# EXERCISE 2.7

Solve the following simultaneous equations.

**Q. 1** $x + y = 5$

$x^2 - 2y - 14 = 0$

**Solution:**

$x + y = 5$ .............. (i)

$x^2 - 2y - 14 = 0$ ....... (ii)

From equation (i)

$x + y = 5$

$x = 5 - y$

Put it in equation (ii)

$(5 - y)^2 - 2y - 14 = 0$

$25 + y^2 - 10y - 2y - 14 = 0$

$y^2 - 12y + 11 = 0$

$y^2 - 11y - y + 11 = 0$

$y(y - 11) - 1(y - 11) = 0$

$(y - 11)(y - 1) = 0$

Either $y - 11 = 0$ or $y - 1 = 0$

$y = 11$ or $y = 1$

Putting the values of y in eq .....(i)

| When $y = 11$ | When $y = 1$ |
|---|---|
| $x + y = 5$ | $x + y = 5$ |
| $x + 11 = 5$ | $x + 1 = 5$ |
| $x = 5 - 11$ | $x = 5 - 1$ |
| $x = -6$ | $x = 4$ |

Solution Set is $\{(-6, 11), (4, 1)\}$

**Q. 2** $3x - 2y = 1$

$x^2 + xy - y^2 = 1$

**Solution:**

$3x - 2y = 1$ ............... (i)

$x^2 + xy - y^2 = 1$ ...........(ii)

From equation (i)

$3x = 1 + 2y$

$x = \frac{1+2y}{3}$ .................(iii)

Put it in equation (ii)

$$\left(\frac{1+2y}{3}\right)^2 + \left(\frac{1+2y}{3}\right)y - y^2 = 1$$

$$\frac{1+4y^2+4y}{9} + \frac{y+2y^2}{3} - y^2 = 1$$

Multiplying by '9' on both sides we have

$$\frac{\cancel{9}(1+4y^2+4y)}{\cancel{9}} + \frac{{}^{3}\cancel{9}(y+2y^2)}{\cancel{3}} - 9(y^2) = 9\times 1$$

$1 + 4y^2 + 4y + 3y + 6y^2 - 9y^2 = 9$

$y^2 + 7y - 8 = 0$

$y^2 + 8y - y - 8 = 0$

$y(y + 8) - 1(y + 8) = 0$

$(y + 8)(y - 1) = 0$

Either

$y + 8 = 0$ or $y - 1 = 0$

$y = -8$ or $y = 1$

Putting these values in equation (iii)

| when $y = -8$ | when $y = 1$ |
|---|---|
| $x = \frac{1+2y}{3}$ | $x = \frac{1+2y}{3}$ |
| $x = \frac{1+2(-8)}{3}$ | $x = \frac{1+2(1)}{3}$ |
| $x = \frac{1-16}{3}$ | $x = \frac{1+2}{3}$ |
| $x = \frac{-15}{3} = -5$ | $x = \frac{3}{3} = 1$ |

*Solution Set is* $\{(-5,-8),(1,1)\}$

**Q. 3** $x - y = 7$

$\frac{2}{x} - \frac{5}{y} = 2$

**Solution:**

$x - y = 7$ ........... (i)

$\frac{2}{x} - \frac{5}{y} = 2$ ............ (ii)

Multiplying equation (ii) by "xy" we have

$2y - 5x = 2xy$........ (iii)

From equation (i)

$x = 7 + y$

Put it in equation (iii)

$2y - 5(7 + y) = 2(7 + y)y$

$2y - 35 - 5y = 14y + 2y^2$

$\Rightarrow$ $2y^2 + 17y + 35 = 0$

$2y^2 + 10y + 7y + 35 = 0$

$2y(y + 5) + 7(y + 5) = 0$

$(y + 5)(2y + 7) = 0$

Either

$y + 5 = 0$ or $2y + 7 = 0$

$y = -5$ or $y = \frac{-7}{2}$

Now putting these values of y in equation (i)

