

EXERCISE 3.2

Q.1 If y varies directly as x , and $y = 8$ when $x = 2$, find

- (i) y in terms of x
- (ii) y when $x = 5$
- (iii) x when $y = 28$

Solution: $y \propto x$ $y = 8$ $y = ?$ $x = ?$
 $x = 2$ $x = 5$ $y = 28$

(i) $y \propto x$
 $y = kx$ (i)

To find k ,
 Put $y = 8$ and $x = 2$ in equation (i)

$$8 = k(2)$$

$$\frac{8}{2} = k$$

$$4 = k \Rightarrow \boxed{k = 4}$$

(ii) To find y ,

Put $k = 4$ in equation (i)

$$y = kx$$

$$y = 4x$$
(ii)

Now Put $x = 5$ in equation (ii)

$$y = 4x$$

$$y = 4(5)$$

$$\boxed{y = 20}$$

(iii) To find x ,

Put $y = 28$ in equation (i)

$$y = 4x$$

$$28 = 4(x)$$

$$28 = 4(x)$$

$$7 = x \Rightarrow \boxed{x = 7}$$

Q.2 If $y \propto x$, and $y = 7$ when $x = 3$ find

- (i) y in terms of x
- (ii) x when $y = 35$ and y when $x = 18$

Solution:

$y \propto x$ $y = 7$ $x = ?$ $x = 18$
 $x = 3$ $y = 35$ $y = ?$

(i) $y \propto x$
 $y = kx$ (i)

To find k ,

Put $y = 7$ and $x = 3$ in equation (i)

$$7 = k(3)$$

$$\frac{7}{3} = k \Rightarrow \boxed{k = \frac{7}{3}}$$

Put it in equation (i)

$$y = kx$$

$$y = \frac{7}{3}x$$
(ii)

To find x ,

Put $y = 35$ in equation (ii)

$$y = \frac{7}{3}x$$

$$35 = \frac{7}{3}x$$

$$\frac{3 \times 35}{7} = x$$

$$3 \times 5 = x$$

$$15 = x \Rightarrow \boxed{x = 15}$$

To find y ,

Put $k = \frac{7}{3}$ and $x = 18$ in equation (i)

$$y = kx$$

$$y = \frac{7}{3}(18)$$

$$y = 7(6)$$

$$\boxed{y = 42}$$

Q.3 If $R \propto T$ and $R = 5$ when $T = 8$, find the equation connecting R and T . Also find R when $T = 64$ and T when $R = 20$.

Solution: $R \propto T$ $R = 5$ $R = ?$ $R = 20$
 $T = 8$ $T = 64$ $T = ?$

$$R \propto T$$

$$R = KT$$
(i)

To find k ,

Put $R = 5$ and $T = 8$ in equation (i)

$$5 = K(8)$$

$$\frac{5}{8} = k \Rightarrow \boxed{k = \frac{5}{8}}$$

Put $k = \frac{5}{8}$ in equation (i)

$$R = kT$$

$$R = \frac{5}{8}T \dots\dots\dots(ii)$$

To find R,

Put $T = 64$ in equation (ii)

$$R = \frac{5}{8}T$$

$$R = \frac{5}{8}(64)$$

$$R = 5(8)$$

$$\boxed{R = 40}$$

To find T,

Put $K = \frac{5}{8}$ and $R = 20$ in equation (i)

$$R = KT$$

$$20 = \frac{5}{8}(T)$$

$$T = \frac{20 \times 8}{5}$$

$$4 \times 8 = T$$

$$32 = T \Rightarrow \boxed{T = 32}$$

Q.4 If $R \propto T^2$ and $R = 8$ when $T = 3$, find R when $T = 6$.

Solution: $\boxed{R \propto T^2}$ $\boxed{R = 8}$ $\boxed{R = ?}$
 $\boxed{T = 3}$ $\boxed{T = 6}$

$$R \propto T^2$$

$$R = kT^2 \dots\dots\dots(i)$$

To find k,

Put $R = 8$ and $T = 3$ in equation (i)

$$8 = k(3)^2$$

$$8 = k(9)$$

$$\frac{8}{9} = k \Rightarrow \boxed{k = \frac{8}{9}}$$

To find R,

Put $k = \frac{8}{9}$ and $T = 6$ in equation (i)

$$R = kT^2$$

$$R = \frac{8}{9}(6)^2$$

$$R = \frac{8(36)}{9}$$

$$R = 8(4) \Rightarrow \boxed{R = 32}$$

Q.5 $V \propto R^3$ and $V = 5$ when $R = 3$, find R, when $V = 625$.

