(d) **Complement of a set**:

If U is a universal set and A is a subset of U, then the complement of A is the set of those elements of U, which are not contained in A and is denoted by A' or A^c.

$$A' = U - A = \{x | x \in U \text{ and } x \notin A\}$$

For example, if U = {1, 2, 3, …, 10} and A= {2, 4, 6, 8}, then

$A' = U - A$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8\}$

$= \{1, 3, 5, 7, 9, 10\}$

Example:

If $U = \{1, 2, 3, \ldots, 10\}$,

$A = \{2, 3, 5, 7\}, B = \{3, 5, 8\}$, then

Find (i) $A \cup B$ (ii) $A \cap B$

(iii) $A - B$ (iv) A' and B'

Solution:

(i) $A \cup B = \{2,3,5,7\} \cup \{3,5,8\}$

$= \{2,3,5,7,8\}$

(ii) $A \cap B = \{2,3,5,7\} \cap \{3,5,8\}$

$= \{3,5\}$

(iii) $A - B = \{2,3,5,7\} - \{3,5,8\}$

$= \{2,7\}$

(iv) $A' = U - A$

$= \{1,2,3\ldots,10\} - \{2,3,5,7\}$

$= \{1,4,6,8,9,10\}$

$B' = U - B$

$= \{1,2,3\ldots,10\} - \{3,5,8\}$

$= \{1,2,4,6,7,9,10\}$

EXERCISE 5.1

Q.1 If X = {1, 4, 7, 9} and Y = {2, 4, 5, 9} then find:

(i) $X \cup Y$ (ii) $X \cap Y$

(iii) $Y \cup X$ (iv) $Y \cap X$

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(ii) $X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$

$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$

$= \{4, 9\}$

Q.2 If X = Set of Prime numbers less than or equal to 17.
Y = Set of first 12 natural numbers, then find.

(i) $X \cup Y$ (ii) $X \cap Y$

(iii) $Y \cup X$ (iv) $Y \cap X$

Solution:

X = {2, 3, 5, 7, 11, 13, 17}

Y = {1, 2, 3, 4, …, 12}

(i) $X \cup Y$

$= \{2, 3, 5, 7, 11, 13, 17\} \cup \{1, 2, 3, 4, \ldots, 12\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17\}$

(ii) $Y \cup X$

$= \{1,2,3,4, \ldots, 12\} \cup \{2,3,5,7, 11, 13, 17\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17\}$

(iii) $X \cap Y$

$= \{2,3,5,7,11,13,17\} \cap \{1,2,3,4, 5, \ldots, 12\}$

$= \{2,3,5, 7, 11\}$

(iv) $Y \cap X$

$= \{1,2,3,5, \ldots, 12\} \cap \{2,3,5,7,11,13, 17\}$

$= \{2, 3, 5, 7, 11\}$

Q.3 **If $X = \phi, Y = Z^+, T = O^+$, then find.**

(i) $X \cup Y$ (ii) $X \cup T$ (iii) $Y \cup T$

(iv) $X \cap Y$ (v) $X \cap T$ (vi) $Y \cap T$

Solution:

(i) $X \cup Y = \phi \cup Z^+$

$= Z^+ = Y$

(ii) $X \cup T = \phi \cup O^+$

$= O^+ = T$

(iii) $Y \cup T = Z^+ \cup O^+$

$= \{1, 2, 3, 4, 5 \ldots.\} \cup \{1, 3, 5, 7, \ldots.\}$

$= \{1, 2, 3, 4, 5, \ldots\} = Z^+ = Y$

(iv) $X \cap Y = \phi \cap Z^+$

$= \phi = X$

(v) $X \cap T = \phi \cap O^+$

$= \phi = X$

(vi) $Y \cap T = Z^+ \cap O^+$

$= \{1, 2, 3, 4, 5, \ldots\} \cap \{1, 3, 5, 7, \ldots\}$

$= \{1, 3, 5, 7, \ldots\} = O^+ = T$

Q.4 **If $U = \{x | x \in N \wedge 3 < x \leq 25\}$**
$X = \{x | x \text{ is Prime} \wedge 8 < x < 25\}$
$Y = \{x | x \in W \wedge 4 \leq x \leq 17\}$
then find the value of:

(i) $(X \cup Y)'$ (ii) $X' \cap Y'$

(iii) $(X \cap Y)'$ (iv) $X' \cup Y'$

Solution:

$U = \{4, 5, 6, 7, \ldots, 25\}$

$X = \{11, 13, 17, 19, 23\}$

$Y = \{4, 5, 6, 7, \ldots, 17\}$

(i) $(X \cup Y)'$

$X \cup Y = \{11,13,17,19,23\} \cup \{4,5,6,7,\ldots,17\}$

$= \{4,5,6,7,8,9,10,11,12,13,14,15,16,17,19,23\}$

$(X \cup Y)' = U - (X \cup Y)$

$= \{4,5,6,7,\ldots,25\} - \{4,5,6,7,8,9,10,11,12,13,14,15,16,17,19,23\}$

$= \{18, 20, 21, 22, 24, 25\}$

(ii) $X' \cap Y'$

$X' = U - X$

$= \{4, 5, 6, 7, \ldots, 25\} - \{11,13,17,19,23\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\}$

$Y' = U - Y$

$= \{4,5,6,7, \ldots, 25\} - \{4,5,6,7,\ldots, 17\}$

$= \{18, 19, 20, 21, 22, 23, 24, 25\}$

$X' \cap Y' = \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\} \cap \{18,19,20,21,22,23,24,25\}$

