EXERCISE 3.5

Q.1 If s varies directly as u^2 and inversely as v and s = 7, when u = 3, v = 2. Find the value of s when u = 6 and v = 10

Solution: $s \propto \dfrac{u^2}{v}$ | s = 7, u = 3, v = 2 | s = ?, u = 6, v = 10

$$s \propto \frac{u^2}{v}$$

$$s = \frac{ku^2}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put s = 7 = , u = 3 , v = 2 in equation (i)

$$7 = \frac{k(3)^2}{2}$$

$$7 \times 2 = k(9)$$

$$\frac{14}{9} = k \quad \Rightarrow \quad k = \frac{14}{9}$$

Put $k = \dfrac{14}{9}$ in equation (i)

$$s = \frac{14u^2}{9v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

To find s,

Now put, u=6 and v=10 in equation (i)

$$s = \frac{14u^2}{9v}$$

$$s = \frac{14(6)^2}{9(10)}$$

$$s = \frac{14 \times \cancel{36}^{\,2}}{\cancel{90}_{\,5}} \quad \text{(dividing by 18)}$$

$$s = \frac{14 \times 2}{5}$$

$$\boxed{s = \frac{28}{5}}$$

Q.2 If w varies jointly as x, y^2 and z and w = 5 when x = 2, y = 3, z = 10. Find w when x = 4, y = 7 and z = 3.

Solution:

$w \propto xy^2z$ | w = 5, x = 2, y = 3, z = 10 | w = ?, x = 4, y = 7, z = 3

$$w \propto xy^2z$$

$$w = kxy^2z \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put w = 5, x = 2, y = 3 and z = 10 in equation (i)

$$5 = k(2)(3)^2(10)$$

$$5 = k(20)(9)$$

$$5 = k(180)$$

$$\frac{5}{180} = k$$

$$\Rightarrow \quad k = \frac{\cancel{5}^{\,1}}{\cancel{180}_{\,36}}$$

$$k = \frac{1}{36}$$

Put $k = \dfrac{1}{36}$ in equation (i)

$$w = \frac{1}{36}xy^2z \ldots\ldots\ldots\ldots (ii)$$

To find w,

Now put x = 4, y = 7, z = 3 in equation (ii)

$$w = \frac{1}{36}xy^2z$$

$$w = \frac{1}{36}(4)(7)^2(3)$$

$$w = \frac{1}{36}(4)(49)(3)$$

$$w = \frac{1}{\cancel{36}_{\,3}}(49) \times \cancel{12}^{\,1}$$

$$w = \frac{1}{3}(49)$$

$$\boxed{w = \frac{49}{3}}$$

Q.3 If y varies directly as x^3 and inversely as z^2 and t, and y = 16 when x = 4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

Solution:

$y \propto \frac{x^3}{z^2 t}$	$y = 16, x = 4$; $z = 2, t = 3$	$y = ?, x = 2$; $z = 3, t = 4$

$$y \propto \frac{x^3}{z^2 t}$$

$$y = \frac{kx^3}{z^2 t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

To find k,

Put y = 16, x = 4, z = 2 and t = 3 in equation (i)

$$16 = \frac{k(4)^3}{(2)^2(3)}$$

$$16 = \frac{k(64)}{4\times 3}$$

$$\frac{16\times 4\times 3}{64} = k$$

$$\frac{\cancel{64}}{\cancel{64}}\times 3 = k \quad \Rightarrow \quad k = 3$$

Put k = 3, in equation (i)

$$y = \frac{3x^3}{z^2 t} \ldots\ldots\ldots\ldots\text{(ii)}$$

To find y,

Now, put x = 2, z = 3 and t = 4 in equation (ii)

$$y = \frac{3\times(2)^3}{(3)^2(4)}$$

$$y = \frac{3\times 8}{9\times 4}$$

$$y = \frac{24}{36}$$

$$y = \frac{\overset{2}{\cancel{24}}}{\underset{3}{\cancel{36}}} \ (\textit{dividing by } 12)$$

$$\boxed{y = \frac{2}{3}}$$

Q.4 If u varies directly as x^2 and inversely as product yz^3, and u=2, when x = 8, y = 7, z = 2. Find the value of u when x = 6, y = 3, z = 2.