Solution: $\boxed{V \propto R^3}$ $\boxed{V = 5}$ $\boxed{R = ?}$
 $\boxed{R = 3}$ $\boxed{V = 625}$

$$V \propto R^3$$

$$V = kR^3 \dots\dots\dots(i)$$

To find k,

Put $V = 5$ and $R = 3$ in equation (i)

$$5 = k(3^3)$$

$$5 = k(27)$$

$$\frac{5}{27} = k \Rightarrow k = \frac{5}{27}$$

Put $k = \frac{5}{27}$ in equation (i)

$$V = kR^3$$

$$V = \frac{5}{27}R^3 \dots\dots\dots(ii)$$

To find R,

Now put $V = 625$ in equation (ii)

$$(625) = \frac{5}{27}R^3$$

$$\frac{125}{27 \times 5} = R^3$$

$$27 \times 125 = R^3$$

$$R^3 = 3^3 \times 5^3$$

$$R^3 = (3 \times 5)^3$$

$$R^3 = (15)^3$$

Taking cube root of both sides

$$\sqrt[3]{R^3} = \sqrt[3]{15^3}$$

$$R^{\frac{3 \times 1}{3}} = 15^{\frac{3 \times 1}{3}}$$

$$\boxed{R = 15}$$

Q.6 If w varies directly as u^3 and $w = 81$ when $u = 3$. Find w , when $u = 5$.

Solution: $w \propto u^3$ $w = 81$ $w = ?$
 $u = 3$ $u = 5$

$$w \propto u^3$$

$$w = ku^3 \dots\dots\dots(i)$$

To find k,

Put $w = 81$ and $u = 3$ in equation (i)

$$81 = k(3)^3$$

$$81 = k(27)$$

$$\frac{81}{27} = k$$

$$3 = k \Rightarrow \boxed{k = 3}$$

Put $k = 3$ in equation (i)

$$w = ku^3$$

$$w = 3u^3 \dots\dots\dots(ii)$$

To find w,

Now put $u = 5$ in equation (ii)

$$w = 3(5)^3$$

$$w = 3(125)$$

$$\boxed{w = 375}$$

Q.7 If y varies inversely as x and $y = 7$ when $x = 2$, find y , when $x = 126$.

Solution: $y \propto \frac{1}{x}$ $y = 7$ $y = ?$
 $x = 2$ $x = 126$

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x} \dots\dots\dots(i)$$

To find k,

Put $y = 7$ and $x = 2$

$$7 = \frac{k}{2}$$

$$7 \times 2 = k$$

$$14 = k \Rightarrow \boxed{k = 14}$$

Put $k = 14$ in equation (i)

$$y = \frac{k}{x} = \frac{14}{x}$$

$$\Rightarrow y = \frac{14}{x} \dots\dots\dots(ii)$$

To find y,

Now put $x = 126$ in equation (ii)

$$y = \frac{14}{x} = \frac{14}{126} = \frac{14^1}{126^9}$$

$$\boxed{y = \frac{1}{9}}$$

Q.8 If $y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$, find x when $y = 24$.

Solution: $y \propto \frac{1}{x}$ $y = 4$ $x = ?$
 $x = 3$ $y = 24$

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x} \dots\dots\dots(i)$$

To find k,

Put $y = 4$ and $x = 3$ in equation (i)

$$4 = \frac{k}{3}$$

$$4 \times 3 = k$$

$$12 = k \Rightarrow \boxed{k = 12}$$

Put $k = 12$ in equation (i)

$$y = \frac{k}{x}$$

$$y = \frac{12}{x} \dots\dots\dots(ii)$$

To find x,

Now put $y = 24$ in equation (ii)

$$24 = \frac{12}{x}$$

$$24x = 12$$

$$x = \frac{12}{24} = \frac{12^1}{24^2}$$

$$\boxed{x = \frac{1}{2}}$$

Q.9 If $w \propto \frac{1}{z}$ and $w = 5$ when $z = 7$, find w when $z = \frac{175}{4}$.