When $y = -5$

$x = 7 + y$

$x = 7 + (-5)$

$x = 7 - 5$

$x = 2$

When $y = \frac{-7}{2}$

$x = 7 + y$

$x = 7 + \left(\frac{-7}{2}\right)$

$x = 7 - \frac{7}{2}$

$x = \frac{14-7}{2}$

$x = \frac{7}{2}$

*Solution Set is* $\left\{(2,-5), \left(\frac{7}{2}, \frac{-7}{2}\right)\right\}$

**Q. 4** $x + y = a - b$

$\frac{a}{x} - \frac{b}{y} = 2$

**Solution:**

$x + y = a - b$ ..........(i)

$\frac{a}{x} - \frac{b}{y} = 2 \Rightarrow \frac{ay - bx}{xy} = 2$

$ay - bx = 2xy$ ............(ii)

From equation (i)

$x = a - b - y$ ............(iii)

Put it in equation (ii)

$ay - bx = 2xy$

$ay - b(a - b - y) = 2(a - b - y)y$

$ay - ba + b^2 + by = 2ay - 2by - 2y^2$

$2y^2 - 2ay + ay + 2by + by + b^2 - ab = 0$

$2y^2 - ay + 3by + b^2 - ab = 0$

$2y^2 - y(a - 3b) + (b^2 - ab) = 0$

By using quadratic formula

$a = 2, \quad b = -(a - 3b), \; c = (b^2 - ab)$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-[-(a-3b)] \pm \sqrt{[-(a-3b)]^2 - 4(2)(b^2 - ab)}}{2(2)}$$

$$y = \frac{(a-3b) \pm \sqrt{a^2 + 9b^2 - 6ab - 8b^2 + 8ab}}{4}$$

$$y = \frac{(a-3b) \pm \sqrt{a^2 + b^2 + 2ab}}{4}$$

$$y = \frac{(a-3b) \pm \sqrt{(a+b)^2}}{4}$$

$$y = \frac{(a-3b) \pm (a+b)}{4}$$

$y = \frac{\not{a} - 3b - \not{a} - b}{4}$ or $y = \frac{a - 3b + a + b}{4}$

$y = \frac{-4b}{4}$ or $y = \frac{2a - 2b}{4}$

$y = \frac{2(a-b)}{4}$

$y = -b$ or $y = \frac{a-b}{2}$

Putting these values of y in equation (iii)

when $y = -b$

$x = a - b - y$

$x = a - b - (-b)$

$x = a - \not{b} + \not{b}$

$x = a$

when $y = \frac{a-b}{2}$

$x = a - b - y$

$x = a - b - \frac{a-b}{2}$

$x = \frac{2a - 2b - a + b}{2}$

$x = \frac{a-b}{2}$

*Solution Set is* $\left\{(a, -b), \left(\frac{a-b}{2}, \frac{a-b}{2}\right)\right\}$

**Q. 5** $x^2 + (y-1)^2 = 10$
$x^2 + y^2 + 4x = 1$

**Solution:**

$x^2 + (y-1)^2 = 10$ ........... (i)

$x^2 + y^2 + 4x = 1$ .............(ii)

Subtracting equation (ii) from (i)

$$\cancel{x^2} + \cancel{y^2} + 1 - 2y = 10$$
$$\pm \cancel{x^2} \pm \cancel{y^2} \quad \pm 4x = \pm 1$$

$$-4x - 2y + 1 = 9$$
$$-4x - 2y = 9 - 1$$
$$-4x - 2y = 8$$
$$-2(2x + y) = 8$$
$$\Rightarrow \quad 2x + y = \frac{8}{-2}$$
$$2x + y = -4$$
$$y = -4 - 2x \quad \text{..........(iii)}$$

Put in equation (ii)

$$x^2 + (-4 - 2x)^2 + 4x = 1$$
$$x^2 + [-(4 + 2x)]^2 + 4x = 1$$
$$x^2 + [16 + 4x^2 + 16x] + 4x = 1$$
$$5x^2 + 20x + 16 - 1 = 0$$
$$5x^2 + 20x + 15 = 0$$
$$5(x^2 + 4x + 3) = 0$$
$$\Rightarrow \quad x^2 + 4x + 3 = 0 \qquad (\because 5 \neq 0)$$
$$x^2 + 3x + x + 3 = 0$$
$$x(x + 3) + 1(x + 3) = 0$$
$$(x + 3)(x + 1) = 0$$

