EXERCISE 4.1

Resolve into partial fractions.

Q.1 $\dfrac{7x-9}{(x+1)(x-3)}$

Solution: $\dfrac{7x-9}{(x+1)(x-3)}$

Let $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{A}{x+1} + \dfrac{B}{x-3}$ ...... (i)

Multiplying equation (i) by (x + 1) (x – 3)

$7x - 9 = A(x-3) + B(x+1)$ ...... (ii)

As equation (ii) is an identity which is true for all values of x.

Put $x - 3 = 0$ i.e $x = 3$ and

Put $x + 1 = 0$ i.e $x = -1$

Putting $x = 3$ and $x = -1$ in (ii) we get

For $x = 3$	For $x = -1$
$7(3)-9 = +B(3+1)$	$7(-1)-9 = A(-1-3)$
$21 - 9 = 4B$	$-7-9 = -4A$
$12 = 4B$	$-16 = -4A$
$\Rightarrow \boxed{B=3}$	$\Rightarrow \boxed{A=4}$

Putting the value of A and B in equation (i)

We get the required partial fractions as.

$$\frac{4}{x+1} + \frac{3}{x-3}$$

Thus $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{4}{x+1} + \dfrac{3}{x-3}$

Q.2 $\dfrac{x-11}{(x-4)(x+3)}$

Solution: $\dfrac{x-11}{(x-4)(x+3)}$

Let $\dfrac{x-11}{(x-4)(x+3)} = \dfrac{A}{x-4} + \dfrac{B}{x+3}$ .....(i)

Multiplying by (x–4)(x+3) on both sides, we get

$x - 11 = A(x+3) + B(x-4)$....... (ii)

As equation (ii) is an identity which is true for all value of x.

Putting $x + 3 = 0$ i.e $x = -3$

and $x - 4 = 0$ i.e $x = 4$ in (ii) we get

For $x = -3$	For $x = 4$
$-3-11 = B(-3-4)$	$4-11 = A(4+3)$
$-14 = -7B$	$-7 = 7A$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{x-4} + \frac{2}{x+3}$$

Q.3 $\dfrac{3x-1}{x^2-1}$

Solution: $\dfrac{3x-1}{x^2-1}$

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)}$$

Let $\dfrac{3x-1}{(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$ ......(i)

Multiplying both sides by (x–1)(x+1), we get

$3x - 1 = A(x+1) + B(x-1)$ .... (ii)

As equation (ii) is an identity which is true for all values of x.

Let $x + 1 = 0$ i.e $x = -1$ and

$x - 1 = 0$ i.e $x = 1$

Putting $x = -1$ and $x = 1$ in (ii) We get

For $x = 1$	For $x = -1$
$3(1) - 1 = A(1+1)$	$3(-1) - 1 = B(-1-1)$
$3 - 1 = 2A$	$-3 - 1 = -2B$
$2 = 2A$	$-4 = -2B$
$\Rightarrow \boxed{A=1}$	$\Rightarrow \boxed{B=2}$

Hence the required partial fractions are

$$\frac{3x-1}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{2}{x+1}$$

Q.4 $\dfrac{x-5}{x^2+2x-3}$

Solution: $\dfrac{x-5}{x^2+2x-3}$

$$\frac{x-5}{x^2+2x-3}=\frac{x-5}{x^2+3x-x-3}$$

$$=\frac{x-5}{x(x+3)-1(x+3)}$$

$$=\frac{x-5}{(x-1)(x+3)}$$

$$\frac{x-5}{(x-1)(x+3)}=\frac{A}{x-1}+\frac{B}{x+3}\text{ .... (i)}$$

Multiplying both sides by (x–1) (x+3), we get

$$x-5=A(x+3)+B(x-1)\text{........ (ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x+3=0 \Rightarrow x=-3$

and $x-1=0 \Rightarrow x=1$

Putting $x=-3$ and $x=1$ in equation (ii) we get

For $x=-3$	For $x=1$
$-3-5 = +B(-3-1)$	$1-5 = A(1+3)$
$-8 = -4B$	$-4 = 4A$
$B=\dfrac{-8}{-4}$	$A=\dfrac{-4}{4}$
$\Rightarrow \boxed{B=2}$	$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-5}{x^2+2x-3}=\frac{-1}{x-1}+\frac{2}{x+3}$$

