EXERCISE 4.2

Resolve into partial fractions:

Q.1 $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}$

Solution:

Let $\dfrac{x^2-3x+1}{(x-1)^2(x-2)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x-2}$ ...(i)

Multiplying both sides by $(x-1)^2(x-2)$, we get

$x^2-3x+1=A(x-1)(x-2)+B(x-2)+C(x-1)^2$ ..(ii)

$x^2-3x+1=A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$(1)^2-3(1)+1=\ (1-2)$$
$$1-3+1=B(-1)$$
$$-1=-B$$
$$\Rightarrow B=1$$

Putting $x-2=0$ i.e $x=2$ in (ii) we get

$$(2)^2-3(2)+1=C(2-1)^2$$
$$4-6+1=C$$
$$-1=C$$

Equating the coefficient of x^2 in (ii) we get

$$1=A+B$$
$$1=A-1$$
$$\Rightarrow \quad A=1+1$$
$$\boxed{A=2}$$

Hence the required partial fractions are

$$\frac{x^2-3x+1}{(x-1)^2(x-2)}=\frac{2}{x-1}+\frac{1}{(x-1)^2}-\frac{1}{x-2}$$

Q.2 $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}$

Solution:

Let $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}=\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}+\dfrac{C}{x+3}$ ...(i)

Multiplying both sides by $(x+2)^2(x+3)$

$\Rightarrow x^2+7x+11=A(x+2)(x+3)+B(x+3)+C(x+2)^2$

$x^2+7x+11=A(x^2+5x+6)+B(x+3)+C(x^2+4x+4)$..(ii)

Putting $x+2=0$ i.e $x=-2$ in (ii) we get

$$(-2)^2+7(-2)+11=B(-2+3)$$
$$4-14+11=B$$
$$\Rightarrow \quad \boxed{B=1}$$

Putting $x+3=0$ i.e $x=-3$ in (ii) we get

$$(-3)^2+7(-3)+11=C(-3+2)^2$$
$$9-21+11=C(-1)^2$$
$$20-21=C(1)$$
$$-1=C \Rightarrow \boxed{C=-1}$$

Equating coefficient of x^2 in (ii) we get

$$A+C=1$$
$$A-1=1$$
$$A=1+1$$
$$\boxed{A=2}$$

Hence the required partial fractions are:

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{2}{x+2}+\frac{1}{(x+2)^2}-\frac{1}{x+3}$$

Q.3 $\dfrac{9}{(x-1)(x+2)^2}$

Solution:

Let $\dfrac{9}{(x-1)(x+2)^2}=\dfrac{A}{x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$ .... (i)

Multiplying both sides by $(x-1)(x+2)^2$, we get

$9=A(x+2)^2+B(x-1)(x+2)+C(x-1)$..(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$9=A(1+2)^2$$
$$9=A(3)^2$$
$$9=9A$$
$$\Rightarrow \boxed{A=1}$$

Putting $x+2=0$ i.e $x=-2$ in (ii) we get

$$9=C(-2-1)$$
$$9=-3C$$
$$\Rightarrow \boxed{C=-3}$$

Equating the coefficient of x^2 in (ii) we get

$$A+B=0$$
$$B=-A$$
$$\boxed{B=-1}$$

Hence the partial fractions are

$$\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$$

Q.4 $\dfrac{x^4+1}{x^2(x-1)}$

Solution: $\dfrac{x^4+1}{x^2(x-1)}=\dfrac{x^4+1}{x^3-x^2}$ is an improper fraction. First we resolve it into proper fraction.

$$\begin{array}{r|l} & x+1 \\ x^3-x^2 & x^4+1 \\ & \pm x^4 \mp x^3 \\ \hline & x^3+1 \\ & \pm x^3 \mp x^2 \\ \hline & x^2+1 \end{array}$$

$$\frac{x^4+1}{x^2(x-1)}=(x+1)+\frac{x^2+1}{x^2(x-1)} \quad \ldots\ldots\ldots \text{(i)}$$

Let $\dfrac{x^2+1}{x^2(x-1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}$ .... (ii)

