EXERCISE 3.7

Q. 1 The surface area A of a cube varies directly as the square of the length ℓ of an edge and A = 27 square units when ℓ = 3 units. Find (i) A when ℓ = 4 units (ii) ℓ when A = 12 sq. units.

Solution: $\boxed{A \propto \ell^2}$ $\boxed{A = 27 \text{ units}, \ \ell = 3 \text{ units}}$

$\boxed{\text{(i)} \ A = ?, \ \ell = 4 \text{units}}$

$\boxed{\text{(ii)} \ \ell = ?, \ A = 12 \text{ units}}$

$A \propto \ell^2$

$A = k\ell^2$ .... (i)

To find k,

Put A = 27 and $\ell = 3$ in equation (i)

$27 = k(3)^2$

$27 = k(9)$

$\frac{27}{9} = k$

$3 = k$

$\Rightarrow$ $\boxed{k = 3}$

(i) To find A,

Put k = 3 and $\ell = 4$ in equation (i)

$A = 3(4)^2$

$A = 3(16)$

$\boxed{A = 48 \text{ square units}}$

(ii) To find ℓ,

Put k = 3 and A = 12 in equation (i)

$A = K\ell^2$

$12 = 3\ell^2$

$\frac{12}{3} = \ell^2$

$\ell^2 = 4$

$\sqrt{\ell^2} = \pm\sqrt{4}$

$\ell = \pm 2$ units

Since length is always taken positive so

$\boxed{\ell = 2 \text{ units}}$

Q. 2 The surface area S of the sphere varies directly as the square of radius r, and S = 16 π when r = 2. Find r when S = 36π.

Solution:

$\boxed{S \propto r^2}$ $\boxed{S = 16\pi,\ r = 2}$ $\boxed{r = ?\ ,\ S = 36\pi}$

$$S \propto r^2$$

$$S = kr^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put s = 16 π and r = 2 in equation (i)

$$16\pi = k(2)^2$$

$$16\pi = k(4)$$

$$\frac{\overset{4}{\cancel{16}}\pi}{\underset{1}{\cancel{4}}} = k$$

$$4\pi = k$$

$\Rightarrow$ $\boxed{k = 4\pi}$

To find s,

Put k = 4π in equation (i)

$$S = kr^2$$

$$S = 4\pi r^2 \ldots\ldots\ldots \text{(ii)}$$

To find r,

Now put S = 36π in equation (ii)

$$36\pi = 4\pi r^2$$

$$\frac{36\pi}{4\pi} = r^2$$

$$\frac{\overset{9}{\cancel{36}}\cancel{\pi}}{\underset{1}{\cancel{4}}\cancel{\pi}} = r^2$$

$$r^2 = 9$$

$$r^2 = 9$$

Taking square root

$$\sqrt{r^2} = \pm\sqrt{9}$$

$$r = \pm 3$$

Since r is radius of sphere which is positive quantity so $\boxed{r = 3 \text{ units}}$

Q.3 In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S, and F = 32 ℓb when S = 1.6 in. Find (i) S when F = 50 ℓb (ii) F when S=0.8 in.

Solution: $\boxed{F \propto S}$ $\boxed{F = 32\ \ell b\ ,\ S = 1.6 \text{ inch}}$

$\boxed{\text{(i)}\ S = ?\ ,\ F = 50\ \ell b}$

$\boxed{\text{(ii)}\ F = ?,\ S = 0.8 \text{inch}}$

$$F \propto S$$

$$F = kS \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

put F = 32 and s = 1.6 in equation (i)

$$32 = k(1.6)$$

$$\frac{32}{1.6} = k$$

$$20 = k$$

$$k = 20$$

(i) **To find s,**

Put k = 20 and F = 50 in equation. (i)

$$F = kS$$

$$50 = 20\,(S)$$

$$\frac{50}{20} = S$$

$$S = \frac{5}{2}$$

$$S = 2.5 \text{ inch}$$

(ii) To find F,

Put K = 20 and S = 0.8 inch in equation (i)

$$F = kS$$

$$F = 20(0.8)$$

$$F = \overset{2}{\cancel{20}}\,\frac{8}{\underset{1}{\cancel{10}}}$$

$$F = 2 \times 8$$

$$F = 16\ \ell b$$

Q.4 The intensity I of the light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12 ft. from the source, Find the intensity at a point 8ft. from the source.

