

## EXERCISE 3.1

**Q.1** Express the following as a ratio  $a : b$  and as a fraction in its simplest (lowest) form.

(i) Rs. 750 : Rs. 1250

$$= \frac{\text{Rs. } 750}{\text{Rs. } 1250} = \frac{75 \div 25}{125 \div 25} = \frac{3}{5} = 3 : 5$$

(ii) 450 cm : 3 m ( $\because 1 \text{ m} = 100 \text{ cm}$ )

$$450 \text{ cm} : 300 \text{ cm}$$

$$= \frac{450 \cancel{\text{cm}}}{300 \cancel{\text{cm}}} = \frac{45}{30} = \frac{45 \div 15}{30 \div 15} = \frac{3}{2} = 3 : 2$$

(iii) 4kg : 2kg 750g ( $\because 1 \text{ kg} = 1000 \text{ g}$ )

$$= 4000 \text{ g} : (2000 \text{ g} + 750 \text{ g})$$

$$= 4000 \text{ g} : 2750 \text{ g}$$

$$= 4000 \text{ g} : 2750 \text{ g}$$

$$= \frac{4000 \cancel{\text{g}}}{2750 \cancel{\text{g}}} = \frac{400 \div 25}{275 \div 25} = \frac{16}{11} = 16 : 11$$

(iv) 27min. 30 Second , 1 h

27min. 30 Second : 1 h

$$\boxed{\because 1 \text{ h} = 60 \text{ min} = 60 \times 60 \text{ s} = 3600 \text{ s}}$$

$$= (27 \times 60 \text{ s} + 30 \text{ s}) : 3600 \text{ s}$$

$$= (1620 \text{ s} + 30 \text{ s}) : 3600 \text{ s}$$

$$= 1650 \text{ s} : 3600 \text{ s}$$

$$= \frac{1650}{3600} = \frac{165}{360} \text{ (dividing by 15)}$$

$$= \frac{11}{24} = 11 : 24$$

(v)  $75^\circ : 225^\circ$

$$= \frac{75 \times 1^\circ}{225 \times 1^\circ} = \frac{75 \div 25}{225 \div 25} = \frac{3}{9}$$

$$= \frac{1}{3} = 1 : 3$$

**Q.2** In a class of 60 students, 25 students are girls and remaining students are boys.

Compute the ratio of

(i). Boys to total students

(ii). Boys to girls

**Solution:**

Number of students = 60

Number of girls = 25

Number of boys =  $60 - 25 = 35$

(i) Ratio of boys to the total students

Number of boys : Total students

$35 : 60$

$$= \frac{35}{60} = \frac{35 \div 5}{60 \div 5} = \frac{7}{12} = 7 : 12$$

(ii) Ratio of boys to the girls

Number of boys : Number of girls.

$35 : 25$

$$= \frac{35}{25} = \frac{35 \div 5}{25 \div 5} = \frac{7}{5} = 7 : 5$$

**Q.3** If  $3(4x-5y) = 2x-7y$ , find the ratio  $x : y$ .

**Solution:**  $3(4x-5y) = 2x-7y$

$$12x - 15y = 2x - 7y$$

$$12x - 2x = 15y - 7y$$

$$10x = 8y$$

$$\frac{x}{y} = \frac{8}{10}$$

$$\frac{x}{y} = \frac{8 \div 2}{10 \div 2}$$

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{x}{y} = \frac{4}{5}$$

$$x : y = 4 : 5$$

**Q.4** Find the value of  $p$ , if the ratios

$2p + 5 : 3p + 4$  and  $3 : 4$  are equal.

**Solution:** 1<sup>st</sup> ratio =  $2p + 5 : 3p + 4$

2<sup>nd</sup> ratio =  $3 : 4$

According to given condition

$$2p + 5 : 3p + 4 = 3 : 4$$

$$\frac{2p+5}{3p+4} = \frac{3}{4}$$

$$4(2p+5) = 3(3p+4)$$

$$8p+20 = 9p+12$$

$$20-12 = 9p-8p$$

$$8 = p$$

$\Rightarrow$

$$\boxed{p = 8}$$

**Q.5** If the ratios  $3x + 1 : 6 + 4x$  and  $2 : 5$  are equal. Find the value of  $x$ .

**Solution:** 1<sup>st</sup> Ratio =  $3x + 1 : 6 + 4x$

$$2^{\text{nd}} \text{ Ratio} = 2 : 5$$

According to given condition

$$3x + 1 : 6 + 4x = 2 : 5$$

$$\frac{3x+1}{6+4x} = \frac{2}{5}$$

$$5(3x+1) = 2(6+4x)$$

$$15x+5 = 12+8x$$

$$15x-8x = 12-5$$

$$7x = 7$$

$$x = \frac{7}{7} = 1$$

$$\boxed{x = 1}$$

**Q.6** Two numbers are in the ratio  $5 : 8$ . If 9 is added to each number, we get a new ratio  $8 : 11$ . Find the numbers.

