EXERCISE 2.5

Q.1 Write the quadratic equation having following roots.

(a). 1, 5 (b) 4, 9 (c) −2, 3
(d) 0, −3 (e) 2, −6 (f). −1, −7
(g). 1+i, 1−i (h) $3+\sqrt{2}$, $3-\sqrt{2}$

(a) **1, 5**

Solution: Since 1 and 5 are the roots of the required quadratic equation, therefore

Sum of roots = S = 1 + 5 = 6

Product of roots = P = 1×5 = 5

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - 6x + 5 = 0$

(b) **4, 9**

Solution: Since 4 and 9 are the roots of the required quadratic equation, therefore

Sum of roots = S = $4+9=13$

Product of roots = P = $4\times 9=36$

As $x^2 - Sx + P = 0$

So the required equation is

$x^2 - 13x + 36 = 0$

(c) **−2, 3**

Since −2, 3 are the roots of required quadratic equation, therefore

Sum of roots = S = − 2 + 3 = 1

Product of roots = P = −2 ×3 = −6

As $x^2 - Sx + P = 0$

Therefore the required quadratic equation is

$x^2 - x - 6 = 0$

(d) **0, −3**

Since 0, −3 are the roots of required quadratic equation therefore

Sum of roots = S = 0 + (−3) = −3

Product of roots = P = $0\times(-3)=0$

As $x^2 - Sx + P = 0$

Therefore the required quadratic equation is

$x^2 + 3x + 0 = 0 \Rightarrow$

$\Rightarrow x^2 + 3x = 0$

(e) **2, −6**

Solution: Since 2 and −6 are the roots of the required quadratic equation therefore

Sum of roots = S = 2 + (−6) = 2 − 6 = − 4

Product of roots = P = 2 × (−6) = −12

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 + 4x - 12 = 0$

(f) **−1, −7**

Solution: Since −1 and −7 are the roots of the required quadratic equation therefore

Sum of roots = S = (−1) + (−7)

$= -1 - 7 = -8$

Product of roots = P = $(-1)(-7) = 7$

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 + 8x + 7 = 0$

(g) $1+i,\ 1-i$

Solution: Since 1 + *i* and 1 − *i* are the roots of the required quadratic equation therefore

Sum of roots = S = $1+\not{i}+1-\not{i}=2$

Product of roots = P = $(1+i)(1-i)$

$P = (1)^2 - (i)^2$

$P = 1 - (-1)$

$P = 1 + 1 = 2$

As $x^2 - Sx + P = 0$ so the required equation is

$x^2 - 2x + 2 = 0$

(h) $\mathbf{3+\sqrt{2}, 3-\sqrt{2}}$

Solution: $3+\sqrt{2}$ and $3-\sqrt{2}$ are the roots of the required quadratic equation therefore

Sum of roots = S = $3+\sqrt{2}+3-\sqrt{2}=6$

Product of roots

$$P = \left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$$

$$P = (3)^2 - \left(\sqrt{2}\right)^2$$

$$P = 9 - 2 = 7$$

As $x^2 - Sx + P = 0$, so the required equation is

$x^2 - 6x + 7 = 0$

Q.2 If α, β are the roots of the equation $x^2 - 3x + 6 = 0$. Form equation whose roots are (a) $2\alpha+1, 2\beta+1$ (b) α^2, β^2

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$ (d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

(e) $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \; b = -3, \; c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \qquad \Rightarrow \boxed{\alpha\beta = 6}$$

(a) $2\alpha+1, 2\beta+1$

Sum of roots

$S = 2\alpha + 1 + 2\beta + 1$

$S = 2\alpha + 2\beta + 2$

$S = 2(\alpha+\beta) + 2$

$S = 2(3) + 2 = 6 + 2 = 8$

$\boxed{S = 8}$

Product of roots

$P = (2\alpha+1)(2\beta+1)$

$P = 4\alpha\beta + 2\alpha + 2\beta + 1$

$P = 4\alpha\beta + 2(\alpha+\beta) + 1$

$P = 4(6) + 2(3) + 1$

$P = 24 + 6 + 1 = 31$

$\boxed{P = 31}$

Using $x^2 - Sx + P = 0$, we have

$x^2 - 8x + 31 = 0$

(b) α^2, β^2

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \; b = -3, \; c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \qquad \Rightarrow \boxed{\alpha\beta = 6}$$

Sum of roots $= S = \alpha^2 + \beta^2$

$S = (\alpha+\beta)^2 - 2\alpha\beta$

$S = (3)^2 - 2(6)$

$S = 9 - 12 = -3$

$\boxed{S = -3}$

Product of roots $= P = \alpha^2\beta^2$

$P = (\alpha\beta)^2$

$P = (6)^2 = 36$

$\boxed{P = 36}$

Using $x^2 - Sx + P = 0$, we have

$x^2 + 3x + 36 = 0$

(c) $\frac{1}{\alpha}, \frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \; b = -3, \; c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \qquad \Rightarrow \boxed{\alpha\beta = 6}$$

Sum of roots $= S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$S = (\alpha+\beta).\frac{1}{\alpha\beta}$

