EXERCISE 4.4

Q.1 $\dfrac{x^3}{(x^2+4)^2}$

Solution: $\dfrac{x^3}{(x^2+4)^2}$

Let $\dfrac{x^3}{(x^2+4)^2} = \dfrac{Ax+B}{x^2+4}+\dfrac{Cx+D}{(x^2+4)^2}$ .... (i)

Multiplying both sides by $(x^2 + 4)^2$, we get

$$x^3 = (Ax+B)(x^2+4)+(Cx+D)$$

$$x^3 = Ax(x^2+4) + B(x^2+4) + (Cx+D) \text{ ....(ii)}$$

Equating the coefficients of x^3, x^2, x and constants, we get

Coefficients of x^3: $A = 1$

Coefficients of x^2: $B = 0$

Coefficients of x: $4A + C = 0$

$4(1) + C = 0$

$\Rightarrow C = -4$

Constants: $4B + D = 0$

$4(0)+D = 0$

$\Rightarrow D = 0$

Putting the value of A,B,C and D in equation(i) we get required partial fractions.

$$\frac{x^3}{(x^2+4)^2} = \frac{x}{x^2+4}-\frac{4x}{(x^2+4)^2}$$

Q.2 $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Solution: $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Let $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$ ...(i)

Multiplying both sides by $(x+1)(x^2+1)^2$ we get

$x^4+3x^2+x+1 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$

$$+(Dx+E)(x+1)\ldots\ldots(ii)$$

$$x^4+3x^2+x+1 = A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)$$
$$+C(x^3+x^2+x+1)+Dx(x+1)+E(x+1)$$

$$x^4+3x^2+x+1 = A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$$
$$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1)\ldots\ (iii)$$

Putting $x+1=0$ i.e $x=-1$ in eq.(ii), we get

$$(-1)^4+3(-1)^2+(-1)+1 = A[(-1)^2+1]^2$$
$$1+3(1)-1+1 = A(1+1)^2$$
$$4 = 4A$$
$$\Rightarrow \boxed{A = 1}$$

Now equating the coefficients of x^4, x^3, x^2, x and constants, we get from equation (iii)

Coefficients of x^4: $A+B=1$
$$1+B=1$$
$$B=1-1$$
$$\Rightarrow \boxed{B=0}$$

Coefficients of x^3: $B+C=0$
$$0+C=0$$
$$\Rightarrow \boxed{C=0}$$

Coefficients of x^2: $2A+B+C+D=3$
$$2(1)+0+0+D=3$$
$$D=3-2$$
$$\boxed{D=1}$$

Coefficients of x: $B+C+D+E=1$
$$0+0+1+E=1$$
$$E=1-1$$
$$\Rightarrow \boxed{E=0}$$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{1}{x+1}+\frac{x}{(x^2+1)^2}$$

Q.3 $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Solution: $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Let $\dfrac{x^2}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$ .. (i)

Multiply both sides by $(x+1)(x^2+1)^2$ we get

$$x^2 = A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)$$
$$+(Dx+E)(x+1)\ldots\ldots.(ii)$$

$$x^2 = A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)$$
$$+C(x^3+x^2+x+1)+Dx(x+1)+E(x+1)$$

$$x^2 = A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$$
$$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1)\ldots(iii)$$

Putting $x+1=0$ i.e $x=-1$ in equation (ii) we get

$$(-1)^2 = A[(-1)^2+1]^2$$
$$1 = A(1+1)^2$$
$$1 = 4A \Rightarrow \boxed{A=\frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2, x and constants we get from equation (iii)

Coefficients of x^4: $A+B=0$
$$\frac{1}{4}+B=0 \Rightarrow \boxed{B=-\frac{1}{4}}$$

Coefficients of x^3: $B+C=0$
$$-\frac{1}{4}+C=0 \Rightarrow \boxed{C=\frac{1}{4}}$$

Coefficients of x^2: $2A+B+C+D=1$
$$2\left(\frac{1}{4}\right)-\frac{1}{4}+\frac{1}{4}+D=1$$
$$\frac{1}{2}+D=1$$
$$D=1-\frac{1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x: B + C + D + E = 0

$$-\frac{1}{4}+\frac{1}{4}+\frac{1}{2}+E = 0$$

$$\frac{1}{2}+E = 0$$

$$\boxed{E = -\frac{1}{2}}$$

Putting the value of A, B, C, D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} - \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

Q.4 $\quad \dfrac{x^2}{(x-1)(x^2+1)^2}$

Solution: $\dfrac{x^2}{(x-1)(x^2+1)^2}$

Let $\dfrac{x^2}{(x-1)(x^2+1)^2} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2}$ ..(i)