Solution: $I \propto \frac{1}{d^2}$ | I = 20 cp, d = 12 ft | I = ?, d = 8 ft

$$I \propto \frac{1}{d^2}$$

$$I = \frac{k}{d^2} \quad \ldots\ldots\ldots\ldots (i)$$

To find k,

Put I = 20 and d = 12 ft in equation (i)

$$20 = \frac{k}{(12)^2}$$

$$20 = \frac{k}{144}$$

$$20 \times 144 = k$$

$$2880 = k$$

$$k = 2880$$

To find I,

Now Put k = 2880 and d = 8 ft in equation (i)

$$I = \frac{k}{d^2}$$

$$I = \frac{2880}{(8)^2}$$

$$I = \frac{2880}{64}$$

$I = 45$ Candle power

Q.5 The pressure P in a body of fluid, varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must he fluid be to exert a pressure of 9 lb/sq.in?

Solution:

$p \propto d$

d = 5ft, p = 2.25 ℓb/sq.inch

d = ?, p = 9 ℓb/sq.inch

$$p \propto d$$

$$p = kd \quad \ldots\ldots\ldots\ldots (i)$$

To find k,

Put p = 2.25 and d = 5 in equation (i)

$$2.25 = k(5)$$

$$\frac{2.25}{5} = k$$

$$\Rightarrow \quad k = 0.45$$

To find d,

Put k = 0.45 and p = 9 in equation (i)

$$p = kd$$

$$9 = 0.45\,(d)$$

$$\frac{9}{0.45} = d$$

$$d = \frac{900}{45} \Rightarrow \boxed{d = 20\text{ft}}$$

Q.6 Labour costs c varies jointly as the number of workers n and the average number of days d, if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600 workers for 18 days.

Solution: $c \propto nd$

c = Rs.286000	c = ?
n = 800, d = 13	n = 600, d = 18

$$c = knd \quad \ldots\ldots (i)$$

To find k,

Put the values of c, n and d in equation (i)

$$286000 = k(800)(13)$$

$$\frac{286000}{800 \times 13} = k$$

$$27.5 = k \Rightarrow \boxed{k = 27.5}$$

To find C,

Put k = 27.5, n = 600 and d = 18 in equation (i)

$$c = knd$$

$$c = 27.5 \times 600 \times 18$$

$$\boxed{c = \text{Rs. } 297000}$$

Q.7 The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length ℓ. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?

Solution: $c \propto \frac{d^4}{\ell^2}$

d = 6 inch	ℓ = ?
ℓ = 30ft	d = 4 inch
c = 63 tons	c = 28 tons

$$c \propto \frac{d^4}{\ell^2}$$

$$c = \frac{kd^4}{\ell^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put the values of d, ℓ and c in equation (i)

$$63 = \frac{k(6)^4}{(30)^2}$$

$$63 = \frac{k(1296)}{900}$$

$$\frac{63 \times 900}{1296} = k$$

$$\frac{56700}{1296} = k$$

$$43.75 = k$$

$$k = 43.75$$

To find ℓ,

Put K = 43.75 , d = 4 and c = 28 in equation (i)

$$c = \frac{kd^4}{\ell^2}$$

$$28 = \frac{43.75 \times (4)^4}{\ell^2}$$

$$\ell^2 \times 28 = 43.75 \times 256$$

$$\ell^2 = \frac{11200}{28}$$

$$\ell^2 = 400$$

Taking square root of both side

$$\sqrt{\ell^2} = \pm\sqrt{400}$$

$$\ell = \pm 20$$

Since length is always positive so

$$\boxed{\ell = 20\,ft}$$

Q.8 The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 ℓb through 40 ft, what power is required to lift 800 ℓb, through 120 ft in 40 sec?

Solution: $T \propto \frac{wd}{P}$

T = 25s, p = 4hp	p = ?, w = 800ℓb
w = 500ℓb, d = 40ft	d = 120ft, T = 40sec

$$T \propto \frac{wd}{p}$$

$$T = \frac{kwd}{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To find k,

Put the values of T, P, W and d in equation (i)

$$25 = \frac{k(500)(40)}{4}$$

$$\frac{25 \times 4}{500 \times 40} = k$$

$$\frac{100}{20000} = k$$

$$\frac{1}{200} = k \quad \Rightarrow \quad \boxed{k = \frac{1}{200}}$$

To find P,

Put, $k = \frac{1}{200}$, w = 800 ℓb, d = 120 ft,
T = 40s in equation (i)

$$T = \frac{kwd}{p}$$

$$40 = \frac{1}{200} \times \frac{800 \times 120}{p}$$

$$40 = \frac{480}{p}$$

$$p = \frac{480}{40}$$

$$p = \frac{480}{40} = 12$$

$$\Rightarrow \boxed{p = 12hp}$$