$S = 3 \times \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$\boxed{S = \frac{1}{2}}$

Product of roots $= P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$

$P = \frac{1}{\alpha\beta} = \frac{1}{6} \; \boxed{P = \frac{1}{6}}$

Using $x^2 - Sx + P = 0$, we have

$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$

Multiplying by '6' on both side, we have

$6x^2 - 3x + 1 = 0$

(d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \ b = -3, \ c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta = 6}$$

Sum of roots = S $= \frac{\alpha}{\beta} + \frac{\beta}{a}$

$$S = \frac{\alpha^2+\beta^2}{\beta\alpha}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(3)^2 - 2(6)}{6}$$

$$S = \frac{9-12}{6}$$

$$S = \frac{-3}{6}$$

$$\boxed{S = \frac{-1}{2}}$$

Product of roots = P $= \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$

Using $x^2 - Sx + P = 0$, we have

$$x^2 + \frac{1}{2}x + 1 = 0$$

Multiplying both sides by '2', we have

$$2x^2 + x + 2 = 0$$

(e) $\alpha+\beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution: As α, β are the roots of the equation $x^2 - 3x + 6 = 0$

$a = 1, \ b = -3, \ c = 6$

Therefore,

$$\alpha+\beta = \frac{-b}{a} = \frac{-(-3)}{1} = 3 \Rightarrow \boxed{\alpha+\beta = 3}$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6 \Rightarrow \boxed{\alpha\beta = 6}$$

Sum of roots = S $= (\alpha+\beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$S = (\alpha+\beta) + \frac{\beta+\alpha}{\alpha\beta}$$

$$S = (\alpha+\beta) + \frac{(\alpha+\beta)}{a\beta}$$

$$S = 3 + \frac{3}{6}$$

$$S = 3 + \frac{1}{2}$$

$$S = \frac{6+1}{2}$$

$$\boxed{S = \frac{7}{2}}$$

Product of roots = P $= (\alpha+\beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$P = (\alpha+\beta)\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

$$P = (\alpha+\beta)\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$P = 3\left(\frac{3}{6}\right)$$

$$\boxed{P = \frac{3}{2}}$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying both sides by '2' we have

$$2x^2 - 7x + 3 = 0$$

Q.3 If α, β are the roots of the equation $x^2 + px + q = 0$. From equation whose roots are

(a). α^2, β^2 (b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$x^2 + px + q = 0$

$ax^2 + bx + c = 0$

By comparing the coefficients of these equations, we have

$$a = 1, \ b = p, \ c = q$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p \Rightarrow \boxed{\alpha + \beta = -p}$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q \Rightarrow \boxed{\alpha\beta = q}$$

(a) α^2, β^2

Sum of roots = S = $\alpha^2 + \beta^2$

$$S = (\alpha + \beta)^2 - 2\alpha\beta$$

$$S = (-p)^2 - 2q$$

$$S = p^2 - 2q$$

Product of roots = P = $\alpha^2\beta^2$

$$P = (\alpha\beta)^2$$

$$P = q^2$$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - (p^2 - 2q)x + q^2 = 0$$

(b) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since α, β are the roots of the equation

$x^2 + px + q = 0$

$ax^2 + bx + c = 0$

By comparing the coefficients of these equations, we have

$$a = 1, \ b = p, \ c = q$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-p}{1} = -p \Rightarrow \boxed{\alpha + \beta = -p}$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q \Rightarrow \boxed{\alpha\beta = q}$$

Sum of roots = S = $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$S = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(-p)^2 - 2(q)}{q}$$

$$S = \frac{p^2 - 2q}{q}$$

Product of roots = P = $\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$

Using $x^2 - Sx + P = 0$, we have

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

Multiplying by q

$$qx^2 - (p^2 - 2q)x + q = 0$$

Synthetic Division

Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In fact synthetic division is simply a shortcut of long division method.

Example 1:

Using synthetic division, divide the polynomial P(x)= $5x^4 + x^3 - 3x$ by $x - 2$

Solution:

$$(5x^4 + x^3 - 3x) \div (x - 2)$$

From divisor, x–a, here a =2

Now write the co–efficient of the dividend in row with zero as the co–efficient of the missing powers of x in the descending order as shown below.

Dividend

$P(x) = 5x^4 + 1\times x^3 + 0\times x^2 - 3x + 0\times x^0$

Now write the co–efficient of x from dividend in a row and a = 2 on the left side.

	5	1	0	–3	0
2	↓	10	22	44	82
	5	11	22	41	82

(i) Write 5 the first co–efficient as it is in the row under horizontal line.

(ii) Multiply 5 with 2 and write the result 10 under 1 write the sum of 1 + 10 = 11 under the line.

(iii) Multiply 11 with 2 and place the result 22 under 0. Add 0 and 22 and write the result 22 under the line.

(iv) Multiply 22 with 2, place the result 44 under– 3. Write 41 as the sum of 44 and –3 under the line.