Multiplying both sides by $x^2(x-1)$ we get

$x^2+1=A(x)(x-1)+B(x-1)+Cx^2$......(iii)

Putting $x=0$ in (iii) we get

$$0+1=B(0-1)$$
$$1=-B$$
$$\Rightarrow \boxed{B=-1}$$

Putting $x-1=0$ i.e $x=1$ in (iii) we get

$$(1)^2+1=C(1)^2$$
$$1+1=C(1)$$
$$2=C$$
$$\Rightarrow \boxed{C=2}$$

Equating the coefficient of x^2 in (iii) we get

$$A+C=1$$
$$A+2=1$$
$$A=1-2$$
$$\Rightarrow \boxed{A=-1}$$

Putting the value of A, B and C in equation(ii)

Thus required partial fractions are

$$\frac{x^4+1}{x^2(x-1)}=(x+1)-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$$

Q.5 $\dfrac{7x+4}{(3x+2)(x+1)^2}$

Solution: $\dfrac{7x+4}{(3x+2)(x+1)^2}$

Let $\dfrac{7x+4}{(3x+2)(x+1)^2}=\dfrac{A}{3x+2}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}$ .....(i)

Multiplying both sides by $(3x+2)(x+1)^2$ we get

$7x+4=A(x+1)^2+B(3x+2)(x+1)+C(3x+2)$... (ii)

Putting $3x+2=0$ i.e $x=\dfrac{-2}{3}$ in (ii) we get

$$7\left(\frac{-2}{3}\right)+4 = A\left(\frac{-2}{3}+1\right)^2$$

$$\frac{-14}{3}+4 = A\left(\frac{-2+3}{3}\right)^2$$

$$\frac{-14+12}{3} = A\left(\frac{1}{3}\right)^2$$

$$\frac{-2}{3} = \frac{1}{9}A$$

$$-18 = 3A$$

$$A = \frac{-18}{3}$$

$$\Rightarrow \boxed{A=-6}$$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$$7(-1)+4 = C(3(-1)+2)$$

$$-7+4 = -C$$

$$\Rightarrow -3 = -C$$

$$\Rightarrow \boxed{C=3}$$

Equating the coefficient of x^2 we get

$$A+3B=0$$

$$-6+3B=0$$

$$3B=6$$

$$B=\frac{6}{3} \Rightarrow \boxed{B=2}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{7x+4}{(3x+2)(x+1)^2} = \frac{-6}{3x+2}+\frac{2}{x+1}+\frac{3}{(x+1)^2}$$

Q.6 $\frac{1}{(x-1)^2(x+1)}$

Solution: $\frac{1}{(x-1)^2(x+1)}$

Let $\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}$ .....(i)

Multiplying both sides by $(x-1)^2(x+1)$ we get

$$1 = A(x-1)(x+1)+B(x+1)+C(x-1)^2 \text{...(ii)}$$

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$$1 = B(1+1)$$

$$1 = 2B \Rightarrow \boxed{B=\frac{1}{2}}$$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$$1 = C(-1-1)^2$$

$$1 = C(-2)^2$$

$$1 = 4C \Rightarrow \boxed{C=\frac{1}{4}}$$

Equating the coefficient of x^2 in (ii) we get

$$A+C=0$$

$$A=-C$$

$$A=-\left(\frac{1}{4}\right) \Rightarrow \boxed{A=\frac{-1}{4}}$$

Putting the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{1}{(x-1)^2(x+1)} = \frac{-1}{4(x-1)}+\frac{1}{2(x-1)^2}+\frac{1}{4(x+1)}$$

Q.7 $\frac{3x^2+15x+16}{(x+2)^2}$

Solution: $\frac{3x^2+15x+16}{(x+2)^2} = \frac{3x^2+15x+16}{x^2+4x+4}$

The given fraction is improper fraction. First we resolve it into proper fraction.