Solution:

$u \propto \frac{x^2}{yz^3}$	$u = 2, x = 8$; $y = 7, z = 2$	$u = ?, x = 6$; $y = 3, z = 2$

$$u \propto \frac{x^2}{yz^3}$$

$$u = \frac{kx^2}{yz^3} \quad \ldots\ldots\ldots\ldots\text{(i)}$$

To find k,

Put u = 2, x = 8, y = 7 and z = 2 in equation(i)

$$2 = \frac{k(8)^2}{7(2)^3}$$

$$2 = \frac{k64}{7(8)}$$

$$\frac{2(8)(7)}{64} = k$$

$$\frac{\cancel{16}^{1}(7)}{\cancel{64}\,4} = k$$

$$\frac{7}{4} = k \quad \Rightarrow \quad k = \frac{7}{4}$$

Put $k = \frac{7}{4}$ in equation (i)

$$u = \frac{7x^2}{4yz^3} \ldots\ldots\ldots\ldots\text{(ii)}$$

To find u,

Now put x = 6, y = 3, z = 2 in equation (ii)

$$u = \frac{7x^2}{4yz^3} = \frac{7(6)^2}{4(3)(2)^3}$$

$$u = \frac{7(\cancel{36}^{3})}{{}_{1}\cancel{12}\times 8}$$

$$u = \frac{7(3)}{8}$$

$$\boxed{u = \frac{21}{8}}$$

Q.5 If v varies directly as the product xy^3 and inversely as z^2 and v = 27 when x = 7, y = 6, z = 7. Find the value of V when x = 6; y = 2, z = 3.

Solution: $v \propto \frac{xy^3}{z^2}$ | $v = 27, x = 7$, $y = 6, z = 7$ | $v = ?, x = 6$, $y = 2, z = 3$

$$v \propto \frac{xy^3}{z^2}$$

$$v = \frac{kxy^3}{z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

To find k,

Put v = 27, x = 7, y = 6, z = 7 in equation(i)

$$27 = \frac{k(7)(6)^3}{(7)^2}$$

$$27 = \frac{k(216)}{7}$$

$$\frac{\cancel{27}^{1} \times 7}{\cancel{216}_{8}} = k$$

$$\Rightarrow \quad k = \frac{7}{8}$$

Put $k = \frac{7}{8}$ in equation (i)

$$v = \frac{7xy^3}{8z^2} \quad \ldots\ldots\ldots\ldots (ii)$$

To find v,

Now, put x = 6, y = 2, z = 3 in equation (ii).

$$v = \frac{7}{8}\frac{(6)(2)^3}{(3)^2}$$

$$v = \frac{7}{\cancel{8}}\frac{(6)(\cancel{8})}{9}$$

$$v = \frac{\cancel{42}^{14}}{\cancel{9}_{3}}$$

$$\Rightarrow \quad \boxed{v = \frac{14}{3}}$$

Q.6 If w varies inversely as the cube of U, and w = 5 when U = 3. Find w, when U = 6.

Solution: $w \propto \frac{1}{u^3}$ | $w = 5$, $u = 3$ | $w = ?$, $u = 6$

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

To find k,

Put w = 5 and u = 3 in equation (i)

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$\Rightarrow \quad k = 27 \times 5$$

$$k = 135$$

Put k = 135 in equation (i)

$$w = \frac{k}{u^3}$$

$$w = \frac{135}{u^3} \quad \ldots\ldots\ldots\ldots (ii)$$

To find w,

Now, Put u = 6 in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5 \times \cancel{27}}{8 \times \cancel{27}}$$

$$\boxed{w = \frac{5}{8}}$$

K–Method

If $a : b :: c : d$ is a proportion, then putting each ratio equal to k.

i.e $\frac{a}{b} = \frac{c}{d} = k$

$\frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk$ and $c = dk$

Using the above equations, we can solve certain problems relating to proportions more easily. This method is known as k–method.