Q.5 $\dfrac{3x+3}{(x-1)(x+2)}$

Solution: $\dfrac{3x+3}{(x-1)(x+2)}$

Let $\dfrac{3x+3}{(x-1)(x+2)}=\dfrac{A}{x-1}+\dfrac{B}{x+2}$ .... (i)

Multiplying both sides by (x–1)(x+2), we get

$$3x+3=A(x+2)+B(x-1)\text{....... (ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x-1=0$ i.e $x=1$

and $x+2=0$ i.e $x=-2$

Putting $x=1$ and $x=-2$ in equation (ii) we get

For $x=1$	For $x=-2$
$3(1)+3 = A(1+2)$	$3(-2)+3 = B(-2-1)$
$3+3 = 3A$	$-6+3 = -3B$
$6 = 3A$	$-3 = -3B$
$A=\dfrac{6}{3}$	$B=\dfrac{-3}{-3}$
$\Rightarrow \boxed{A=2}$	$\Rightarrow \boxed{B=1}$

Hence the required partial fractions are

$$\frac{3x+3}{(x-1)(x+2)}=\frac{2}{x-1}+\frac{1}{x+2}$$

Q.6 $\dfrac{7x-25}{(x-4)(x-3)}$

Solution: $\dfrac{7x-25}{(x-4)(x-3)}$

Let $\dfrac{7x-25}{(x-4)(x-3)}=\dfrac{A}{x-4}+\dfrac{B}{x-3}$

Multiplying both sides by(x–4) (x – 3), we get

$$7x-25=A(x-3)+B(x-4)\text{..... (ii)}$$

As equation (ii) is an identity which is true for all values of x.

Let $x-3=0$ i.e $x=3$

and $x-4=0$ i.e $x=4$

Putting $x=3$ and $x=4$ in equation (ii) we get

For $x=3$	For $x=4$
$7(3)-25 = B(3-4)$	$7(4)-25 = A(4-3)$
$21-25 = -B$	$28-25 = 1A$
$-4 = -B$	$3 = A$
$\Rightarrow \boxed{B=4}$	$\Rightarrow \boxed{A=3}$

Hence the required partial fractions are

$$\frac{7x-25}{(x-4)(x-3)}=\frac{3}{x-4}+\frac{4}{x-3}$$

Q.7 $\dfrac{x^2+2x+1}{(x-2)(x+3)}$

Solution: $\dfrac{x^2+2x+1}{(x-2)(x+3)}$ is an improper fraction. First we resolve it into proper fraction.

By long division we get

$$\begin{array}{r} 1 \\ x^2+x-6\,\overline{)\,x^2+2x+1} \\ \pm x^2 \pm x \mp 6 \\ \hline x+7 \end{array}$$

We have $\dfrac{x^2+2x+1}{x^2+x-6} = 1+\dfrac{x+7}{x^2+x-6}$

Let $\dfrac{x+7}{(x-2)(x+3)} = \dfrac{A}{x-2}+\dfrac{B}{x+3}$ ....(i)

Multiplying both sides by (x–2) (x +3), we get

$$x + 7= A\,(x +3) + B(x - 2)\text{.........} \;(ii)$$

As equation (ii) is an identity which is true for all values of x.

Let $x + 3 = 0$ i.e $x = -3$

and $x - 2 = 0$ i.e $x = 2$

Putting $x = -3$ and $x = 2$ in equation (ii) we get

For $x = -3$	For $x = 2$
$-3 + 7 = B(-3 - 2)$	$2 + 7 = A\,(2 + 3)$
$4 = -5B$	$9 = 5A$
$\Rightarrow \boxed{B = -\frac{4}{5}}$	$\Rightarrow \boxed{A = \frac{9}{5}}$

Hence the required partial fractions are

$$\frac{x^2+2x+1}{(x-2)(x+3)} = 1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Q.8 $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

Solution: $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ is an improper fraction.