Either $x + 3 = 0$ or $x + 1 = 0$

$x = -3$ or $x = -1$

Putting these values of x in equation (iii)

| when $x = -3$ | when $x = -1$ |
|---|---|
| $y = -4 - 2x$ | $y = -4 - 2x$ |
| $y = -4 - 2(-3)$ | $y = -4 - 2(-1)$ |
| $y = -4 + 6$ | $y = -4 + 2$ |
| $y = 2$ | $y = -2$ |

So, the solution Set is $\{(-3, 2), (-1, -2)\}$

**Q.6** $(x+1)^2 + (y+1)^2 = 5$, $(x+2)^2 + y^2 = 5$

**Solution:** $(x+1)^2 + (y+1)^2 = 5$ ..........(i)

$(x+2)^2 + y^2 = 5$ ...............(ii)

From equation (i)

$$x^2 + 1 + 2x + y^2 + 1 + 2y = 5$$
$$x^2 + y^2 + 2x + 2y + 2 = 5$$
$$x^2 + y^2 + 2x + 2y = 3 \quad \text{.......... (iii)}$$

From equation (ii)

$$(x+2)^2 + y^2 = 5$$
$$x^2 + 4 + 4x + y^2 = 5$$
$$x^2 + y^2 + 4x = 5 - 4$$
$$x^2 + y^2 + 4x = 1 \quad \text{................. (iv)}$$

Subtracting equation (iv) from (iii)

$$\cancel{x^2} + \cancel{y^2} + 2x + 2y = 3$$
$$\pm \cancel{x^2} \pm \cancel{y^2} \pm 4x \qquad = \pm 1$$

$$-2x + 2y = 2$$
$$-2(x - y) = 2$$
$$x - y = \frac{2}{-2}$$
$$x - y = -1$$
$$x = y - 1 \quad \text{..................(v)}$$

Put it in equation (iv)

$$(y-1)^2 + y^2 + 4(y-1) = 1$$
$$y^2 + 1 - 2y + y^2 + 4y - 4 = 1$$
$$2y^2 + 2y - 4 + \cancel{1} - \cancel{1} = 0$$
$$2y^2 + 2y - 4 = 0$$
$$2(y^2 + y - 2) = 0$$
$$\Rightarrow \quad y^2 + y - 2 = 0 \qquad (\because 2 \neq 0)$$
$$y^2 + y - 2 = 0$$
$$y^2 + 2y - y - 2 = 0$$
$$y(y + 2) - 1(y + 2) = 0$$
$$(y + 2)(y - 1) = 0$$

Either $y + 2 = 0$ or $y - 1 = 0$

$y = -2$ $\quad$ $y = 1$

Putting these values of y in equation (v)

| when $y = -2$ | when $y = 1$ |
|---|---|
| $x = y - 1$ | $x = y - 1$ |
| $x = -2 - 1$ | $x = 1 - 1$ |
| $x = -3$ | $x = 0$ |

So, the solution Set is: $\{(-3, -2), (0, 1)\}$

**Q. 7** $x^2 + 2y^2 = 22$

$5x^2 + y^2 = 29$

**Solution:**

$x^2 + 2y^2 = 22$ ............ (i)

$5x^2 + y^2 = 29$ ............. (ii)

Multiplying equation (ii) by "2" we have

$10x^2 + 2y^2 = 58$ .......... (iii)

Subtracting equation (i) from (iii)

$10x^2 + 2y^2 = 58$

$\pm x^2 \pm 2y^2 = \pm 22$

$9x^2 = 36$

$x^2 = \frac{36}{9}$

$x^2 = 4$

Taking square root, we have

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\Rightarrow x = -2$ or $x = 2$

