EXERCISE 2.6

Q.1 Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) \div (x + 1)$

As $x + 1 = x - (-1)$ So $a = -1$

Now write the co–efficient of dividend in a row and $a = -1$ on the left side

	1	7	−1
−1	↓	−1	−6
	1	6	−7

Quotient $Q(x) = x + 6$

Remainder $R = -7$

(ii) $(4x^3 - 5x + 15) \div (x + 3)$

$(4x^3 + 0x^2 - 5x + 15) \div (x + 3)$

or As $x + 3 = x - (-3)$, So $a = -3$

Now write the co–efficient of dividend in a row and $a = -3$ on the left side

	4	0	−5	15
−3	↓	−12	36	−93
	4	−12	31	−78

Quotient = $Q(x) = 4x^2 - 12x + 31$

Remainder = $R = -78$

(iii) $(x^3 + x^2 - 3x + 2) \div (x - 2)$

As $(x - 2)$ So $a = 2$

Now write the co–efficient of dividend in a row and $a = 2$ on the left side.

	1	1	−3	2
2	↓	2	6	6
	1	3	3	8

Quotient $Q(x) = x^2 + 3x + 3$

Remainder $R = 8$

Q.2 Find the value of h using synthetic division, if 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$

(i) **Solution:** Let $P(x) = 2x^3 - 3hx^2 + 0x + 9$ and its zero is 3. Then by synthetic division.

	2	$-3h$	0	9
3	↓	6	$18-9h$	$54-27h$
	2	$6-3h$	$18-9h$	$63-27h$

Remainder = $63 - 27h$

Since 3 is the zero of the polynomial, therefore Remainder = 0

$$63 - 27h = 0$$

$$63 = 27h$$

$$\Rightarrow h = \frac{63}{27} \quad \Rightarrow \boxed{h = \frac{7}{3}}$$

(ii) **Find the value of h using synthetic division, if 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$**

Solution:

Let $P(x) = x^3 - 2hx^2 + 0x + 11$ and its zero is 1. Then by synthetic division

	1	$-2h$	0	11
1	↓	1	$1-2h$	$1-2h$
	1	$1-2h$	$1-2h$	$12-2h$

Remainder = $12 - 2h$

Since 1 is the zero of the polynomial

So, Remainder = 0 that is

$$12 - 2h = 0$$

$$12 = 2h$$

$$\Rightarrow h = \frac{12}{2}$$

$$\boxed{h = 6}$$

(iii) Find the value of h using synthetic division, if -1 is the zero of the Polynomial $2x^3+5hx-23$

Solution:

Let $P(x) = 2x^3 + 5hx - 23$

$P(x) = 2x^3 + 0x^2 + 5hx - 23$

If -1 is zero of p(x) then by Synthetic division

	2	0	5h	-23
-1	↓	-2	2	$-5h-2$
	2	-2	$5h+2$	$-5h-25$

Remainder $= -5h - 25$

Since -1 is the zero of the Polynomial

So Remainder $= 0$

$-5h - 25 = 0$

$-5h = 25$

$h = \frac{25}{-5}$ $\Rightarrow$ $\boxed{h = -5}$

Q.3 Use synthetic division to find the values of l and m,

(i). if $(x+3)$ and $(x-2)$ are the factors of the polynomial $x^3+4x^2+2lx+m$

Solution: Since $(x+3)$ and $(x-2)$ are the factors of $P(x) = x^3 + 4x^2 + 2\ell x + m$

Therefore -3 and 2 are the zeros of polynomial P(x). Now by synthetic division.