First we resolve it into proper fraction.

$$\begin{array}{r} 2x+3 \\ 3x^2-2x-1\,\overline{)\,6x^3+5x^2-7} \\ \pm 6x^3 \mp 4x^2 \mp 2x \\ \hline 9x^2+2x-7 \\ \pm 9x^2 \mp 6x \mp 3 \\ \hline 8x-4 \end{array}$$

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3)+\frac{8x-4}{(3x+1)(x-1)}$$

Now, Let $\dfrac{8x-4}{(3x+1)(x-1)} = \dfrac{A}{3x+1}+\dfrac{B}{x-1}$ ....(i)

Multiplying both sides by(3x+1) (x–1), we get

$$8x - 4 = A\,(x - 1) + B(3x + 1)\text{......} \;(ii)$$

As equation (ii) is an identity which is true for all values of x.

Let $x - 1 = 0$ i.e $x = 1$

and $3x + 1 = 0$ i.e $x = -\dfrac{1}{3}$

Putting $x = 1$ and $x = \dfrac{-1}{3}$ in equation (ii) we get

For $x = 1$	For $x = \frac{-1}{3}$
$8(1) - 4 = B\,[3(1) + 1]$	$8\left(\frac{-1}{3}\right)-4 = A\left(\frac{-1}{3}-1\right)$
$-4 = 4B$	$\frac{-8}{3}-4 = \frac{A(-1-3)}{3}$
$4 = 4B$	$\frac{-8-12}{3} = \frac{A(-4)}{3}$
$\Rightarrow 4B = 4$	$\frac{-20}{3} = \frac{-4}{3}A$
$B = \frac{4}{4}$	
$\Rightarrow \boxed{B = 1}$	$\Rightarrow \boxed{A = 5}$

Hence the required partial functions are

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{5}{3x+1}+\frac{1}{x-1}$$

Rule II:

Resolution of a fraction when D(x) consists of repeated linear factors:

If a linear factor (ax + b) occurs n times as a factor of D(x), then there are n partial fractions of the form.

$$\frac{A_1}{(ax+b)}+\frac{A_2}{(ax+b)^2}+\ldots.+\frac{A_n}{(ax+b)^n}$$, where

$A_1, A_2, \ldots, A_n$ are constants and $n \geq 2$ is a positive integer.

$$\therefore \quad \frac{N(x)}{D(x)}=\frac{A_1}{(ax+b)}+\frac{A_2}{(ax+b)^2}+\ldots+\frac{A_n}{(ax+b)^n}$$

Example:

Resolve $\frac{1}{(x-1)^2(x-2)}$ into partial fractions.

Solution: Let,

$$\frac{1}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$$

Multiplying both sides by $(x-1)^2(x-2)$, we get

$$1=A(x-1)(x-2)+B(x-2)+C(x-1)^2$$

$$A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)=1 \ldots.(i)$$

Since (i) is an identity and is true for all values of x

Put $x-1=0$ or $x=1$ in (i), we get

$$B(1-2)=1$$

$$\Rightarrow -B=1 \text{ or } \boxed{B=-1}$$

Put $x-2=0$ or $x=2$ in (i), we get

$$C(2-1)^2=1$$

$$C(1)=1 \quad \Rightarrow \quad \boxed{C=1}$$

Equating coefficients of x^2 on both the sides of (i)

$$A+C=0$$

$$\Rightarrow \quad A=-C$$

$$A=-(1) \quad \Rightarrow \quad \boxed{A=-1}$$

Hence required partial fractions are

$$\frac{-1}{x-1}-\frac{1}{(x-1)^2}+\frac{1}{(x-2)}$$

Thus, $$\frac{1}{(x-1)^2(x-2)}=\frac{1}{x-2}-\frac{1}{(x-1)}-\frac{1}{(x-1)^2}$$