**Solution:** Ratio between two numbers =  $5 : 8$

Number to be added = 9

New ratio =  $8 : 11$

Let 1<sup>st</sup> Number =  $5x$

2<sup>nd</sup> Number =  $8x$

**By condition:**

$$5x + 9 : 8x + 9 = 8 : 11$$

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$

$$11(5x+9) = 8(8x+9)$$

$$55x+99 = 64x+72$$

$$99-72 = 64x-55x$$

$$27 = 9x$$

$$\frac{27}{9} = x$$

$$3 = x \Rightarrow x = 3$$

Thus 1<sup>st</sup> number =  $5x = 5(3) = 15$

2<sup>nd</sup> number =  $8x = 8(3) = 24$

Required numbers are 15 and 24.

**Q.7** If 10 is added in each number of the ratio  $4 : 13$ , we get a new ratio  $1 : 2$ . What are the numbers?

**Solution:** Ratio of two numbers =  $4 : 13$

Number to be added = 10

New ratio =  $1 : 2$

Let 1<sup>st</sup> number =  $4x$

2<sup>nd</sup> number =  $13x$

**By condition:**

$$4x + 10 : 13x + 10 = 1 : 2$$

$$\frac{4x+10}{13x+10} = \frac{1}{2}$$

$$2(4x+10) = 1(13x+10)$$

$$8x+20 = 13x+10$$

$$20-10 = 13x-8x$$

$$10 = 5x$$

$$\frac{10}{5} = x$$

$$2 = x \Rightarrow \boxed{x = 2}$$

Thus 1<sup>st</sup> number =  $4x = 4(2) = 8$

2<sup>nd</sup> number =  $13x = 13(2) = 26$

Required numbers are 8 and 26.

**Q.8** Find the cost of 8 kg of mangoes, if 5 kg of mangoes cost Rs. 250.

**Solution:** Mangoes = 5kg

Price = Rs. 250

Mangoes = 8kg

Let Price =  $x = ?$

We know that

$$5\text{kg} : 8\text{kg} = \text{Rs. } 250 : \text{Rs. } x$$

$$\frac{5}{8} = \frac{250}{x}$$

$$x \times 5 = 8 \times 250$$

$$x = \frac{8 \times 250}{5}$$

$$x = 8 \times 50$$

$$x = \text{Rs. } 400$$

**Q.9** If  $a : b = 7 : 6$ , find the value of  $3a + 5b : 7b - 5a$ .

**Solution:**  $a : b = 7 : 6$

$$\frac{a}{b} = \frac{7}{6}$$

$$3a + 5b : 7b - 5a = \frac{3a + 5b}{7b - 5a}$$

Dividing numerator and denominator by "b".

$$\frac{3a + 5b}{7b - 5a} = \frac{\frac{3a}{b} + 5}{7 - 5\frac{a}{b}}$$

$$= \frac{3\frac{a}{b} + 5}{7 - 5\frac{a}{b}}$$

Putting  $\frac{a}{b} = \frac{7}{6}$ , we get

$$\begin{aligned} &= \frac{3\left(\frac{7}{6}\right) + 5}{7 - 5\left(\frac{7}{6}\right)} \\ &= \frac{\frac{21}{6} + 5}{7 - \frac{35}{6}} = \frac{\frac{21 + 30}{6}}{\frac{42 - 35}{6}} = \frac{51}{7} \end{aligned}$$

$$3a + b : 7b - 5a = 51 : 7$$

**Q.10** Complete the following:

(i) If  $\frac{24}{7} = \frac{6}{x}$ , then  $4x = \dots\dots\dots$

(ii) If  $\frac{5a}{3x} = \frac{15b}{y}$ , then  $ay = \dots\dots\dots$

(iii) If  $\frac{9pq}{2\ell m} = \frac{18p}{5m}$ , then  $5q = \dots\dots\dots$

**Solution:**

(i)  $\frac{24}{7} = \frac{6}{x}$   
 $24x = 7 \times 6$   
 $6 \times 4x = 7 \times 6$