Solution: $w \propto \frac{1}{z}$ $w = 5$ $w = ?$
 $z = 7$ $z = \frac{175}{4}$

$w \propto \frac{1}{z}$ (given)

$w = \frac{k}{z}$ (i)

To find k,

Put $w = 5$ and $z = 7$ in equation (i)

$5 = \frac{k}{7}$

$5 \times 7 = k$

$35 = k \Rightarrow \boxed{k = 35}$

Put $K = 35$ in equation (i)

$w = \frac{k}{z} = \frac{35}{z} \Rightarrow w = \frac{35}{z}$(ii)

To find w,

Now put $z = \frac{175}{4}$ in equation (ii)

$w = \frac{35}{z} = 35 \div z$

$w = 35 \div \frac{175}{4}$

$w = 35 \times \frac{4}{175}$

$w = \frac{140}{175} = 1 \times \frac{4}{175}$

$w = \frac{4}{5}$

Q.10 $A \propto \frac{1}{r^2}$ and $A = 2$ when $r = 3$, find r when $A = 72$.

Solution: $A \propto \frac{1}{r^2}$ $A = 2$ $A = 72$
 $r = 3$ $r = ?$

$A \propto \frac{1}{r^2}$

$A = \frac{k}{r^2}$ (i)

To find k,

Put $A = 2$ and $r = 3$ in equation (i)

$2 = \frac{k}{(3)^2}$

$2 = \frac{k}{9}$

$2 \times 9 = k$

$18 = k \Rightarrow \boxed{k = 18}$

Put $k = 18$ in equation (i)

$A = \frac{k}{r^2}$

$A = \frac{18}{r^2}$(ii)

To find r,

Now put $A = 72$ in equation (ii)

$72 = \frac{18}{r^2}$

$72r^2 = 18$

$r^2 = \frac{18}{72}$

$r^2 = \frac{1}{4}$

Taking square root of both sides

$\sqrt{r^2} = \pm \sqrt{\frac{1}{4}}$

$r = \pm \frac{1}{2}$

Q.11 $a \propto \frac{1}{b^2}$ and $a = 3$ when $b = 4$, find a , when $b = 8$.

Solution: $a \propto \frac{1}{b^2}$ $a = 3$ $a = ?$
 $b = 4$ $b = 8$

$a \propto \frac{1}{b^2}$

$a = \frac{k}{b^2}$(i)

To find k,

Put $a = 3$ and $b = 4$ in equation (i)

$$3 = \frac{k}{(4)^2}$$

$$3 = \frac{k}{16}$$

$$3 \times 16 = k$$

$$48 = k \Rightarrow \boxed{k = 48}$$

Put $k = 48$ in equation(i)

$$a = \frac{k}{b^2} \Rightarrow a = \frac{48}{b^2} \text{(ii)}$$

To find a,

Now, put $b = 8$ in equation(ii)

$$a = \frac{48}{b^2} = \frac{48}{(8)^2} = \frac{48}{64}$$

$$\boxed{a = \frac{3}{4}}$$

Q.12 $V \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$, find V

when $r = 6$ and r when $V = 320$.

Solution:

$V \propto \frac{1}{r^3}$	$V = 5$ $r = 3$	$V = ?$ $r = 6$	$V = 320$ $r = ?$
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$$V \propto \frac{1}{r^3}$$

$$V = \frac{k}{r^3} \text{(i)}$$

To find k,

Put $V = 5$ and $r = 3$ in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$5 \times 27 = k \Rightarrow \boxed{k = 135}$$

Put $k = 135$ in equation(i)

$$V = \frac{135}{r^3} \text{(ii)}$$

To find V, Put $r = 6$ in equation (ii)

$$V = \frac{135}{(6)^3} = \frac{135}{216}$$

$$\boxed{V = \frac{5}{8}} \text{ (Dividing by 27)}$$

To find r, put $V = 320$ in equation (ii)

$$V = \frac{135}{r^3}$$

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320} = \frac{27}{64}$$

$$r^3 = \frac{3^3}{4^3}$$

$$r^3 = \left(\frac{3}{4}\right)^3 \text{ taking cube root}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3}{4}\right)^3}$$

$$\boxed{r = \frac{3}{4}}$$

Q.13 $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$ find m

when $n = 6$ and n when $m = 432$.

