EXERCISE 3.6

Q.1 If a:b=c:d, (a,b,c,d ≠0), then show that

(i) $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

(ii) $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

(iii) $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $a^6+c^6 : b^6+d^6 = a^3c^3 : b^3d^3$

(v) $p(a+b)+qb : p(c+d)+qd = a : c$

(vi) $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

(vii) $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Solution:

(i) $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k, \text{ and } \frac{c}{d}=k$$

$\boxed{a = bk}$ and $\boxed{c = dk}$

$$L.H.S=\frac{4a-9b}{4a+9b}$$

$$=\frac{4bk-9b}{4bk+9b}$$

$$=\frac{b(4k-9)}{b(4k+9)}$$

$$L.H.S=\frac{4k-9}{4k+9} \qquad \text{............... (i)}$$

Now , taking

$$R.H.S=\frac{4c-9d}{4c+9d}$$

$$=\frac{4dk-9d}{4dk+9d}$$

$$=\frac{d(4k-9)}{d(4k+d)}$$

$$R.H.S=\frac{4k-9}{4k+9} \qquad \text{............... (ii)}$$

From equation (i) and (ii)

L.H.S = R.H.S

So $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

Hence proved

(ii) $\mathbf{\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}}$

Let a : b = c : d = k

$\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \frac{a}{b}=k \quad : \quad \frac{c}{d}=k$

$\boxed{a=bk}; \quad \boxed{c=dk}$

Let $L.H.S=\frac{6a-5b}{6a+5b}$

$=\frac{6bk-5b}{6bk+5b}$

$=\frac{b(6k-5)}{b(6k+5)}$

$L.H.S=\frac{6k-5}{6k+5}$ .... (i)

Now $R.H.S=\frac{6c-5d}{6c+5d}$

$=\frac{6dk-5d}{6dk+5d}$

$=\frac{d(6k-5)}{d(6k+5)}$

$R.H.S=\frac{6k-5}{6k+5}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So, $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

Hence proved

(iii) $\mathbf{\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}}$

Let a : b = c : d = k

$\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k \quad : \quad \frac{c}{d}=k$

$\boxed{a=bk}; \quad \boxed{c=dk}$

Let $L.H.S=\frac{a}{b}=\frac{\cancel{b}k}{\cancel{b}}$

L.H.S = k ———————— (i)

Now $R.H.S=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

$=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$=\sqrt{\frac{k^2\cancel{(b^2+d^2)}}{\cancel{(b^2+d^2)}}}$

$=\sqrt{k^2}$

$R.H.S=k$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

Hence proved

So $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $\mathbf{a^6+c^6:b^6+d^6=a^3c^3:b^3d^3}$

Let a : b = c : d = k

$\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k;\frac{c}{d}=k$

Let L.H.S = $a^6+c^6:b^6+d^6$

$=\frac{a^6+c^6}{b^6+d^6}=\frac{(bk)^6+(dk)^6}{b^6+d^6}$

$=\frac{b^6k^6+d^6k^6}{b^6+d^6}=\frac{k^6\cancel{(b^6+d^6)}}{\cancel{(b^6+d^6)}}$

L.H.S = k^6 .... (i)

Now, R.H.S = $a^3c^3:b^3d^3$

$=\frac{a^3c^3}{b^3d^3}=\frac{(bk)^3.(dk)^3}{b^3d^3}$

$=\frac{b^3k^3.d^3k^3}{b^3d^3}=\frac{\cancel{b^3d^3}.k^{3+3}}{\cancel{b^3d^3}}$

R.H.S = k^6 .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So $a^6+c^6:b^6+d^6=a^3c^3:b^3d^3$