$$4x = \frac{7 \times 6}{6}$$

$$\boxed{4x = 7}$$

(ii)  $\frac{5a}{3x} = \frac{15b}{y}$

$$5ay = 15b(3x)$$

$$5ay = 45bx$$

$$ay = \frac{45bx}{5}$$

$$\boxed{ay = 9bx}$$

(iii)  $\frac{9pq}{2\ell m} = \frac{18p}{5m}$

$$5m(9pq) = 18p(2\ell m)$$

$$5q = \frac{36 p \ell m}{9pm}$$

$$5q = \frac{\cancel{36}^4 \cancel{p} \cancel{\ell} \cancel{m}}{\cancel{9} \cancel{p} \cancel{m}}$$

$$\boxed{5q = 4\ell}$$

**Q.11** Find x in the following proportions.

(i)  $3x - 2 : 4 :: 2x + 3 : 7$

(ii)  $\frac{3x - 1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

(iii)  $\frac{x - 3}{2} : \frac{5}{x - 1} :: \frac{x - 1}{3} : \frac{4}{x + 4}$

(iv)  $p^2 + pq + q^2 : x :: \frac{p^3 - q^3}{p + q} : (p - q)^2$

(v)  $8 - x : 11 - x :: 16 - x : 25 - x$

**Solution:**

(i)  $3x - 2 : 4 :: 2x + 3 : 7$

Product of Extremes = Product of Means

$$7(3x - 2) = 4(2x + 3)$$

$$21x - 14 = 8x + 12$$

$$21x - 8x = 12 + 14$$

$$13x = 26$$

$$x = \frac{26}{13}$$

$$\boxed{x = 2}$$

$$(ii) \quad \frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$$

Product of Extremes = Product of Means

$$\left(\frac{3x-1}{7}\right)\left(\frac{7}{5}\right) = \left(\frac{2x}{3}\right)\left(\frac{3}{5}\right)$$

$$\frac{3x-1}{5} = \frac{2x}{5}$$

$$3x-1 = \frac{2x}{5} \times 5$$

$$3x-1 = 2x$$

$$3x-2x = 1$$

$$\boxed{x = 1}$$

$$(iii) \quad \frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$$

Product of Extremes = Product of Means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right) = \left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$

$$\frac{4x-12}{2x+8} = \frac{5}{3}$$

$$3(4x-12) = 5(2x+8)$$

$$12x-36 = 10x+40$$

$$12x-10x = 40+36$$

$$2x = 76$$

$$x = \frac{76}{2}$$

$$\boxed{x = 38}$$

$$(iv) \quad p^2 + pq + q^2 : \frac{p^3 - q^3}{p+q} :: (p-q) : (p-q)$$

Product of Means = Product of Extremes

$$(x) \cdot \frac{(p^3 - q^3)}{(p+q)} = (p^2 + pq + q^2)(p-q)^2$$

$$x = \frac{(p^2 + pq + q^2)(p-q)^2(p+q)}{p^3 - q^3}$$

$$x = \frac{(p^2 + pq + q^2)(p-q)(p-q)(p+q)}{(p-q)(p^2 + pq + q^2)}$$

$$x = (p-q)(p+q)$$

$$\boxed{x = p^2 - q^2}$$

$$(v) \quad 8-x : 11-x :: 16-x : 25-x$$

Product of Extremes = Product of Means

$$(8-x)(25-x) = (11-x)(16-x)$$

$$200 - 8x - 25x + x^2 = 176 - 11x - 16x + x^2$$

$$200 - 33x + x^2 = 176 - 27x + x^2$$

$$200 - 176 = 33x - 27x$$

$$24 = 6x$$

$$\frac{24}{6} = x$$

$$4 = x$$

$$\Rightarrow \boxed{x = 4}$$

### Variation:

There are two types of variations:

(i) Direct variation

(ii) Inverse variation

#### (i) Direct variation:

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity in the same ratio, then this variation is called direct variation.

If a quantity  $y$  varies directly with regard to a quantity  $x$ , we say that  $y$  is directly proportional to  $x$  and is written as

$$y \propto x \text{ or } y = kx \text{ where } k \neq 0.$$

**Note:**

The sign  $\propto$  read as "varies as" is called the sign of proportionality or variation, while  $k \neq 0$  is known as constant of variation.

**Examples:**

(i) Faster the speed of a car, longer the distance it covers.

(ii) The smaller is the radius of the circle, smaller is the circumference.

**Example 1:** Find the relation between distance  $d$  of a body falling from rest varies directly as the square of the time  $t$ , neglecting air resistance. Find  $k$ , if  $d = 16$  feet for  $t = 1$  sec. Also derive a relation between  $d$  and  $t$ .