Now putting these values of x in equation (i)

| When $x = -2$ | When $x = 2$ |
|---|---|
| $x^2 + 2y^2 = 22$ | $x^2 + 2y^2 = 22$ |
| $(-2)^2 + 2y^2 = 22$ | $(2)^2 + 2y^2 = 22$ |
| $4 + 2y^2 = 22$ | $4 + 2y^2 = 22$ |
| $2y^2 = 22 - 4$ | $2y^2 = 22 - 4$ |
| $2y^2 = 18$ | $2y^2 = 18$ |
| $y^2 = \frac{18}{2}$ | $y^2 = \frac{18}{2}$ |
| $y^2 = 9$ | $y^2 = 9$ |
| $\Rightarrow y = \pm 3$ | $\Rightarrow y = \pm 3$ |

So, the solution set is $\{(\pm 2, \pm 3)\}$

**Q.8** $4x^2 - 5y^2 = 6$

$3x^2 + y^2 = 14$

**Solution:**

$4x^2 - 5y^2 = 6$ .............. (i)

$3x^2 + y^2 = 14$ .............. (ii)

Multiplying equation (ii) by 5 and add in equation (i)

$4x^2 - 5y^2 = 6$

$15x^2 + 5y^2 = 70$

$19x^2 = 76$

$x = \frac{76}{19}$

$x^2 = 4$

$\Rightarrow x = \pm 2$

Either $x = 2$ or $x = -2$

Putting these values of x in equation (ii)

| When x = 2, | When x = −2 |
|---|---|
| $3(2)^2 + y^2 = 14$ | $3(-2)^2 + y^2 = 14$ |
| $3(4) + y^2 = 14$ | $3(4) + y^2 = 14$ |
| $12 + y^2 = 14$ | $12 + y^2 = 14$ |
| $y^2 = 14 - 12$ | $y^2 = 14 - 12$ |
| $y^2 = 2$ | $y^2 = 2$ |
| $y = \pm\sqrt{2}$ | $y = \pm\sqrt{2}$ |

So the solution set is $\{(\pm 2, \pm\sqrt{2})\}$

**Q. 9** $7x^2 - 3y^2 = 4$

$2x^2 + 5y^2 = 7$

**Solution:**

$7x^2 - 3y^2 = 4$ ———— (i)

$2x^2 + 5y^2 = 7$ ———— (ii)

Multiply equation (i) by 5 and equation (ii) by 3 and add them

$35x^2 - 15y^2 = 20$

$6x^2 + 15y^2 = 21$

$41x^2 = 41$

$x^2 = \frac{41}{41}$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

Either $x = 1$ or $x = -1$

Putting these values of x in equation (i)

When $x = 1$

$7(1)^2 - 3y^2 = 4$

$7 - 3y^2 = 4$

$-3y^2 = 4 - 7$

$-3y^2 = -3$

$y^2 = \frac{-3}{-3}$

$y^2 = 1$

$y = \pm\sqrt{1}$

$y = \pm 1$

When $x = -1$

$7(-1)^2 - 3y^2 = 4$

$7(1) - 3y^2 = 4$

$7 - 3y^2 = 4$

$-3y^2 = 4 - 7$

$-3y^2 = -3$

$y^2 = \frac{-3}{-3}$

$y^2 = 1$

$y = \pm 1$

So the solution set is $\{(\pm 1, \pm 1)\}$

**Q. 10** $\mathbf{x^2 + 2y^2 = 3}$

$\mathbf{x^2 + 4xy - 5y^2 = 0}$

**Solution:**

$x^2 + 2y^2 = 3$ ———— (i)

$x^2 + 4xy - 5y^2 = 0$ ———— (ii)

Factorizing equation (ii) we get

$x^2 + 4xy - 5y^2 = 0$

$x^2 + 5xy - xy - 5y^2 = 0$

$x(x+5y) - y(x+5y) = 0$

$(x+5y)(x-y) = 0$

Either $x + 5y = 0$ or $x - y = 0$

$x = -5y$ —— (iii) $\quad x = y$ —— (iv)

Putting these values of x in equation (i)

When $x = -5y$

$(-5y)^2 + 2y^2 = 3$

$25y^2 + 2y^2 = 3$

$27y^2 = 3$

$y^2 = \frac{3}{27}$

$y^2 = \frac{1}{9}$

$y = \pm\sqrt{\frac{1}{9}}$

$y = \pm\frac{1}{3}$

$y = \frac{1}{3}$ or $y = \frac{-1}{3}$

When $x = y$

$y^2 + 2(y^2) = 3$

$3y^2 = 3$

$y^2 = \frac{3}{3}$

$y^2 = 1$

$y = \pm\sqrt{1}$

$y = -1$, or $y = 1$

Putting the value of $y = \pm\frac{1}{3}$ in equation (iii)