Hence proved

(v) $\mathbf{p(a+b)+qb : p(c+d)+qd = a : c}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k \;;\; \frac{c}{d}=k$$

$$\boxed{a=bk} \;;\; \boxed{c=dk}$$

Let $L.H.S = p(a+b)+qb : p(c+d)+qd$

$$=\frac{p(a+b)+qb}{p(c+d)+qd}$$

$$=\frac{p(bk+b)+qb}{p(dk+d)+qd}$$

$$=\frac{pb(k+1)+qb}{pd(k+1)+qd}$$

$$=\frac{b[p(k+1)+q]}{d[p(k+1)+q]}$$

$$\text{L.H.S}=\frac{b}{d} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\text{R.H.S} = a : c = \frac{a}{c}$$

$$R.H.S=\frac{b\cancel{k}}{d\cancel{k}}$$

$$R.H.S=\frac{b}{d} \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

$$p(a+b)+qb : p(c+d)+qd = a : b$$

Hence proved

(vi) $\mathbf{a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k \;;\; \frac{c}{d}=k$$

$$\boxed{a=bk} \;;\; \boxed{c=dk}$$

Let $L.H.S=a^2+b^2 : \frac{a^3}{a+b}$

$$=\left[(bk)^2+b^2\right]\div\frac{(bk)^3}{bk+b}$$

$$=(b^2k^2+b^2)\times\frac{(bk+b)}{b^3k^3}$$

$$=b^2(k^2+1)\times\frac{b(k+1)}{b^3k^3}$$

$$=\frac{\cancel{b^3}(k^2+1)(k+1)}{\cancel{b^3}k^3}$$

$$L.H.S=\frac{(k^2+1)(k+1)}{k^3} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now $R.H.S=c^2+d^2 : \frac{c^3}{c+d}$

$$=\left[(dk)^2+d^2\right]\div\frac{(dk)^3}{(dk+d)}$$

$$=(d^2k^2+d^2)\times\frac{(dk+d)}{d^3k^3}$$

$$=d^2(k^2+1)\times\frac{d(k+1)}{d^3k^3}$$

$$=\frac{\cancel{d^3}(k^2+1)(k+1)}{\cancel{d^3}k^3}$$

$$R.H.S=\frac{(k^2+1)(k+1)}{k^3} \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

So $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

Hence proved

(vii) $\mathbf{\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}}$

Let $a : b = c : d = k$

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k, \quad \frac{c}{d}=k$$

$$\boxed{a=bk}, \quad \boxed{c=dk}$$

Let $L.H.S = \frac{a}{a-b} : \frac{a+b}{b}$

$$= \frac{bk}{bk-b} \div \frac{bk+b}{b}$$

$$= \frac{\not{b}k}{\not{b}(k-1)} \times \frac{\not{b}}{\not{b}(k+1)}$$

$$= \frac{k}{(k)^2-(1)^2}$$

$$L.H.S = \frac{k}{k^2-1} \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now $R.H.S = \frac{c}{c-d} : \frac{c+d}{d}$

$$= \frac{dk}{dk-d} \div \frac{dk+d}{d}$$

$$= \frac{\not{d}k}{\not{d}(k-1)} \times \frac{\not{d}}{\not{d}(k+1)}$$

$$= \frac{k}{(k)^2-(1)^2}$$

$$R.H.S = \frac{k}{k^2-1} \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

L.H.S = R.H.S

So $\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$

Hence proved

Q.2 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} (a, b, c, d, e, f \neq 0)$, then show that

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{2/3}$

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Solution:

(i) $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k$, $\frac{c}{d} = k$, $\frac{e}{f} = k$

| a = bk , c = dk , e = fk |

$$L.H.S = \frac{a}{b} = \frac{\not{b}k}{\not{b}} = k$$

$$L.H.S = k \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now $R.H.S = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

$$= \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$

$$= \sqrt{\frac{k^2(b^2+d^2+f^2)}{(b^2+d^2+f^2)}}$$

$$= \sqrt{k^2}$$

$$R.H.S = k \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

R.H.S = R.H.S

i.e $\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb} = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k$, $\frac{c}{d} = k$, $\frac{e}{f} = k$

| $a = bk$; $c = dk$, $e = fk$ |

Let $L.H.S = \frac{ac+ce+ea}{bd+df+fb}$

$$= \frac{bk(dk)+dk(fk)+fk(bk)}{bd+df+fb}$$

$$= \frac{k^2bd+k^2df+k^2fb}{bd+df+fb}$$

$$= \frac{k^2(bd+df+fb)}{(bd+df+fb)}$$

$$L.H.S = k^2 \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now, $R.H.S = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