Example 1: If $a : b = c : d$, then show that

$$\frac{3a+2b}{3a-2b} = \frac{3c+2d}{3c-2d}$$

Solution: $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

$\frac{a}{b} = k$ and $\frac{c}{d} = k$

Then $\boxed{a = bk \text{ and } c = dk}$

$$\text{L.H.S} = \frac{3a+2b}{3a-2b} = \frac{3bk+2b}{3bk-2b} = \frac{b(3k+2)}{b(3k-2)}$$

$$= \frac{3k+2}{3k-2} \text{..................(i)}$$

$$\text{R.H.S} = \frac{3c+2d}{3c-2d} = \frac{3dk+2d}{3dk-2d} = \frac{d(3k+2)}{d(3k-2)}$$

$$= \frac{3k+2}{3k-2} \text{...........(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

i.e., $\frac{3a+2b}{3a-2b} = \frac{3c+2d}{3c-2d}$

Example 2: If $a : b = c : d$, then show that

$pa + qb : ma - nb = pc + qd : mc - nd$

Solution: Let $\frac{a}{b} = \frac{c}{d} = k$, then

$\frac{a}{b} = k$ and $\frac{c}{d} = k$

$\boxed{a = bk \text{ and } c = dk}$

$$\text{L.H.S} = pa+qb : ma-nb = \frac{pa+qb}{ma-nb} = \frac{pkb+qb}{mkb-nb}$$

$$= \frac{b(pk+q)}{b(mk-n)} = \frac{pk+q}{mk-n} \text{.................(i)}$$

$$\text{R.H.S} = pc+qd : mc-nd = \frac{pc+qd}{mc-nd} = \frac{pkd+qd}{mkd-nd}$$

$$= \frac{d(pk+q)}{d(mk-n)} = \frac{pk+q}{mk-n} \text{............(ii)}$$

From (i) and (ii) L.H.S = R.H.S

$pa + qb : ma - nb = pc + qd : mc - nd$

Example 3:

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then show that

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$$

Solution: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

When $\frac{a}{b} = k, \frac{c}{d} = k$ and $\frac{e}{f} = k$

$\boxed{a = bk, c = dk \text{ and } e = fk}$

Let

$$\text{L.H.S} = \frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{(bk)^3+(dk)^3+(fk)^3}{b^3+d^3+f^3}$$

$$= \frac{b^3k^3+d^3k^3+f^3k^3}{b^3+d^3+f^3} = k^3\left(\frac{(b^3+d^3+f^3)}{(b^3+d^3+f^3)}\right)$$

$$= k^3 \text{................(i)}$$

$$R.H.S = \frac{ace}{bdf} = \frac{(bk)(dk)(fk)}{bdf} = k^3\frac{(bdf)}{bdf}$$

$R.H.S = k^3$.....(ii)

From (i) and (ii)

L.H.S = R.H.S

i.e $\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$

Example 4:

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then show that

$$\frac{a^2b+c^2d+e^2f}{ab^2+cd^2+ef^2}=\frac{a+c+e}{b+d+f}$$

Solution: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$$\frac{a}{b}=k\,,\frac{c}{b}=k\,,\ \frac{e}{f}=k$$

$$\boxed{a=bk\ ,c=dk \text{ and } e=fk}$$

To prove $a = bk,\quad c=dk,\quad e=fk$

$$\frac{a^2b+c^2d+e^2f}{ab^2+cd^2+ef^2}=\frac{a+c+e}{b+d+f}$$

$$\text{L.H.S} = \frac{a^2b+c^2d+e^2f}{ab^2+cd^2+ef^2}$$

$$=\frac{(bk)^2b+(dk)^2d+(fk^2)f}{(bk)b^2+(dk)d^2+(fk)f^2}=\frac{k^2b^3+k^2d^3+k^2f^3}{kb^3+kd^3+kf^3}$$

$$=\frac{k^2(b^3+d^3+f^3)}{k(b^3+d^3+f^3)}=k\text{..........(i)}$$

$$\text{R.H.S} = \frac{a+c+e}{b+d+f}=\frac{bk+dk+fk}{b+d+f}$$

$$=\frac{k(b+d+f)}{(b+d+f)}=k\text{............(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

Thus $\frac{a^2b+c^2d+e^2f}{ab^2+cd^2+ef^2}=\frac{a+c+e}{b+d+f}$