	1	4	2ℓ	m
-3	↓	-3	-3	$-6\ell+9$
	1	1	$2\ell-3$	$m+(-6\ell+9)$

Since -3 is the zero of polynomial, therefore remainder is zero that is

$m - 6\ell + 9 = 0$

$\Rightarrow$ $m - 6\ell = -9$ ....(i)

And

	1	4	2ℓ	m
2	↓	2	12	$4\ell+24$
	1	6	$2\ell+12$	$m+4\ell+24$

Since 2 is the zero of polynomial, therefore remainder is zero that is

$m + 4\ell + 24 = 0$

$m + 4\ell = -24$ ....(ii)

Subtracting equations (ii) from (i)

$$\begin{array}{r} \cancel{m} - 6\ell = -9 \\ \pm\cancel{m} \pm 4\ell = \mp 24 \\ \hline -10\ell = 15 \end{array}$$

$\ell = \frac{15}{-10} = \frac{3}{-2}$

$\Rightarrow$ $\boxed{\ell = -\frac{3}{2}}$

Put it in equations (i), we get

$m - 6\left(\frac{-3}{2}\right) = -9$

$m + \frac{18}{2} = -9$

$m + 9 = -9$

$m = -9 - 9$

$\boxed{m = -18}$

(ii). Find the values of l and m *if* $(x-1)$ and $(x+1)$ are the factors of the polynomial $x^3-3lx^2+2mx+6$

Solution: Since $(x-1)$ and $(x+1)$ are the factors of $P(x) = x^3 - 3\ell x^2 + 2mx + 6$

Therefore 1 and -1 are zeros of polynomial P(x). Now by synthetic division

	1	-3ℓ	$2m$	6
1	↓	1	$1-3\ell$	$1-3\ell+2m$
	1	$1-3\ell$	$1-3\ell+2m$	$7-3\ell+2m$

Since 1 is the zero of polynomial, therefore remainder is zero that is

$7 - 3\ell + 2m = 0$

$2m - 3\ell = -7$ .... (i)

And

	1	-3ℓ	$2m$	6
-1	↓	-1	$1+3\ell$	$-1-3\ell-2m$
	1	$-1-3\ell$	$1+3\ell+2m$	$5-3\ell-2m$

Since -1 is the zero of polynomial therefore remainder is zero that is

$$5-3\ell-2m=0$$

$$\Rightarrow \quad 2m+3\ell=5 \quad \ldots\ldots\ldots\ldots(ii)$$

Adding equations (i) and (ii)

$$2m-3\ell=-7$$

$$2m+3\ell=5$$

$$4m=-2$$

$$m=\frac{-2}{4}$$

$$\boxed{m=\frac{-1}{2}}$$

Put it in equation (i),

$$2\left(-\frac{1}{2}\right)-3\ell=-7$$

$$-1-3\ell=-7$$

$$-3\ell=-7+1$$

$$-3\ell=-6$$

$$\Rightarrow \quad \ell=\frac{-6}{-3}$$

$$\Rightarrow \quad \boxed{\ell=2}$$

Q.4 Solve by using synthetic division,

(i) If 2 is the root of the equation $x^3-28x+48=0$

Solution: Let $P(x) = x^3 + 0x^2 - 28x + 48$

Since 2 is the root of the equation $x^3-28x+48=0$ then by synthetic division.

	1	0	−28	48
2	↓	2	4	−48
	1	2	−24	0

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The depressed equation is

$$x^2+2x-24=0$$

$$x^2+6x-4x-24=0$$

$$x(x+6)-4(x+6)=0$$

$$(x+6)(x-4)=0$$

Either $x+6=0$ or $x-4=0$

$x=-6$ or $x=4$

Thus 2, −6 and 4 are the roots of the given equation

(ii). If 3 is the root of the equation $2x^3-3x^2-11x+6=0$

Solution: Since 3 is the root of the equation

$$2x^3-3x^2-11x+6=0$$

Then by synthetic division

	2	−3	−11	6
3	↓	6	9	−6
	2	3	−2	0

The depressed equation is

$$2x^2+3x-2=0$$

$$2x^2+4x-x-2=0$$

$$2x(x+2)-1(x+2)=0$$

$$(x+2)(2x-1)=0$$

Either $x+2=0$ or $2x-1=0$

$\boxed{x=-2}$ or $2x=1$

$$\boxed{x=\frac{1}{2}}$$

Thus, 3, −2 and $\frac{1}{2}$ are the roots of the given equation.