Solution:

$m \propto \frac{1}{n^3}$	$m = 2$ $n = 4$	$m = ?$ $n = 6$	$n = ?$ $m = 432$
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$$m \propto \frac{1}{n^3}$$

$$m = \frac{k}{n^3} \text{(i)}$$

To find k,

Put $m = 2$ and $n = 4$ in equation (i)

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$2 \times 64 = k$$

$$128 = k \Rightarrow \boxed{k = 128}$$

Put $k = 128$ in equation(i)

$$m = \frac{128}{n^3} \text{(ii)}$$

To find m,

put $n = 6$ in equation (ii)

$$m = \frac{128}{(6)^3} = \frac{128}{216} = \frac{32}{54} = \frac{16}{27}$$

$$\boxed{m = \frac{16}{27}}$$

$$3 = \frac{k}{(-4)^2}$$

$$3 = \frac{k}{16}$$

$$3 \times 16 = k$$

$$48 = k \Rightarrow \boxed{k=48}$$

Put $k = 48$ in equation(i)

$$a = \frac{k}{b^2} \Rightarrow a = \frac{48}{b^2} \dots\dots\dots(ii)$$

To find a,

Now, put $b = 8$ in equation(ii)

$$a = \frac{48}{b^2} = \frac{48}{(8)^2} = \frac{48}{64}$$

$$\boxed{a = \frac{3}{4}}$$

Q.12 $V \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$, find V

when $r = 6$ and r when $V = 320$.

Solution:

$V \propto \frac{1}{r^3}$	$V = 5$ $r = 3$	$V = ?$ $r = 6$	$v = 320$ $r = ?$
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$$V \propto \frac{1}{r^3}$$

$$V = \frac{k}{r^3} \dots\dots\dots(i)$$

To find k,

Put $V = 5$ and $r = 3$ in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$5 \times 27 = k \Rightarrow \boxed{k=135}$$

Put $k = 135$ in equation(i)

$$V = \frac{135}{r^3} \dots\dots\dots(ii)$$

To find V, Put $r = 6$ in equation (ii)

$$V = \frac{135}{(6)^3} = \frac{135}{216}$$

$$\boxed{V = \frac{5}{8}} \text{ (Dividing by 27)}$$

To find r, put $v = 320$ in equation (ii)

$$V = \frac{135}{r^3}$$

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320} = \frac{27}{64}$$

$$r^3 = \frac{3^3}{4^3}$$

$$r^3 = \left(\frac{3}{4}\right)^3 \text{ taking cube root}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3}{4}\right)^3}$$

$$\boxed{r = \frac{3}{4}}$$

Q.13 $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$ find

when $n = 6$ and n when $m = 432$.

Solution:

$m \propto \frac{1}{n^3}$	$m = 2$ $n = 4$	$m = ?$ $n = 6$	$n = ?$ $m = 432$
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$$m \propto \frac{1}{n^3}$$

$$m = \frac{k}{n^3} \dots\dots\dots(i)$$

To find k,

Put $m = 2$ and $n = 4$ in equation (i)

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$2 \times 64 = k$$

$$128 = k \Rightarrow \boxed{k = 128}$$

Put $k = 128$ in equation(i)

$$m = \frac{128}{n^3} \dots\dots\dots(ii)$$

To find m,

put $n = 6$ in equation (ii)

$$m = \frac{128}{(6)^3} = \frac{128}{216} = \frac{32}{54} = \frac{16}{27}$$

$$\boxed{m = \frac{16}{27}}$$

$$3 = \frac{k}{(-4)^2}$$

$$3 = \frac{k}{16}$$

$$3 \times 16 = k$$

$$48 = k \Rightarrow \boxed{k=48}$$

Put $k = 48$ in equation(i)

$$a = \frac{k}{b^2} \Rightarrow a = \frac{48}{b^2} \dots\dots\dots(ii)$$

To find a,

Now, put $b = 8$ in equation(ii)

$$a = \frac{48}{b^2} = \frac{48}{(8)^2} = \frac{48}{64}$$

$$\boxed{a = \frac{3}{4}}$$

Q.12 $V \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$, find V

when $r = 6$ and r when $V = 320$.

Solution:

$V \propto \frac{1}{r^3}$	$V = 5$ $r = 3$	$V = ?$ $r = 6$	$v = 320$ $r = ?$
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$$V \propto \frac{1}{r^3}$$

$$V = \frac{k}{r^3} \dots\dots\dots(i)$$

To find k,

Put $V = 5$ and $r = 3$ in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$5 \times 27 = k \Rightarrow \boxed{k=135}$$

Put $k = 135$ in equation(i)

$$V = \frac{135}{r^3} \dots\dots\dots(ii)$$

To find V, Put $r = 6$ in equation (ii)

$$V = \frac{135}{(6)^3} = \frac{135}{216}$$

$$\boxed{V = \frac{5}{8}} \text{ (Dividing by 27)}$$

To find r, put $v = 320$ in equation (ii)

$$V = \frac{135}{r^3}$$

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320} = \frac{27}{64}$$

$$r^3 = \frac{3^3}{4^3}$$

$$r^3 = \left(\frac{3}{4}\right)^3 \text{ taking cube root}$$

$$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3}{4}\right)^3}$$

$$\boxed{r = \frac{3}{4}}$$

Q.13 $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$ find

when $n = 6$ and n when $m = 432$.

