Steps of Construction:

i. Let C, D, E and F be the four points on the given arc AB.

ii. Draw chord $\overline{CD}$ and $\overline{EF}$.

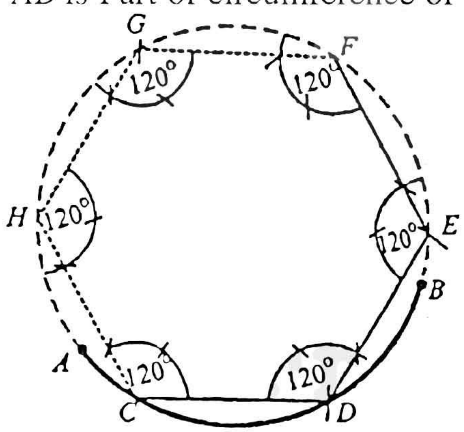
iii. Draw $\overleftrightarrow{PQ}$ as perpendicular bisector of $\overline{CD}$ and $\overleftrightarrow{LM}$ as perpendicular bisector of $\overline{EF}$

iv. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at O. Therefore, O is equidistant from points A, B, C, D, E and F.

v. Complete the circle with centre O and radius ($m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$). This will pass through all the points A, B, C, D, E and F on the given part of the circumference.

4. To complete the circle without finding the center when a part of its circumference is given.

Given: $\overset{\frown}{AB}$ is Part of circumference of a circle.



Steps of Construction:

i. Take a chord $\overline{CD}$ of reasonable length on the arc AB.

ii. Construct an internal angle of 120° at point D and draw a line segment $\overline{DE}$ equal to the length of $\overline{CD}$.

iii. At point E again construct an internal angle of 120° and from point E draw line segment $\overline{EF}$ of length equal to $\overline{CD}$ etc.

iv. Continue this practice until we reach at the starting point.

v. Now join the points D, E, F, G, H and C by arcs DE, EF and FG, GH and HC all having length equal to the length of arc CD.

As a result we get a circle including the given part of circumference.

EXERCISE 13.1

Q.1 Divide an arc of any length

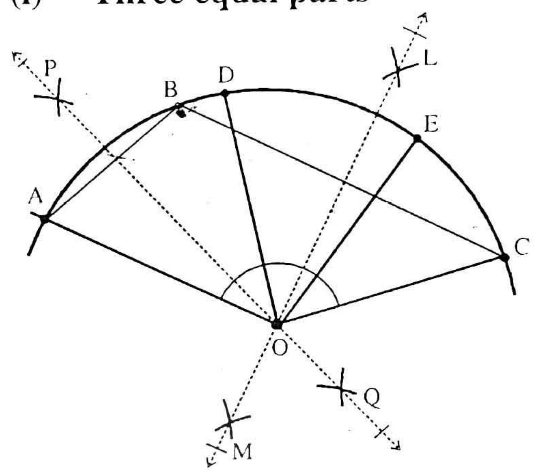
(i) Into three equal parts

(ii) Into four equal parts

(iii) Into six equal parts

Solution:

(i) Three equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

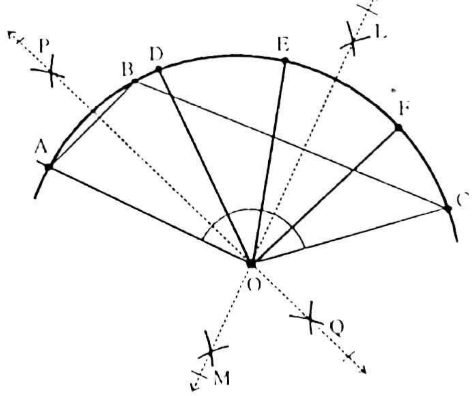
iii. Draw right bisectors $\overline{PQ}$ and $\overline{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into three equal central angles cutting the arc AC at points D and E.

vi. Arcs of same radii corresponding to equal central angles are equal. Thus three equal parts of the arc ABC are $m\overset{\frown}{AD} = m\overset{\frown}{DE} = m\overset{\frown}{EC}$.

(ii) Four equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

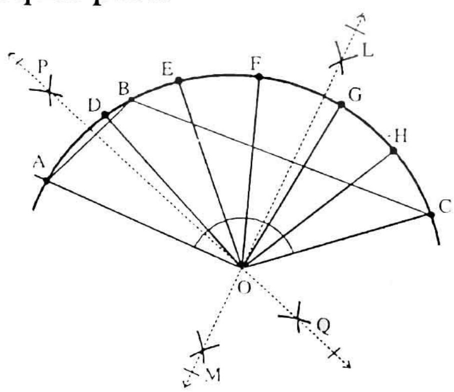
iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into four equal central angles cutting the arc AC at points D, E and F.

vi. Arcs of same radii corresponding to equal central angles are equal. Thus four equal parts of the arc ABC are $m\overset{\frown}{AD} = m\overset{\frown}{DE} = m\overset{\frown}{EF} = m\overset{\frown}{FC}$.