When $y = \frac{1}{3}$

$x = -5y$

$x = -5\left(\frac{1}{3}\right)$

$x = \frac{-5}{3}$

When $y = \frac{-1}{3}$

$x = -5y$

$x = -5\left(\frac{-1}{3}\right)$

$x = \frac{5}{3}$

Now putting the values of $y = \pm 1$ in equation (iv)

$x = y$

When $y = 1$ then $x = 1$

When $y = -1$ then $x = -1$

Solution Set is $\left\{(-1,-1), (1,1), \left(\frac{5}{3}, \frac{-1}{3}\right), \left(\frac{-5}{3}, \frac{1}{3}\right)\right\}$

**Q. 11** $\mathbf{3x^2 - y^2 = 26}$

$\mathbf{3x^2 - 5xy - 12y^2 = 0}$

**Solution:**

$3x^2 - y^2 = 26$ ———— (i)

$3x^2 - 5xy - 12y^2 = 0$ ———— (ii)

Factorizing equation (ii)

$3x^2 - 5xy - 12y^2 = 0$

$3x^2 - 9xy + 4xy - 12y^2 = 0$

$3x(x-3y) + 4y(x-3y) = 0$

$(x-3y)(3x+4y) = 0$

Either

$x - 3y = 0$ or $3x + 4y = 0$

$x = 3y$ ———— (iii) $\quad 3x = (-4y)$

$x = \frac{-4y}{3}$ ....(iv)

From equation (iii) putting the value of x in equation (i)

$3(3y)^2 - y^2 = 26$

$3(9y)^2 - y^2 = 26$

$27y^2 - y^2 = 26$

$26y^2 = 26$

$y^2 = 1$

$\Rightarrow y = \pm 1$

$y = 1$ or $y = -1$

Putting these value of y in equation (iii)

| When $y = 1$ | When $y = -1$ |
|---|---|
| $x = 3y$ | $x = 3y$ |
| $x = 3(1)$ | $x = 3(-1)$ |
| $x = 3$ | $x = -3$ |
| $(x, y) = (3, 1)$ | $(x, y) = (-3, -1)$ |

From equation (iv) putting the values of x in equation (i)

$$3\left(\frac{-4y}{3}\right)^2 - y^2 = 26$$

$$3\times\frac{16y^2}{9} - y^2 = 26$$

$$\frac{48y^2 - 9y^2}{9} = 26$$

$$39y^2 = 26\times 9$$

$$y^2 = \frac{234}{39}$$

$$y^2 = 6$$

$$\Rightarrow \boxed{y = \pm\sqrt{6}}$$

$$y = \sqrt{6} \quad or \quad y = -\sqrt{6}$$

Putting these values of y in equation (iv)

| When $y = \sqrt{6}$ | When $y = -\sqrt{6}$ |
|---|---|
| $x = \frac{-4y}{3}$ | $x = \frac{-4y}{3}$ |
| $x = \frac{-4\sqrt{6}}{3}$ | $x = \frac{-4(-\sqrt{6})}{3}$ |
| $(x, y) = \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right)$ | $x = \frac{4\sqrt{6}}{3}$ |
| | $(x, y) = \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)$ |

So, the Solution set is

$$\left\{(3,1), (-3,-1), \left(\frac{-4\sqrt{6}}{3}, \sqrt{6}\right), \left(\frac{4\sqrt{6}}{3}, -\sqrt{6}\right)\right\}$$

**Q. 12** $x^2 + xy = 5$ ——— (i)

$y^2 + xy = 3$ ——— (ii)

Multiply equation (i) by 3 and equation (ii) by 5 and subtract them

$$3x^2 + 3xy = 15$$

$$\pm 5xy \pm 5y^2 = -15$$

$$3x^2 - 2xy - 5y^2 = 0$$

$$3x^2 - 5xy + 3xy - 5y^2 = 0$$

$$x(3x - 5y) + y(3x - 5y) = 0$$

Either $(3x - 5y)(x + y) = 0$

3x–5y=0 or $x + y = 0$

3x=5y or $\boxed{x = -y}$ ...(iv)

