

## EXERCISE 5.2

**Q.1** If  $X = \{1, 3, 5, 7, \dots, 19\}$

$Y = \{0, 2, 4, 6, 8, \dots, 20\}$

$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$ , then find the following:

(i)  $X \cup (Y \cup Z)$       (ii)  $(X \cup Y) \cup Z$

(iii)  $X \cap (Y \cap Z)$       (iv)  $(X \cap Y) \cap Z$

(v)  $X \cup (Y \cap Z)$       (vi)  $(X \cup Y) \cap (X \cup Z)$

(vii)  $X \cap (Y \cup Z)$       (viii)  $(X \cap Y) \cup (X \cap Z)$

**Solution:**

(i)  $X \cup (Y \cup Z)$

$$\begin{aligned} &= X \cup (\{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \\ &\quad \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}) \\ &= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \cup \\ &\quad \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, \\ &\quad 16, 17, 18, 19, 20, 23\} \\ &= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, \\ &\quad 14, 15, 16, 17, 18, 19, 20, 23\} \end{aligned}$$

(ii)  $(X \cup Y) \cup Z$

$$\begin{aligned} &= (\{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\}) \cup Z \\ &= \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 20\} \cup \\ &\quad \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\ &= \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 20, 23\} \end{aligned}$$

(iii)  $X \cap (Y \cap Z)$

$$\begin{aligned} &= X \cap (\{0, 2, 4, 6, 8, \dots, 20\} \cap \\ &\quad \{2, 3, 5, 7, 11, 13, 17, 19, 23\}) \\ &= \{1, 3, 5, 7, \dots, 19\} \cap \{2\} \\ &= \phi \end{aligned}$$

(iv)  $(X \cap Y) \cap Z$

$$\begin{aligned} &= (\{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\}) \cap Z \\ &= \{ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\ &= \phi \end{aligned}$$

(v)  $X \cup (Y \cap Z)$

$$\begin{aligned} &= X \cup (\{0, 2, 4, 6, 8, \dots, 20\} \cap \\ &\quad \{2, 3, 5, 7, 11, 13, 17, 19, 23\}) \\ &= \{1, 3, 5, 7, \dots, 19\} \cup \{2\} \\ &= \{1, 2, 3, 5, 7, \dots, 19\} \end{aligned}$$

(vi)  $(X \cup Y) \cap (X \cup Z)$

$$\begin{aligned} X \cup Y &= \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\} \\ &= \{0, 1, 2, 3, 4, 5, \dots, 20\} \end{aligned}$$

$$\begin{aligned} X \cup Z &= \{1, 3, 5, 7, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\ &= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\} \end{aligned}$$

$(X \cup Y) \cap (X \cup Z)$

$$\begin{aligned} &= \{0, 1, 2, 3, 4, \dots, 20\} \cap \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\} \\ &= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \end{aligned}$$

(vii)  $X \cap (Y \cup Z)$

$$\begin{aligned} X \cap (Y \cup Z) &= X \cap (\{0, 2, 4, 6, 8, \dots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}) \\ &= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, \\ &\quad 12, 13, 14, 16, 17, 18, 19, 20, 23\} \end{aligned}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

(viii)  $(X \cap Y) \cup (X \cap Z)$

$$\begin{aligned} X \cap Y &= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\} \\ &= \{ \} \end{aligned}$$

$$\begin{aligned} X \cap Z &= \{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\ &= \{3, 5, 7, 11, 13, 17, 19\} \end{aligned}$$

$$\begin{aligned} (X \cap Y) \cup (X \cap Z) &= \{ \} \cup \{3, 5, 7, 11, 13, 17, 19\} \\ &= \{3, 5, 7, 11, 13, 17, 19\} \end{aligned}$$

**Q. 2.** If  $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 4, 6, 8\}$   $C = \{1, 4, 8\}$  Prove the following identities:

(i)  $A \cap B = B \cap A$

(ii)  $A \cup B = B \cup A$

(iii)  $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

(iv)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

**Solution:**

(i)  $A \cap B = B \cap A$

L.H.S =  $A \cap B$   
=  $\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$   
=  $\{2, 4, 6\}$