(iii)Six equal parts



Steps of Construction:

i. Take an arc AC of any length.

ii. Take any point B on the arc AC and join A to B and B to C.

iii. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{LM}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which meet each other at point "O". Point O is the centre of circle having the arc AC.

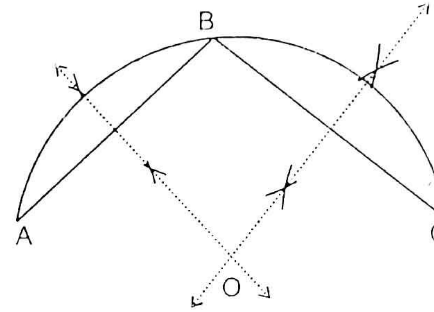
iv. Join end points of arc AC with centre O to form central angle AOC.

v. Measure the central angle and divide it into six equal central angles cutting the arc AC at points D, E, F, G and H.

Arcs of same radii corresponding to equal central angles are equal. Thus six equal parts of the arc ABC are

$m\overset{\frown}{AD} = m\overset{\frown}{DE} = m\overset{\frown}{EF} = m\overset{\frown}{FG} = m\overset{\frown}{GH} = m\overset{\frown}{HC}$

Q.2 Practically find the centre of an arc ABC



Steps of Construction:

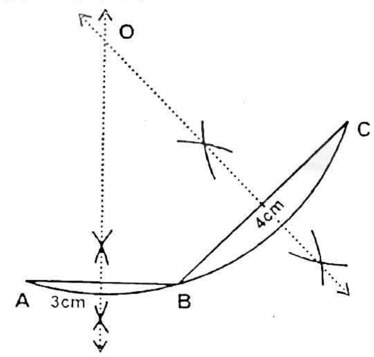
i. We draw an arc ABC of any length.

ii. We draw line segments $\overline{AB}$ and $\overline{BC}$.

iii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$, intersecting each other at point O.

iv. Point 'O' is the required centre of arc ABC.

Q. 3 (i) If $|\overline{AB}| = 3cm$ and $|\overline{BC}| = 4cm$ arc the lengths of two chords of an arc, then locate the centre of the arc.



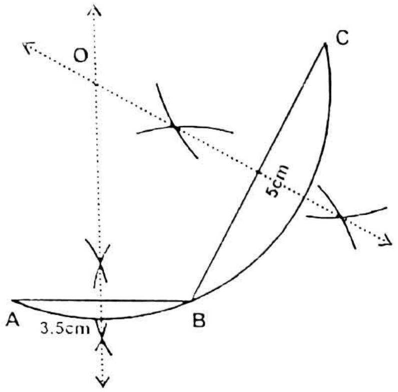
Steps of Construction:

i. We draw $|\overline{AB}| = 3cm$ and $|\overline{BC}| = 4cm$, inclined at any angle.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$ intersecting each other at point O, which is the required centre of arc ABC.

iii. Taking centre 'O', we draw an arc ABC of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$.

(ii) If $|\overline{AB}| = 3.5cm$ and $|\overline{BC}| = 5cm$ arc the lengths of two chords of an arc, then locate the centre of the arc.



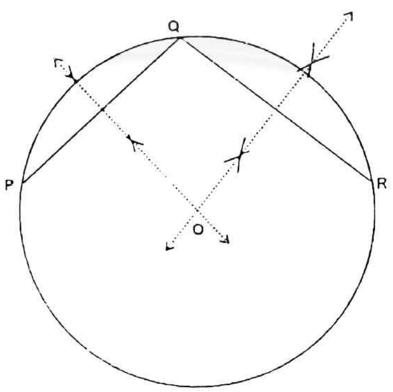
Steps of Construction:

i. We draw $|\overline{AB}| = 3.5cm$ and $|\overline{BC}| = 5cm$, inclined at any angle.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$ intersecting each other at point O, which is the required centre of arc ABC.

iii. Taking centre 'O', we draw an arc ABC of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$.

4. For an arc draw two perpendicular bisectors of the chords $\overline{PQ}$ and $\overline{QR}$ of this arc, construct a circle through P, Q and R.