(iii). If −1 is the root of the equation $4x^3-x^2-11x-6=0$

Solution: Since −1 is the root of the equation.

$$4x^3-x^2-11x-6=0$$

Then by synthetic division

	4	−1	−11	−6
−1	↓	−4	5	6
	4	−5	−6	0

The depressed equation is

$$4x^2-5x-6=0$$

$$4x^2-8x+3x-6=0$$

$$4x(x-2)+3(x-2)=0$$

$$(x-2)(4x+3)=0$$

Either $x-2=0$ or $4x+3=0$

$\boxed{x=2}$ or $4x=-3$

$$\boxed{x=\frac{-3}{4}}$$

Thus −1, 2 and $\frac{-3}{4}$ are the roots of the given equation

Q.5

(i) Solve by using synthetic division, if 1 and 3 are the roots of the equation $x^4-10x^2+9=0$

Solution: Since 1 and 3 are the roots of the equation $x^4-10x^2+9=0$

Then by synthetic division, we get

	1	0	−10	0	9
1	↓	1	1	−9	−9
	1	1	−9	−9	0
3	↓	3	12	9	
	1	4	3	0	

Thus the depressed equation is

$$x^2+4x+3=0$$
$$x+3x+x+3=0$$
$$x(x+3)+1(x+3)=0$$
$$(x+3)(x+1)=0$$

Either $x+3=0$ or $x+1=0$

$\boxed{x=-3}$ $\boxed{x=-1}$

Hence 1, 3, −3 and −1 are the roots of the given equation

(ii) Solve by using synthetic division, if 3 and −4 are the roots of the equation $x^4+2x^3-13x^2-14x+24=0$ 02(084)

Solution: Since 3 and −4 are the roots of the given equation $x^4+2x^3-13x^2-14x+24=0$ then by synthetic division, we get

	1	2	−13	−14	24
3	↓	3	15	6	−24
	1	5	2	−8	0
−4	↓	−4	−4	8	
	1	1	−2	0	

The depressed equation is

$$x^2+x-2=0$$
$$x^2+2x-x-2=0$$
$$x(x+2)-1(x+2)=0$$
$$(x+2)(x-1)=0$$

Either $x+2=0$ or $x-1=0$

$\boxed{x=-2}$ $\boxed{x=1}$

Thus 3, −4, −2 and 1 are the roots of the given equation.

Simultaneous equations

A system of equations having a common solution is called a system of **simultaneous equations**.

The set of all the ordered pairs (x, y), which satisfies the system of equations is called the **solution set** of the system.

(i) Solve a system of two equations in two variables

(a) When one equation is linear and the other is quadratic.

To solve a system of equations in two variables x and y. Find 'y' in terms of x from the given linear equation. Substitute the value of y in the quadratic equation we get an other quadratic equation in one variable x. Solve this equation for x and then find the values of y.

Example 1:

Solve the system of equations

$3x + y = 4$ and $3x^2 + y^2 = 52$.

Solution:

The given equations are

$3x + y = 4$ ....(i)

and $3x^2 + y^2 = 52$ ....(ii)

From equation (i)

$y = 4 - 3x$ ....(iii)

Put value of y in equation (ii)

$$3x^2 + (4-3x)^2 = 52$$
$$3x^2 + [(4)^2 - 2(4)(3x) + (3x)^2] - 52 = 0$$
$$3x^2 + [16 - 24x + 9x^2] - 52 = 0$$
$$3x^2 + 16 - 24x + 9x^2 - 52 = 0$$
$$12x^2 - 24x - 36 = 0$$
$$12(x^2 - 2x - 3) = 0$$

$\Rightarrow$ $x^2 - 2x - 3 = 0$ $(\because 12 \neq 0)$

By factorization

$$x^2 - 3x + x - 3 = 0$$
$$x(x-3)+1(x-3)=0$$
$$\Rightarrow (x-3)(x+1)=0$$

Either $x-3=0$ or $x+1=0$

$x=3$ or $x=-1$

Put the values of x in equation (iii)

When $x = 3$	When $x = -1$
$y = 4 - 3x$	$y = 4 - 3x$
$y = 4 - 3(3)$	$y = 4 - 3(-1)$
$y = 4 - 9$	$y = 4 + 3$
$y = -5$	$y = 7$

∴ The ordered pairs are (3, –5) and (–1, 7)

Thus, the solution set is $\{(3,-5),(-1,7)\}$

(b) When both the equations are quadratic.