Multiplying both sides by $(x - 1)(x^2 + 1)^2$, we get

$$x^2 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \quad \text{....(ii)}$$

$$x^2 = A(x^4+2x^2+1) + Bx(x-1)(x^2+1) + C(x-1)(x^2+1) + Dx(x-1) + E(x-1)$$

$$x^2 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1)+D(x^2-x) + E(x-1) \quad \text{... (iii)}$$

Putting $x - 1 = 0$ i.e $x = 1$ in equation (ii) we get

$$(1)^2 = A[(1)^2+1]^2$$

$$1 = A(1+1)^2$$

$$1 = 4A \Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2 and x in equation (iii) we get

Coefficients of x^4: A + B = 0

$$\frac{1}{4}+B = 0$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3: – B + C = 0

$$-\left(-\frac{1}{4}\right)+C = 0$$

$$\Rightarrow C = -\frac{1}{4}$$

Coefficients of x^2: 2A + B – C + D = 1

$$2\left(\frac{1}{4}\right) - \frac{1}{4} - \left(\frac{-1}{4}\right) + D = 1$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$D = \frac{2-1}{2}$$

$$\boxed{D = \frac{1}{2}}$$

Coefficients of x: – B + C – D + E = 0

$$-\left(-\frac{1}{4}\right) - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{-1}{2} + E = 0$$

$$\boxed{E = \frac{1}{2}}$$

Putting the value of A, B, C, D and E in equation(i) we get required partial fractions.

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

Q.5 $\dfrac{x^4}{(x^2+2)^2}$

Solution: $\dfrac{x^4}{(x^2+2)^2}$

$\dfrac{x^4}{(x^2+2)^2} = \dfrac{x^4}{x^4+4x^2+4}$ is an improper fraction. First we resolve it into proper fraction.

$$x^4+4x^2+4\overline{)\;x^4\;}\;1$$
$$\pm x^4 \pm 4x^2 \pm 4$$
$$-4x^2-4$$

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4x^2-4}{(x^2+2)^2}$$

Let $\dfrac{-4x^2-4}{(x^2+2)^2} = \dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{(x^2+2)^2}$ .....(i)

Multiplying both sides by $(x^2+2)^2$ we get

$-4x^2-4 = (Ax+B)(x^2+2) + (Cx+D)$

$-4x^2-4 = A(x^3+2x) + B(x^2+2) + Cx + D$......(ii)

Equating the coefficients of x^3, x^2, x and constants in equation (ii) we get

Coefficients of x^3: $A = 0$

Coefficients of x^2: $B = -4$

Coefficients of x: $2A + C = 0$

$2(0) + C = 0$

$\Rightarrow \boxed{C = 0}$

Constants : $2B + D = -4$

$2(-4) + D = -4$

$-8 + D = -4$

$D = 8 - 4$

$\boxed{D = 4}$

Putting the value of A, B, C and D in equation(i) we get required partial fractions.

$$\frac{x^4}{(x^2+2)^2} = 1 + \frac{-4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

$$\frac{x^4}{(x^2+2)^2} = 1 - \frac{4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

Q.6 $\dfrac{x^5}{(x^2+1)^2}$

Solution: $\dfrac{x^5}{(x^2+1)^2}$

$\dfrac{x^5}{(x^2+1)^2} = \dfrac{x^5}{x^4+2x^2+1}$ is an improper fraction.

First we resolve it into proper fraction.

$$x^4+2x^2+1\overline{)\;x^5\;}\;x$$
$$\pm x^5 \pm 2x^3 \pm x$$
$$-2x^3-x$$

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x^3-x}{(x^2+1)^2}$$

Let $\dfrac{-2x^3-x}{(x^2+1)^2} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2}$ ....(i)

Multiplying both sides by $(x^2+1)^2$ we get

$-2x^3-x = (Ax+B)(x^2+1) + (Cx+D)$

$-2x^3-x = A(x^3+x) + B(x^2+1) + Cx + D$

Equating the coefficients of x^3, x^2, x and constants we get

Coefficients of x^3 : $A = -2$

Coefficients of x^2 : $B = 0$

Coefficients of x : $A + C = -1$

$-2 + C = -1$

$C = -1 + 2$

$\Rightarrow \boxed{C = 1}$

Constants : $B + D = 0$

$0 + D = 0$

$\Rightarrow D = 0$

Hence the required partial fractions are

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$\Rightarrow \frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$