_3$ are diameters of the circles.