Example 2: **Solve the equations**

$x^2 + y^2 + 2x = 8$ **and** $(x-1)^2 + (y+1)^2 = 8$

Solution: The given equations are

$$x^2 + y^2 + 2x = 8 \quad \text{.................(i)}$$

$$(x-1)^2 + (y+1)^2 = 8 \quad \text{..........(ii)}$$

From equation (ii), we get

$$x^2 - 2x + 1 + y^2 + 2y + 1 = 8$$

$$x^2 + y^2 - 2x + 2y = 8 - 2$$

$$x^2 + y^2 - 2x + 2y = 6 \quad \text{...........(iii)}$$

Subtracting equation (iii) from equation (i), we have

$$4x - 2y = 2$$

$$2(2x - y) = 2$$

$$2x - y = 1$$

$$\Rightarrow \quad y = 2x - 1 \quad \text{.................... (iv)}$$

Put the value of y in equation (ii)

$$(x-1)^2 + (2x - 1 + 1)^2 = 8$$

$$(x)^2 - 2(x)(1) + (1)^2 + (2x)^2 = 8$$

$$x^2 - 2x + 1 + 4x^2 - 8 = 0$$

$$5x^2 - 2x - 7 = 0$$

$$5x^2 - 7x + 5x - 7 = 0$$

or $\quad x(5x - 7) + 1(5x - 7) = 0$

$\Rightarrow \quad (5x - 7)(x + 1) = 0$

Either $5x - 7 = 0$ or $x + 1 = 0$

$5x = 7$ or $x = -1$

$\Rightarrow x = \dfrac{7}{5}$

Now putting the values of x in equation (iv), we have

When $x = \frac{7}{5}$	When $x = -1$
$y = 2\left(\frac{7}{5}\right) - 1$	$y = 2(-1) - 1$
$y = \frac{14}{5} - 1$	$y = -2 - 1$
$y = \frac{14-5}{5} = \frac{9}{5}$	$y = -3$

Thus, the solution set is $\left\{(-1,-3),\left(\frac{7}{5},\frac{9}{5}\right)\right\}$

Example 3: **Solve the equations**

$x^2 + y^2 = 7$ and $2x^2 + 3y^2 = 18$

Solution: Given equations are

$$x^2 + y^2 = 7 \quad \text{.......................(i)}$$

$$2x^2 + 3y^2 = 18 \quad \text{..............(ii)}$$

Multiply equation (i) by 3

$$3x^2 + 3y^2 = 21 \quad \text{..............(iii)}$$

Subtracting equations (ii) from (iii)

$$\begin{array}{l} 3x^2 + 3y^2 = 21 \\ \pm 2x^2 \pm 3y^2 = \pm 18 \\ \hline x^2 \qquad\quad = 3 \end{array}$$

$x^2 = 3 \quad \Rightarrow \quad x = \pm\sqrt{3}$

Now putting the values of $x = \pm\sqrt{3}$, in equation (i)

We have

$$x^2 + y^2 = 7$$

$$3 + y^2 = 7$$

$$y^2 = 7 - 3$$

$$\Rightarrow \quad y^2 = 4$$

$$y = \pm 2$$

Thus, the required solution set is $\left\{\left(\pm\sqrt{3}, \pm 2\right)\right\}$