$\boxed{x = \frac{5y}{3}}$ ———(iii)

From equation (iv) put y = –x in equation (i)

$$(-y)^2 + (-y)y = 5$$

$$y^2 - y^2 = 5$$

$$0 \neq 5$$

Impossible

Now from equation (iii) put $x = \frac{5y}{3}$ in equation (i)

$$\left(\frac{5y}{3}\right)^2 + \frac{5y}{3}\times y = 5$$

$$\frac{25y^2}{9} + \frac{5y^2}{3} = 5$$

Multiply by 9

$$9\times\frac{25y^2}{9} + 9\times\frac{5y^2}{3} = 9\times 5$$

$$25y^2 + 15y^2 = 45$$

$$40y^2 = 45$$

$$y^2 = \frac{45}{40}$$

$$y^2 = \frac{9}{8}$$

$$y = \pm\sqrt{\frac{9}{8}}$$

$$= \pm\sqrt{\frac{3^2}{4\times 2}}$$

$$y = \pm\frac{3}{2\sqrt{2}}$$

$$y = \frac{3}{2\sqrt{2}} \quad or \quad y = \frac{-3}{2\sqrt{2}}$$

Now putting the value of y in equation (iii)

When $y = \frac{3}{2\sqrt{2}}$

Then $x = \frac{5}{3}\times\frac{3}{2\sqrt{2}}$

$$x = \frac{5}{2\sqrt{2}}$$

$$\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right)$$

When $y = \frac{-3}{2\sqrt{2}}$

Then $x = \frac{5}{3}\times\left(\frac{-3}{2\sqrt{2}}\right)$

$$x = \frac{-5}{2\sqrt{2}}$$

$$\left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}}\right)$$

Solution set is $\left\{\left(\frac{5}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}\right), \left(\frac{-5}{2\sqrt{2}}, \frac{-3}{2\sqrt{2}}\right)\right\}$

**Q. 13** $\mathbf{x^2 - 2xy = 7}$

$\mathbf{xy + 3y^2 = 2}$

**Solution:**

$$x^2 - 2xy = 7 \text{ ——— (i)}$$

$$xy + 3y^2 = 2 \text{ ——— (ii)}$$

Multiplying equation (i) by 2 and equation (ii) by 7 and subtracting them, we get

$$2x^2 - 4xy = 14$$

$$\pm 7xy \pm 21y^2 = 14$$

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(x - 7y)(2x + 3y) = 0$$

Either $x - 7y = 0$ or $2x + 3y = 0$

$x = 7y$ ——— (iii) or $2x = -3y$

or $x = \frac{-3}{2}y$ ——— (iv)

From equation (iii) Put $x = 7y$ in equation (i)

$$(7y)^2 - 2(7y)y = 7$$

$$49y^2 - 14y^2 = 7$$

$$35y^2 = 7$$

$$y^2 = \frac{7}{35}$$

$$y^2 = \frac{1}{5}$$

$$y = \pm\frac{1}{\sqrt{5}}$$

Either $y = \frac{1}{\sqrt{5}}$ or $y = \frac{-1}{\sqrt{5}}$

Putting these values of y in equation (iii)

When $y = \frac{1}{\sqrt{5}}$

$x = 7y$

Then $x = 7\left(\frac{1}{\sqrt{5}}\right)$

$$x = \frac{7}{\sqrt{5}}$$

$$(x, y) = \left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

When $y = \frac{-1}{\sqrt{5}}$

$x = 7y$

Then $x = 7\left(\frac{-1}{\sqrt{5}}\right)$

$$x = \frac{-7}{\sqrt{5}}$$

$$(x, y) = \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$$

From equation (iv) putting the value of x in equation (i)

$$\left(\frac{-3}{2}y\right)^2 - 2\left(\frac{-3}{2}y\right)y = 7$$

$$\frac{9}{4}y^2 + 3y^2 = 7$$

$$9y^2 + 12y^2 = 28$$

$$21y^2 = 28$$

$$y^2 = \frac{28}{21}$$

$$y^2 = \frac{4}{3}$$

$$\sqrt{y^2} = \pm\sqrt{\frac{4}{3}}$$

$$y = \pm\frac{2}{\sqrt{3}}$$

Either $y = \frac{2}{\sqrt{3}}$ or $y = \frac{-2}{\sqrt{3}}$

Putting these values of y in equation (iv)