Steps of construction:

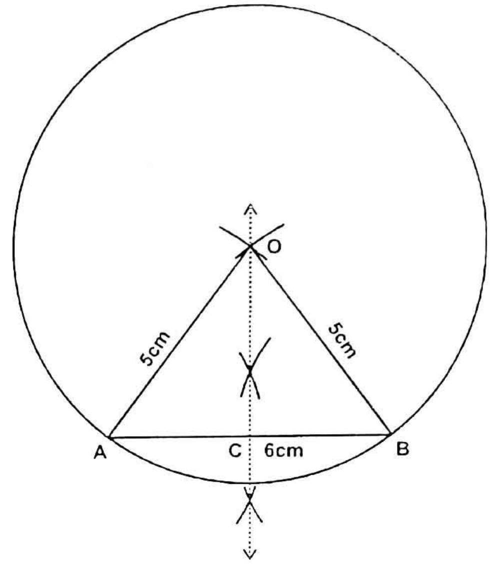
i. We take an arc PQR of any length.

ii. We take two chords $\overline{PQ}$ and $\overline{QR}$ of any lengths of arc PQR.

iii. We draw right bisectors of $\overline{PQ}$ and $\overline{QR}$, intersecting each other at point 'O', which is the centre of arc PQR.

iv. Taking 'O' as centre, we complete the required circle passing through P, Q and R.

5. Describe a circle of radius 5 cm passing through points A and B, 6 cm apart. Also find distance from the centre to line AB.



Steps of Construction:

i. We draw a line segment $\overline{AB}$ of length 6cm.

ii. We draw right bisector of $\overline{AB}$ intersecting it at point 'C'.

iii. From points A and B we draw arcs of radius 5cm each, intersecting the bisector at point O.

iv. Taking 'O' as centre we draw a circle of radius 5 cm passing through the points A and B.

v. To find the distance of centre O from $\overline{AB}$, we consider right angle ΔOAC.

By Pythagorean Theorem

$$(m\overline{OC})^2 + (m\overline{AC})^2 = (m\overline{OA})^2$$

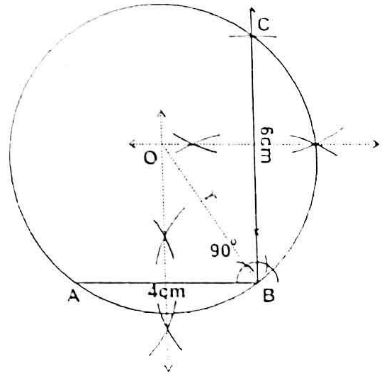
$$(m\overline{OC})^2 + (3)^2 = (5)^2$$

$$(m\overline{OC})^2 = 25 - 9$$

$$(m\overline{OC})^2 = 16$$

$$= 4 \text{ cm } m\overline{OC}$$

6. If $|\overline{AB}| = 4cm$ and $|\overline{BC}| = 6cm$, such that $\overline{AB}$ is perpendicular to $\overline{BC}$, construct a circle through points A, B and C. Also measure its radiu



Steps of construction:

i. We draw $\overline{AB}$ and $\overline{BC}$, 4 cm and 6 cm long respectively, perpendicular to each other.

ii. We draw right bisectors of $\overline{AB}$ and $\overline{BC}$, intersecting each other at point 'O'.

iii. Taking 'O' as centre we draw a circle of radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$ passing through the points A, B and C.

iv. The radius of this circle is measured to be 3.6 cm.

v. By Pythagoras theorem

$r^2 = 2^2 + 3^2$

$r^2 = 4 + 9$

$\sqrt{r^2} = \sqrt{13}$

r = 3.6cm

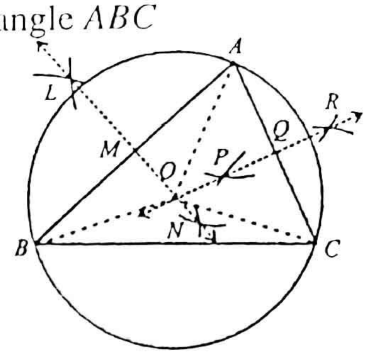
CIRCLES ATTACHED TO POLYGONS

1. Circum circle:

The circle passing through the vertices of triangle ABC is known as circum circle, its radius as circum radius and centre as circum centre.

Circumscribe a circle about a given triangle.