Solution:

$m \propto \frac{1}{n^3}$	$m = 2$ $n = 4$	$m = ?$ $n = 6$	$n = ?$ $m = 432$
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$$m \propto \frac{1}{n^3}$$

$$m = \frac{k}{n^3} \dots\dots\dots(i)$$

To find k,

Put $m = 2$ and $n = 4$ in equation (i)

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$2 \times 64 = k$$

$$128 = k \Rightarrow \boxed{k = 128}$$

Put $k = 128$ in equation(i)

$$m = \frac{128}{n^3} \dots\dots\dots(ii)$$

To find m,

put $n = 6$ in equation (ii)

$$m = \frac{128}{(6)^3} = \frac{128}{216} = \frac{32}{54} = \frac{16}{27}$$

$$\boxed{m = \frac{16}{27}}$$

To find n,

Put $m = 432$ in equation (ii)

$$m = \frac{128}{n^3}$$

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432} \text{ (dividing by 16)}$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \frac{2^2}{3^3}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

Taking cube of root

$$\sqrt[3]{n^3} = \sqrt[3]{\left(\frac{2}{3}\right)^3}$$

$$n = \frac{2}{3}$$

Third Proportional

If three quantities a, b and c are related as $a : b :: b : c$.

Then c is called the third proportion.

Example 1: Find a third proportional of x + y and $x^2 - y^2$.

Solution: Let c be the third proportional.

then $x + y : x^2 - y^2 :: x^2 - y^2 : c$

Product of Extremes = Product of Means

$$c(x + y) = (x^2 - y^2)(x^2 - y^2)$$

$$c = \frac{(x^2 - y^2)(x^2 - y^2)}{x + y} = \frac{(x^2 - y^2)(x - y)(x + y)}{x + y}$$

$$c = (x + y)(x - y)(x - y)$$

$$c = (x + y)(x - y)^2$$

Fourth Proportional

If four quantities a, b, c and d are related as

$$a : b :: c : d$$

then d is called the fourth proportional.

Example 2: Find fourth proportional of $a^3 - b^3$, $a + b$ and $a^2 + ab + b^2$

Solution: Let x be the fourth proportional

then $(a^3 - b^3) : (a + b) :: (a^2 + ab + b^2) : x$

Product of Extremes = Product of Means:

$$x(a^3 - b^3) = (a + b)(a^2 + ab + b^2)$$

$$x = \frac{(a + b)(a^2 + ab + b^2)}{a^3 - b^3}$$

$$x = \frac{(a + b)(\cancel{a^2 + ab + b^2})}{(a - b)(\cancel{a^2 + ab + b^2})}$$

$$x = \frac{a + b}{a - b}$$

Mean Proportional

If three quantities a, b and c are related as $a : b :: b : c$, then b is called the mean proportional.

Example 3: Find the mean proportional of $9p^6q^4$ and r^8 .

Solution: Let m be the mean proportional, then $9p^6q^4 : m :: m : r^8$

Product of Extremes = Product of Means:

$$\text{or } m \cdot m = 9p^6q^4(r^8)$$

$$m^2 = 9p^6q^4r^8$$

$$\sqrt{m^2} = \pm \sqrt{9p^6q^4r^8}$$

$$m = \pm (3^2 p^6 q^4 r^8)^{\frac{1}{2}}$$

$$2 \times \frac{1}{2} \quad 6 \times \frac{1}{2} \quad 4 \times \frac{1}{2} \quad 8 \times \frac{1}{2}$$

$$m = \pm 3 p q r$$

$$m = \pm 3 p^3 q^2 r^4$$

Continued Proportion

If three quantities a, b and c are related as

$$a : b :: b : c$$

Where a is first, b is the mean and c is the third proportional, then a, b and c are in continued proportion.

Example 4: Find p, if 12, p and 3 are in continued proportion.

Solution: Since 12, p and 3 are in continued proportion.

$$\therefore 12 : p :: p : 3 \text{ i.e.}$$

Product of means = Product of extremes

$$p \cdot p = (12)(3)$$

$$\Rightarrow p^2 = 36$$

$$\sqrt{p^2} = \pm \sqrt{36}$$

$$\text{Thus } p = \pm 6$$