$= \{18, 20, 21, 22, 24, 25\}$

(iii) $(X \cap Y)'$

$X \cap Y = \{11,13,17,19,23\} \cap \{4,5,6,7,\ldots,17\}$

$= \{11, 13, 17\}$

$(X \cap Y)' = U - (X \cap Y)$

$= \{4, 5, 6, 7, \ldots, 25\} - \{11,13,17\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,18,19,20,21,22,23,24,25\}$

(iv) $X' \cup Y'$

$X' = U - X = \{4,5,6,7,\ldots,25\} - \{11,13,17,19,23\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\}$

$Y' = U - Y$

$= \{4,5,6,7,\ldots,25\} - \{4,5,6,7,\ldots,17\}$

$= \{18, 19, 20, 21, 22, 23, 24, 25\}$

$X' \cup Y' = \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\} \cup \{18,19,20,21,22,23,24,25\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,18,19,20,21,22,23,24,25\}$

Q.5 **If $X = \{2, 4, 6, \ldots, 20\}$ and $Y = \{4, 8, 12, \ldots 24\}$ then find the following: (i) $X - Y$ (ii) $Y - X$**

Solution:

(i) $X - Y = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} - \{4, 8, 12, 16, 20, 24\}$

$= \{2, 6, 10, 14, 18\}$

(ii) $Y - X = \{4, 8, 12, 16, 20, 24\} - \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$= \{24\}$

Q.6 **If $A = N$ and $B = W$ then find the value of**

(i) $A - B$ (ii) $B - A$

Solution:

(i) $A - B = N - W$

$= \{1, 2, 3, \ldots\} - \{0, 1, 2, 3, \ldots\}$

$= \{\ \}$

(ii) $B - A = W - N$

$= \{0, 1, 2, 3, \ldots\} - \{1, 2, 3, \ldots\}$

$= \{0\}$

Properties of Union and Intersection:

(a) Commutative property of union:

For any two sets A and B, $A \cup B = B \cup A$ is called commutative property of union .

Proof:

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$ (by definition of union of sets)

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$\Rightarrow A \cup B \subseteq B \cup A$ ....(i)

Now let $y \in B \cup A$

$\Rightarrow y \in B$ or $y \in A$ (by definition of union of sets)

$\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow B \cup A \subseteq A \cup B$ .... (ii)

From (i) and (ii), we have $A \cup B = B \cup A$ (by definition of equal sets)

(b) Commutative property of intersection:

For any two sets A and B, $A \cap B = B \cap A$ is called commutative property of intersection.

Proof: Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$ (by definition intersection of sets)

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cap A$

$A \cap B \subseteq B \cap A$ ....(i)

Now let $y \in B \cap A$

$\Rightarrow y \in B$ and $y \in A$ (by definition intersection of sets)

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in A \cap B$

Therefore, $B \cap A \subseteq A \cap B$ ....(ii)

From (i) and (ii), we have $A \cap B = B \cap A$ (by definition of equal sets)

(c) Associative property of union:

For any three sets A,B and C, $(A \cup B) \cup C = A \cup (B \cup C)$ is called associative property of union.

Proof: Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$...............(i)

Similarly $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ ......(ii)

From (i) and (ii), we have

$(A \cup B) \cup C = A \cup (B \cup C)$

(d) Associative property of intersection:

For any three sets A, B and C, $(A \cap B) \cap C = A \cap (B \cap C)$ is called associative property of intersection.

Proof: Let $x \in (A \cap B) \cap C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow (x \in A$ and $x \in B)$ and $x \in C$

$\Rightarrow x \in A$ and $(x \in B$ and $\in C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A \cap (B \cap C)$

$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C)$......(i)

Similarly $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ ... (ii)

From (i) and (ii), we have

$(A \cap B) \cap C = A \cap (B \cap C)$

(e) **Distributive property of union over intersection**:

If A,B and C are three sets then $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ is called distributive property of union over intersection

Proof: Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A\cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.......(i)

Similarly, now let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y\in A$ or $y\in C)$

$\Rightarrow y \in A$ or $(y \in B$ and $y \in C)$

$\Rightarrow y \in A$ or $y \in B \cap C$

$\Rightarrow y \in A$ or $(B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$......(ii)

From (i) and (ii), we have

$A\cup (B\cap C) = (A \cup C) \cap (A \cup C)$

(f) **Distributive property of intersection over union**:

If A, B and C are three sets then

$A \cap (B \cup C) = (A\cap B)\cup(A\cap C)$ is called distributive property of intersection over union.

Proof: Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cup C$

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$

$\Rightarrow (x \in A$ and $x\in B)$ or $(x\in A$ and $x\in C)$

$\Rightarrow (x\in A\cap B)$ or $(x \in A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

Hence by definition of subsets

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$..... (i)

Similarly

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B\cup C)$..... (ii)

From (i) and (ii), we have

$A\cap(B\cup C) = (A \cap B) \cup (A \cap C)$

(g) **De-Morgan's laws**:

If two sets A and B are the sub sets of U then De-Morgan's laws are expressed as

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof:

(i) $(A \cup B)' = A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$ (by definition of complement of set)

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A'\cap B'$ (by definition of intersection of sets)

$\Rightarrow (A \cup B)' \subseteq A' \cap B'$ ....(i)

Similarly $A' \cap B' \subseteq (A \cup B)'$..........(ii)

Using (i) and (ii), we have

$(A\cup B)' = A'\cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Let $x \in (A\cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A'\cup B'$

$\Rightarrow (A \cap B)' \subseteq A' \cup B'$..............(i)

Let $y \in A' \cup B'$

$\Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin A\cap B$

$\Rightarrow y \in (A \cap B)'$

$\Rightarrow A' \cup B' \subseteq (A \cap B)'$ ....(ii)

From (i) and (ii) we have proved that

$(A \cap B)' = A' \cup B'$