Given: Triangle *ABC*



Steps of Construction:

i. Draw $\overleftrightarrow{LMN}$ as perpendicular bisector of side $\overline{AB}$.

ii. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{AC}$.

iii. $\overleftrightarrow{LN}$ and $\overleftrightarrow{PR}$ intersect at point O.

iv. With centre O and radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

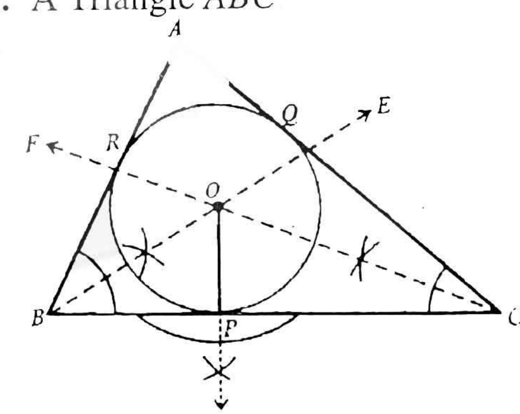
This circle will pass through A, B and C whereas O is the circum centre of the circumscribed circle.

2. Inscribed circle or In-circle:

A circle which touches the three sides of a triangle internally is known as in-circle, its radius as in-radius and centre as in-centre.

Inscribe a circle in a given triangle.

Given: A Triangle *ABC*



Steps of Construction:

i. Draw $\overrightarrow{BE}$ and $\overrightarrow{CF}$ to bisect the angles *ABC* and *ACB* respectively. Rays $\overrightarrow{BE}$ and $\overrightarrow{CF}$ intersect each other at point O.

ii. O is the centre of the inscribed circle.

iii. From O draw $\overrightarrow{OP}$ perpendicular to $\overline{BC}$.

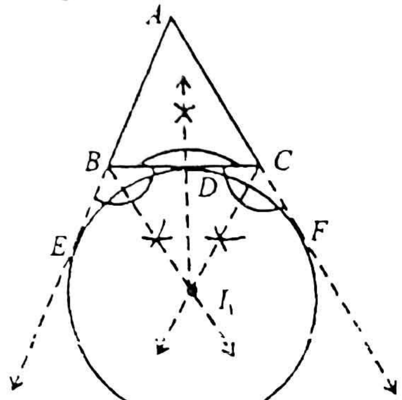
iv. With centre O and radius $\overline{OP}$ draw a circle. This circle is the inscribed circle of triangle *ABC*.

3. Escribed Circle:

The circle touching one side of the triangle externally and other two produced sides internally is called escribed circle (e-circle). The centre of e-circle is called e-centre and radius is called e-radius.

Escribe a circle to a given triangle.

Given: A Triangle ABC



Steps of Construction:

i. Produce the sides $\overline{AB}$ and $\overline{AC}$ of ΔABC.

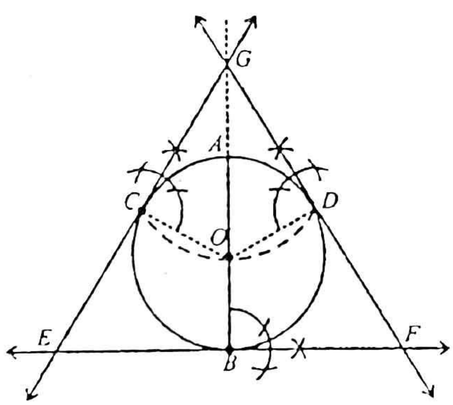
ii. Draw bisectors of exterior angles EBC and FCB. These bisectors of exterior angles meet at I_1

iii. From I_1 draw perpendicular on side $\overline{BC}$ of ΔABC intersecting $\overline{BC}$ at D. $\overline{I_1D}$ is the radius of the escribed circle with centre at I_1.

iv. Draw the circle with radius $\overline{I_1D}$ and centre at I_1 that will touch the side BC of the ΔABC externally and the produced sides AB and AC internally.

4. Circumscribe an equilateral triangle about a given circle.

Given: A circle with centre O of reasonable radius.



Steps of Construction:

i. Draw $\overline{AB}$ the diameter of the circle for locating.

ii. Draw an arc of radius $m\overline{OA}$ with centre at A, to locate points C and D on the circle.

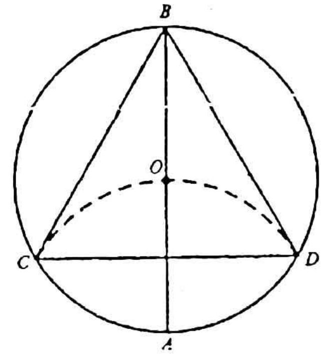
iii. Join O to the points C and D.

iv. Draw tangents to the circle at points B, C and D.

v. These tangents intersect at point E, F and G. Thus ΔEFG is required equilateral triangle.

5. Inscribe an equilateral triangle in a given circle.

Given: A circle with centre at O.