$= \left(\frac{bk.dk.fk}{bdf}\right)^{\frac{2}{3}} = \left(\frac{k^3.bdf}{bdf}\right)^{\frac{2}{3}}$

$= (k^3)^{\frac{2}{3}} = k^{3\times\frac{2}{3}}$

R.H.S $= k^2$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

So, $\frac{ac+ce+ea}{bd+df+fb} = \left(\frac{ace}{bdf}\right)^{\frac{2}{3}}$

Hence proved.

(iii) $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$

$\boxed{a = bk, \quad c = dk, \quad e = fk}$

Let $L.H.S = \frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb}$

$= \frac{bk.dk}{bd} + \frac{dk.fk}{df} + \frac{fk.bk}{fb}$

$= \frac{k^2 \cancel{bd}}{\cancel{bd}} + \frac{k^2 \cancel{df}}{\cancel{df}} + \frac{k^2 \cancel{fb}}{\cancel{fb}}$

$= k^2 + k^2 + k^2$

L.H.S $= 3k^2$ .... (i) Now

$R.H.S = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

$= \frac{\cancel{b^2}k^2}{\cancel{b^2}} + \frac{\cancel{d^2}k^2}{\cancel{d^2}} + \frac{\cancel{f^2}k^2}{\cancel{f^2}}$

$= k^2 + k^2 + k^2$

R.H.S $= 3k^2$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{df} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Real life problems based on variation

Example 1: **The strength "s" of a rectangular beam varies directly as the breadth b and the square of the depth d. If a beam 9cm wide and 12 cm deep with support 1200 ℓb. What weight a beam of 12cm wide and 9 cm deep will support?**

Solution:

By the joint variation, we have

$\boxed{s \propto bd^2}$

$s = 1200\ell b$	$s = ?$
$b = 9cm$	$b = 12cm$
$d = 12cm$	$d = 9cm$

$s \propto bd^2$

i.e $s = kbd^2$ ....(i)

To find k,

put s = 1200, b = 9 and d = 12 in equation (i)

$1200 = k(9)(12)^2$

$1200 = k(9)(144)$

$k = \frac{1200}{9\times144}$

$k = \frac{1200}{1296} = \frac{25}{27}$ (dividing by 48)

$\Rightarrow k = \frac{25}{27}$

Put $k = \frac{25}{27}$ in equation (i)

$s = \frac{25}{27}bd^2$..................(ii)

To find s,

Now put b = 12 and d = 9 in equation (ii)

$s = \frac{25}{27}(12)(9)^2$

$s = \frac{25(12)(9)(9)}{27}$

$s = \frac{24300}{27}$

$\boxed{s = 900\ \ell b}$

Example 2: The current in a wire varies directly as the electromotive force E and inversely as the resistance R. If I = 32 amperes, when E = 128 volts and R = 8 ohms. Find I, when E = 150 volts and R = 18 ohms.

Solution:

In joint variation, we have

$I \propto \frac{E}{R}$	I = 32 amp E = 128 volts R = 8 ohms	I = ? E = 150 volts R = 18 ohms

$$I \propto \frac{E}{R}, \qquad \text{i.e} \qquad I = \frac{kE}{R} \quad \ldots\ldots\ldots\text{(i)}$$

To find k,

Put I = 32, E = 128 and R = 8 in equation (i)

$$32 = \frac{k(128)}{8} \quad \Rightarrow \quad \frac{32\times8}{128} = k$$

$$\frac{256}{128} = k \quad \Rightarrow \quad k = 2$$

Put k = 2 in equation (i)

$$I = \frac{2E}{R} \quad \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

To find I,

Now put E = 150 and R = 18 in equation (ii)

$$I = \frac{2E}{R}$$

$$I = \frac{2(150)}{18} = \frac{\cancel{300}\,50}{\cancel{18}\,3} \quad \text{(dividing by "6")}$$

$$\boxed{I = \frac{50}{3}\text{ amp}}$$