EXERCISE 12.1

Q.1 Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.

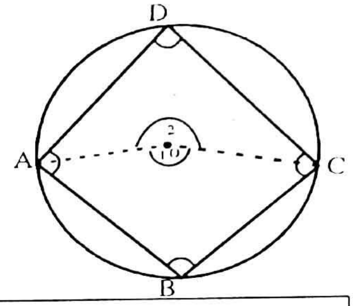
Given: A circle with centre "O"

ABCD is a cyclic quadrilateral

To Prove: $m\angle B + m\angle D = 180^\circ$

$m\angle BCD + m\angle DAB = 180^\circ$

Construction: Join O with A and C



Proof:

Statements	Reasons
$m\angle 1 = 2m\angle D$..........(i)	$\angle 1$, $\angle 2$ are central angles and $\angle D$, $\angle B$ are
$m\angle 2 = 2m\angle B$..........(ii)	circum angles in Arcs
$m\angle 1 + m\angle 2 = 2m\angle D + 2m\angle B$	Adding (i) and (ii)
$m\angle 1 + m\angle 2 = 2(m\angle D + m\angle B)$	
or $2(m\angle D + m\angle B) = m\angle 1 + m\angle 2$	By symmetric property
$2(m\angle D + m\angle B) = 360^\circ$	Sum of all central angles is 360°
$m\angle D + m\angle B = \dfrac{360^\circ}{2}$	Dividing by 2
$m\angle D + m\angle B = 180^\circ$	
Similarly $m\angle BCD + m\angle DAB = 180^\circ$	

Q.2 Show that parallelogram inscribed in a circle will be a rectangle.

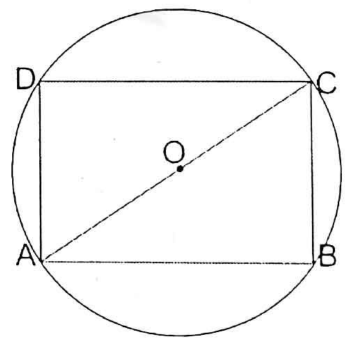
Given: ABCD is a parallelogram inscribed in the circle with centre "O"

$m\overline{AB} = m\overline{DC}$ and $\overline{AB} \parallel \overline{DC}$

$m\overline{AD} = m\overline{BC}$ and $\overline{AD} \parallel \overline{BC}$

To Prove: ABCD is a rectangle

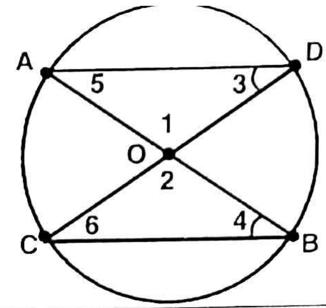
Construction: Join A with C



Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta ADC$	
$m\overline{AC} = m\overline{AC}$	Common
$m\overline{AB} = m\overline{DC}$	Given
$m\overline{BC} = m\overline{AD}$	Given
$\therefore\ \Delta ABC \cong \Delta ADC$	S.S.S $\cong$ S.S.S
Thus, $m\angle B = m\angle D$........................(i)	Corresponding angles of congruent triangles
$m\angle B + m\angle D = 180^\circ$..........(ii)	Opposite angles of parallelogram
$\Rightarrow$ $m\angle B = m\angle D = 90^\circ$	From (i)
Similarly $m\angle BAD = m\angle BCD = 90^\circ$	From (i) and (ii)
Hence ABCD is rectangle	

Q.3 AOB and COD are two intersecting chords of a circle. Show that Δ'AOD and BOC are equiangular.



Given: In a circle $\overline{AOB}$ and $\overline{COD}$ are two intersecting chords at point O.

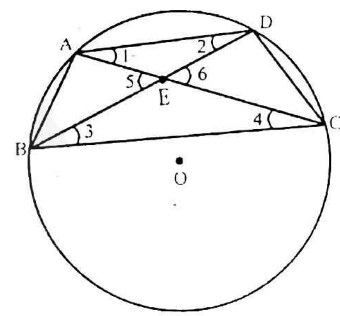
To Prove: ΔAOD and ΔBOC are equiangular

Construction: Join A with C and D. Join B with C and D.

Proof:

Statements	Reasons
$m\angle 1 \cong m\angle 2$........(i)	Vertical angles
$\overline{AC}$ is chord and angles $\angle 3, \angle 4$ are in the same segment.	
$\angle 3 \cong \angle 4$...........(ii)	
Now $\overline{BD}$ is chord and angles $\angle 5, \angle 6$ are in the same segments	
Therefore $\angle 5 \cong \angle 6$...........(iii)	
Thus, ΔAOD and ΔBOC are equiangular	From (i), (ii) and (iii)

Q.4 $\overline{AD}$ and $\overline{BC}$ are two parallel chords of a circle prove that arc AB ≅ are CD and arc AC ≅ arc BD.



Given: A circle with centre "O". Two chords $\overline{AD}$ and $\overline{BC}$ are such that $\overline{AD} \parallel \overline{BC}$.

To Prove: arc AB ≅ arc CD and arc AC ≅ arc BD

Construction: Join A to B and C. Join D to B and C. $\overline{AC}$ and $\overline{BD}$ intersect each other at point E. some angles are named as $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6$.

Proof:

Statements	Reasons
$m\angle 1 = m\angle 3$ ....(i)	Angles inscribed by an arc in the same segment
$m\angle 2 = m\angle 4$ ....(ii)	are equal.
$m\angle 1 = m\angle 4$ ....(iii)	Alternate angles are congruent $\left(\overline{AD} \parallel \overline{BC}\right)$
$m\angle 3 = m\angle 4$ ....(iv)	From (i) and (iii)
$m\angle 1 = m\angle 2$ ....(v)	From (ii) and (iii)
In $\Delta AEB \leftrightarrow \Delta DEC$	
$\overline{AE} \cong \overline{ED}$	Side opposite to equal angles (v)
$m\angle 5 = m\angle 6$	vertical angles
$\overline{BE} \cong \overline{EC}$	Sides opposite to equal angles (iv)
∴ $\Delta AED \cong \Delta DEC$	S.A.S ≅ S.A.S
$\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent.
Thus arc AB ≅ arc CD (Hence Proved)	Arcs corresponding to congruent chords are congruent.
$m\widehat{BC} \cong m\widehat{CB}$	
$m\widehat{BA} + m\widehat{AC} = m\widehat{CD} + m\widehat{DB}$	Self congruent
$\cancel{m\widehat{AB}} + m\widehat{AC} = \cancel{m\widehat{AB}} + m\widehat{BD}$	
$m\widehat{AC} = m\widehat{BD}$	∵ arc $\widehat{AB}$ ≅ arc CD proved
or arc AC ≅ arc BD (Hence proved)	