Steps of Construction:

i. Draw any diameter $\overline{AB}$ of circle.

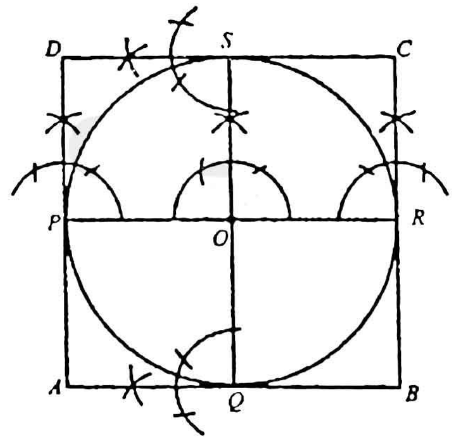
ii. Draw an arc of radius $m\overline{OA}$ from point A. The arc cuts the circle at points C and D.

iii. Join the points B, C and D to form straight line segments $\overline{BC}$, $\overline{CD}$ and $\overline{BD}$.

iv. Triangle BCD is the required inscribed equilateral triangle.

6. Circumscribe a square about a given circle.

Given: A circle with centre at O.



Steps of Construction:

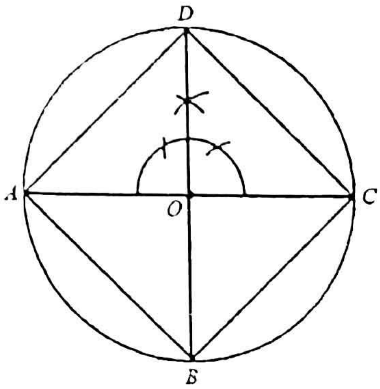
i. Draw two diameters $\overline{PR}$ and $\overline{QS}$ which bisect each other at right tangle.

ii. At points P, Q, R and S draw tangents to the circle.

iii. Produce the tangents to meet each other at A, B, C and D. ABCD is the required circumscribed square.

7. Inscribe a square in a given circle.

Given: A circle with centre at O.



Steps of Construction:

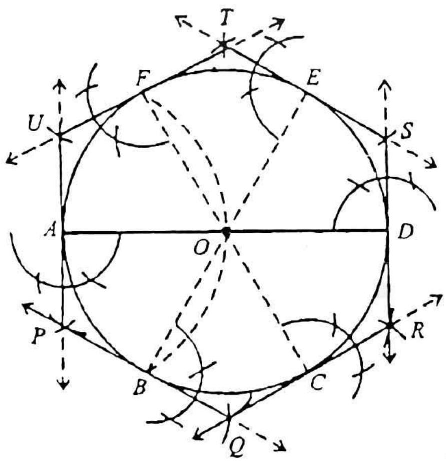
i. Through O draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.

ii. Join A with B, B with C, C with D, and D with A.

iii. $ABCD$ is the required square inscribed in the circle.

8. Circumscribe a regular hexagon about a given circle.

Given: A circle, with centre at O.



Steps of Construction:

i. Draw any diameter $\overline{AD}$.

ii. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at points B and F.

iii. Join B with O and extend it to meet the circle at E.

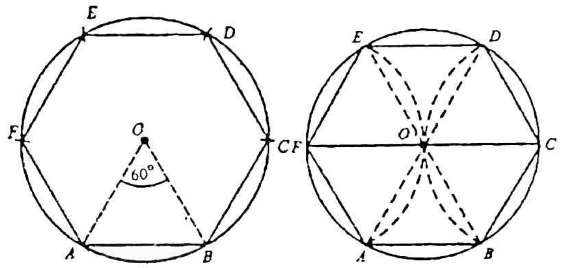
iv. Join F with O and extend it to meet the circle at C.

v. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U respectively.

vi. Thus PQRSTU is the circumscribed regular hexagon.

9. Inscribe a regular hexagon in a given circle:

Given: A circle, with centre at O.



Steps of Construction:

i. Take any point A on the circle with centre O.

ii. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at point B and F.

iii. Join O and A with points B and F.

iv. ΔOAB and ΔOAF are equilateral triangles therefore $\angle AOB$ and $\angle AOF$ are of measure 60° i.e., $m\overline{OA} = m\overline{AB} = m\overline{AF}$.

v. Produce $\overline{FO}$ to meet the circle at C. Join B to C. Since $m\angle BOC = 60$ therefore $m\overline{BC} = m\overline{OA}$.

vi. From C and F, draw arcs of radius $\overline{OA}$, which intersect the circle at points D and E.

vii. Join C to D, D to E to F. So, we have $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$ Thus the figure ABCDEF is a regular hexagon inscribed in the circle.