By long division,

$$\begin{array}{r} 3 \\ x^2+4x+4\overline{)3x^2+15x+16} \\ \pm 3x^2 \pm 12x \pm 12 \\ \hline 3x+4 \end{array}$$

$$\frac{3x^2+15x+16}{(x+2)^2} = 3+\frac{3x+4}{x^2+4x+4} \text{ ....(i)}$$

Let $\frac{3x+4}{(x+2)^2} = \frac{A}{x+2}+\frac{B}{(x+2)^2}$ ....(ii)

Multiplying both sides by $(x+2)^2$ we get

$$3x+4 = A(x+2)+B \text{....... (iii)}$$

Putting $x+2=0$ i.e $x=-2$ in (iii) we get

$$3(-2)+4 = B$$

$$-6+4 = B$$

$$\Rightarrow \boxed{B=-2}$$

Equating the coefficient of 'x' we get

$$3=A$$

$$\Rightarrow \boxed{A=3}$$

Putting the value of A and B in equation (ii) and using equation (i) we get required partial fractions.

$$\frac{3x^2+15x+16}{(x+2)^2} = 3+\frac{3}{x+2}-\frac{2}{(x+2)^2}$$

Q.8 $\frac{1}{(x^2-1)(x+1)}$

Solution: $\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x-1)(x+1)(x+1)}$

$$= \frac{1}{(x-1)(x+1)^2}$$

Let $\frac{1}{(x-1)(x+1)^2} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ ....(i)

Multiplying both sides by $(x-1)(x+1)^2$ we get

$1 = A(x+1)^2 + B(x+1)(x-1)+C(x-1)$ .....(ii)

Putting $x-1=0$ i.e $x=1$ in (ii) we get

$1 = A(1+1)^2$

$1 = A(2)^2$

$1 = 4A$

$\Rightarrow$ $\boxed{A = \frac{1}{4}}$

Putting $x+1=0$ i.e $x=-1$ in (ii) we get

$1 = C(-1-1)$

$1 = -2C$

$\Rightarrow$ $\boxed{C = \frac{-1}{2}}$

Equating the coefficient of x^2 in equation (ii) we get $A + B = 0$

$B = -A$

$B = -\left(\frac{1}{4}\right)$

$\boxed{B = -\frac{1}{4}}$

Putting the value of A and B in equation (ii) we get required partial fractions.

$$\frac{1}{(x-1)(x+1)^2} = \frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$$

Rule III:

Resolution of fraction when D(x) consists of non-repeated irreducible quadratic factors:

If a quadratic factor (ax^2+bx+c) with $a \neq 0$ occurs once as a factor of D(x), the partial fraction is of the form $\frac{Ax+B}{(ax^2+bx+c)}$, where A and B are constants to be found.

Example:

Resolve $\frac{11x+3}{(x-3)(x^2+9)}$ into partial fractions.

Solution:

Let, $\frac{11x+3}{(x-3)(x^2+9)} = \frac{A}{(x-3)}+\frac{Bx+C}{x^2+9}$

Multiplying both sides by $(x-3)(x^2+9)$

$\Rightarrow$ $11x+3 = A(x^2+9)+(Bx+C)(x-3)$

$\Rightarrow$ $11x+3 = A(x^2+9)+B(x^2-3x)+C(x-3)$ ......(i)

Since (i) is an identity, we have on substituting

$x-3=0 \Rightarrow x=3$

Put $x=3$ in equation (i)

$33+3 = A(9+9)$

$36 = A(18)$

$\Rightarrow$ $18A = 36$

$\Rightarrow$ $\boxed{A=2}$

Comparing the coefficients of x^2 and x on both the sides of (i), we get

$A + B = 0$

$B = -A$

$B = -(2)$

$\Rightarrow$ $\boxed{B=-2}$

$-3B + C = 11$

$\Rightarrow$ $-3(-2)+C = 11$

$6 + C = 11$

$C = 11-6$

$\Rightarrow$ $\boxed{C=5}$

Putting the value of A, B and C, we get required partial fractions.

$$\frac{11x+3}{(x-3)(x^2+9)} = \frac{2}{x-3}+\frac{-2x+5}{x^2+9}$$