Example 4: **Solve the equations**

$x^2 + y^2 = 20$ and $6x^2 + xy - y^2 = 0$

Solution: Given equations are:

$$x^2 + y^2 = 20 \quad \text{....................(i)}$$

$$6x^2 + xy - y^2 = 0 \quad \text{.............(ii)}$$

The equation (ii) can be written as

$$y^2 - xy - 6x^2 = 0$$

Here we take "x" as constant and "y" as variable

$a = 1$, $b = -x$, $c = -6x^2$

$$y=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow\quad y=\frac{-(-x)\pm\sqrt{(-x)^2-4\times1\times(-6x^2)}}{2\times1}$$

$$y=\frac{x\pm\sqrt{x^2+24x^2}}{2}=\frac{x\pm\sqrt{25x^2}}{2}$$

$$y=\frac{x\pm5x}{2}$$

Either,

$$y=\frac{x+5x}{2}\quad\text{or}\quad y=\frac{x-5x}{2}$$

$$y=\frac{6x}{2}\quad\text{or}\quad y=\frac{-4x}{2}$$

$\boxed{y=3x}$ ... (iii) or $\boxed{y=-2x}$ .... (iv)

Putting the values of y in equation (i), we get

When $y=3x$	When $y=-2x$
$x^2+(3x)^2=20$	$x^2+(-2x)^2=20$
$x^2+9x^2=20$	$x^2+4x^2=20$
$10x^2=20$	$5x^2=20$
$\Rightarrow\quad x^2=2$	$x^2=4$
$x=\pm\sqrt{2}$	$x=\pm2$
Put $x=\sqrt{2}$ in eq. (iii)	Put $x=2$ in eq. (iv)
$y=3(\sqrt{2})=3\sqrt{2}$	$y=-2(2)=-4$
Now put $x=-\sqrt{2}$ in eq.iii	Now put $x=-2$ in eq. (iv)
$y=3(-\sqrt{2})=-3\sqrt{2}$	$y=-2(-2)=4$

Thus, the solution is

$$\{(\sqrt{2},3\sqrt{2}),(-\sqrt{2},-3\sqrt{2}),(2,-4)(-2,4)\}$$

Example 5: Solve the equations

$$x^2+y^2=40 \text{ and } 3x^2-2xy-y^2=80$$

Solution: Given equations are

$$x^2+y^2=40 \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$3x^2-2xy-y^2=80 \ldots\ldots\ldots\ldots\text{(ii)}$$

Multiplying equation (i) by 2, we have

$$2x^2+2y^2=80 \ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

Subtracting the equation (iii) from equation (ii), we get

$$3x^2-2xy-y^2=80$$

$$\pm2x^2 \qquad \pm2y^2=\pm80$$

$$x^2-2xy-3y^2=0 \ldots\ldots\ldots\ldots\text{(iv)}$$

The equation (iv) can be written as

$$x^2-3xy+xy-3y^2=0$$

or $\quad x(x-3y)+y(x-3y)=0$

$\Rightarrow\quad (x-3y)(x+y)=0$

Either $x-3y=0$ or $x+y=0$

$\boxed{x=3y}$ or $\boxed{x=-y}$

Putting the values of x in equation (i)

When $x = 3y$

$$(3y)^2+y^2=40$$

$$10y^2=40$$

$$y^2=4$$

$$y=\pm2$$

Now putting these values of y in $x = 3y$

When $y = 2$	When $y = -2$
$x=3y$	$x=3y$
$x=3(2)$	$x=3(-2)$
$x=6$	$x=-6$
$(x, y) = (6, 2)$	$(x, y) = (-6, -2)$

Now putting $x=-y$ in equation (i) we get

$$(-y)^2+y^2=40$$

$$2y^2=40$$

$$y^2=20$$

$$y=\pm2\sqrt{5}$$

Now putting these values of y in $x = -y$

When $y=2\sqrt{5}$	When $y=-2\sqrt{5}$
$x=-y$	$x=-y$
$x=-(2\sqrt{5})$	$x=-(-2\sqrt{5})$
$x=-2\sqrt{5}$	$x=2\sqrt{5}$

∴ The solution set is

$$\{(6,2),(-6,-2),(2\sqrt{5},-2\sqrt{5}),(-2\sqrt{5},2\sqrt{5})\}$$