(v) Multiply 41 with 2 and put the result 82 under 0. The sum of 0 and 82 is 82. In the highest power of x in dividend is 4, therefore, the highest power of x in quotient will be 4 –1 =3. Thus

Quotient = $Q(x) = 5x^3 + 11x^2 + 22x + 41$

and the Remainder = R= 82

Use of synthetic division

(a) Find quotient and remainder, when a given polynomial is divided by a linear polynomial.

Example2:

Using synthetic division, divide

$P(x) = x^4 - x^2 + 15$ **by x +1**

Solution: $(x^4 - x^2 + 15) \div (x+1)$

$(x^4 + 0x^3 - x^2 + 0x + 15) \div (x+1)$

As $x+1 = x-(-1)$, so, a = –1

Now write the co–efficient of dividend in a row and a = –1 on the left side.

	1	0	–1	0	15
–1	↓	–1	1	0	0
	1	–1	0	0	15

$\therefore$Quotient= $Q(x) = x^3 - x^2 + 0.x + 0 = x^3 - x^2$

And Remainder = 15

(b) Find the value (s) of unknown (s) , if the zeros of a polynomial are given.

Example 3:

Using synthetic division, find the value of h. If the zero of polynomial $P(x) = 3x^2 + 4x - 7h$ **is 1**

Solution:

$P(x) = 3x^2 + 4x - 7h$ and its zero is 1

Then by synthetic division.

	3	4	–7h
1	↓	3	7
	3	7	7–7h

Remainder = 7 – 7h

Since 1 is the zero of the polynomial, therefore, Remainder = 0 , that is,

7 – 7h = 0

7 = 7h $\Rightarrow$ $\boxed{h = 1}$

(c) Find the value (s) of unknown (s), if the factors of a polynomial are given.

Example 4:

Using synthetic division, find the values of *l* and m, if x–1 and x+1 are the factors of the polynomial P(x) = $x^3 + 3lx^2 + mx - 1$

Solution :

Since x–1 and x+1 are the factors of

$P(x) = x^3 + 3lx^2 + mx - 1$

Therefore, 1and –1 are zeros of polynomial P (x)

Now by synthetic division

	1	$3l$	$-m$	-1
1	↓	1	$3l+1$	$3l+m+1$
	1	$3l+1$	$3l+m+1$	$3l+m$

Since 1 is the zero of polynomial, therefore, remainder is zero, that is, $3l + m = 0$ …….(i)

And

	1	$3l$	m	-1
-1	↓	-1	$-3l+1$	$3l-m-1$
	1	$3l-1$	$-3l+m+1$	$3l-m-2$

Since -1 is the zero of polynomial, therefore, remainder is zero, that is, $3l - m - 2 = 0$ ……(ii)

Adding equations (i) and (ii), we get

$$3l + m = 0$$
$$3l - m - 2 = 0$$
$$6l \quad - 2 = 0$$

$$6l - 2 = 0$$
$$6l = 2$$

$$\Rightarrow \quad l = \frac{2}{6} = \frac{1}{3} \qquad \boxed{l = \frac{1}{3}}$$

Put the value of l in equation (i)

$$3\left(\frac{1}{3}\right) + m = 0$$

$$1 + m = 0 \quad \Rightarrow \quad \boxed{m = -1}$$

Thus $l = \frac{1}{3}$ and $m = -1$

(d) Solve a cubic equation, if one root of the equation is given.

Examples 5:

Using synthetic division, solve the equation $3x^3 - 11x^2 + 5x + 3 = 0$ when 3 is the root of the equation.

Solution:

Since 3 is the root of the equation

$3x^3 - 11x^2 + 5x + 3 = 0$

Then by synthetic division, we get.

	3	-11	5	3
3	↓	9	-6	-3
	3	-2	-1	0

The **depressed equation** is $3x^2 - 2x - 1 = 0$

$$3x^2 - 3x + 1 = 0$$
$$3x(x-1) + 1(x-1) = 0$$
$$(x-1)(3x+1) = 0$$

Either $x - 1 = 0$ or $3x + 1 = 0$,

$x = 1$ or $3x = -1$

$$\Rightarrow x = \frac{-1}{3}$$

Hence 3 , 1 and $\frac{-1}{3}$ are the roots of the given equation.

(e) Solve a biquadratic (quartic) equation, if two of the real roots of the equation are given.

Example 6:

By synthetic division solve the equation $x^4 - 49x^2 + 36x + 252 = 0$ having roots −2 and 6

Solution:

Since −2 and 6 are the roots of the given equation $x^4 - 49x^2 + 36x + 252 = 0$

Then by synthetic division we get.

	1	0	-49	36	252
-2	↓	-2	4	90	-252
	1	-2	-45	126	0
6		6	24	-126	
	1	4	-21	0	

The **depressed equation** is

$$x^2 + 4x - 21 = 0$$
$$x^2 + 7x - 3x - 21 = 0$$
$$x(x+7) - 3(x+7) = 0$$
$$(x+7)(x-3) = 0$$

Either $x + 7 = 0$ or $x - 3 = 0$

$x = -7$ or $x = 3$

Thus −2, 6, −7 and 3 are the roots of the given equation.