When $y=\frac{2}{\sqrt{3}}$

Then $x=\frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right)$

$x=-\sqrt{3}$

$(x,y)=\left(-\sqrt{3},\frac{2}{\sqrt{3}}\right)$

When $y=\frac{-2}{\sqrt{3}}$

Then $x=-\frac{3}{2}\left(\frac{-2}{\sqrt{3}}\right)$

$x=\sqrt{3}$

$(x,y)=\left(\sqrt{3},\frac{-2}{\sqrt{3}}\right)$

So, the Solution set is

$$\left\{\left(\frac{7}{\sqrt{5}},\frac{1}{\sqrt{5}}\right),\left(\frac{-7}{\sqrt{5}},\frac{-1}{\sqrt{5}}\right),\left(-\sqrt{3},\frac{2}{\sqrt{3}}\right),\left(\sqrt{3},\frac{-2}{\sqrt{3}}\right)\right\}$$

## Problems Leading to Quadratic Equations

**Example 1:** Three less than certain number multiplied by 9 less than twice the number is 104: Find the number.

**Solution:** Let the required number be x. Then three less than the number = x – 3

and 9 less than twice the number = 2x – 9

According to the given condition, we have

$$(x-3)(2x-9)=104$$
$$2x^2-9x-6x+27=104$$
$$2x^2-15x+27-104=0$$
$$2x^2-15x-77=0$$

Factorizing, we get

$$2x^2-22x+7x-77=0$$
$$2x(x-11)+7(x-11)=0$$
$$(2x+7)(x-11)=0$$

Either $(2x+7)=0$ or $(x-11)=0$

$\Rightarrow \quad x=-\frac{7}{2}$ or $x=11$

i.e., $x=-\frac{7}{2}$ and 11 are the required numbers.

**Example 2:** The length of a rectangle is 4cm more than its breadth. If the area of the rectangle is $45cm^2$. Find its sides.

**Solution:** Let the breadth in cm be x.

Then the length in cm will be x + 4.

By the given condition

Rectangular area in $cm^2$ = 45, that is,

$$x(x+4)=45$$
$$x^2+4x-45=0$$
$$x^2+9x-5x-45=0$$
$$x(x+9)-5(x+9)=0$$
$$(x+9)(x-5)=0$$

$x+9=0$ or $x-5=0$

$x=-9$ or $x=5$

If x = 5, then $x+4=5+4=9$

(neglecting –ve value)

Thus the breadth is 5cm and length is 9cm.

**Example 3:** The sum of the co–ordinates of a point is 6 and the sum of their squares is 20. Find the co–ordinates of the point.

**Solutions:** Let (x, y) be the co–ordinates of the point. Then by the given conditions, we have

$$x+y=6 \quad \text{.............(i)}$$
$$x^2+y^2=20 \quad \text{.............(ii)}$$

From equation (i)

$$y=6-x$$

Putting $y=6-x$ in equation (ii) we get

$$x^2+(6-x)^2=20$$
$$x^2+36+x^2-12x-20=0$$
$$2x^2-12x+16=0 \quad \text{or}$$
$$2(x^2-6x+8)=0$$
$$\therefore \quad x^2-6x+8=0 \quad (\because 2\neq 0)$$
$$x^2-4x-2x+8=0$$
$$x(x-4)-2(x-4)=0$$
$$(x-4)(x-2)=0$$

Either $x-4=0$ or $x-2=0$

$x=4$ or $x=2$

Putting these values of x in equation (iii)

When x = 4: $y=6-4=2$, $y=2$

or

When x = 2: $y=6-2=4$, $y=4$

∴ the co–ordinates of the point are (4, 2) or (2, 4).