R.H.S =  $B \cap A$   
=  $\{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$   
=  $\{2, 4, 6\}$

L.H.S = R.H.S, so

$A \cap B = B \cap A$

(ii)  $A \cup B = B \cup A$

L.H.S =  $A \cup B$   
=  $\{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$   
=  $\{1, 2, 3, 4, 5, 6, 8\}$

R.H.S =  $B \cup A$   
=  $\{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$   
=  $\{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S,

So,  $A \cup B = B \cup A$

(iii)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S =  $A \cap (B \cup C)$   
=  $A \cap (\{2, 4, 6, 8\} \cup \{1, 4, 8\})$   
=  $\{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$   
=  $\{1, 2, 4, 6\}$

R.H.S =  $(A \cap B) \cup (A \cap C)$

$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$   
=  $\{2, 4, 6\}$

$A \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}$   
=  $\{1, 4\}$

$(A \cap B) \cup (A \cap C) = \{2, 4, 6\} \cup \{1, 4\}$   
=  $\{1, 2, 4, 6\}$

L.H.S = R.H.S

So,  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S =  $A \cup (B \cap C)$   
=  $A \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$   
=  $\{1, 2, 3, 4, 5, 6\} \cup \{4, 8\}$   
=  $\{1, 2, 3, 4, 5, 6, 8\}$

R.H.S =  $(A \cup B) \cap (A \cup C)$

$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$   
=  $\{1, 2, 3, 4, 5, 6, 8\}$

$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}$   
=  $\{1, 2, 3, 4, 5, 6, 8\}$

$(A \cup B) \cap (A \cup C)$   
=  $\{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$   
=  $\{1, 2, 3, 4, 5, 6, 8\}$

L.H.S = R.H.S,

So,  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

**Q.3** If  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 9\}$ ,  $B = \{2, 3, 5, 7\}$  then verify the De Morgan's laws i.e.,

$(A \cup B)' = A' \cap B'$  and  $(A \cap B)' = A' \cup B'$

**Solution:**

(i)  $(A \cup B)' = A' \cap B'$

L.H.S =  $(A \cup B)'$

$A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$   
=  $\{1, 2, 3, 5, 7, 9\}$

$(A \cup B)' = U - (A \cup B)$   
=  $\{1, 2, 3, 4, \dots, 10\} - \{1, 2, 3, 5, 7, 9\}$   
=  $\{4, 6, 8, 10\}$ .....(i)

R.H.S =  $A' \cap B'$

$A' = U - A$   
=  $\{1, 2, 3, 4, \dots, 10\} - \{1, 3, 5, 7, 9\}$   
=  $\{2, 4, 6, 8, 10\}$

$B' = U - B$

$$= \{1, 2, 3, 4, 5, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$$

$$= \{4, 6, 8, 10\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B)' = A' \cap B'$$

$$(ii) (A \cap B)' = A' \cup B'$$

$$\text{L.H.S} = (A \cap B)'$$

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$$

$$= \{3, 5, 7\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$= \{1, 2, 3, 4, \dots, 10\} - \{3, 5, 7\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\} \dots \dots \dots (i)$$

$$\text{R.H.S} = A' \cup B'$$

$$A' = U - A$$

$$= \{1, 2, 3, 4, 5, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B' = U - B$$

$$= \{1, 2, 3, 4, 5, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{So } (A \cap B)' = A' \cup B'$$

**Q.4** If  $U = \{1, 2, 3, \dots, 20\}$

$$X = \{1, 3, 7, 9, 15, 18, 20\}$$

$$Y = \{1, 3, 5, \dots, 17\} \text{ then show that,}$$

$$(i) X - Y = X \cap Y' \quad (ii) Y - X = Y \cap X'$$

**Solution:**

$$(i) X - Y = X \cap Y'$$

$$\text{L.H.S} = X - Y$$

$$= \{1, 3, 7, 9, 15, 18, 20\} -$$

$$\{1, 3, 5, 7, 9, 11, 13, 15, 17\}$$

$$= \{18, 20\} \dots \dots \dots (i)$$

$$\text{R.H.S} = X \cap Y'$$

$$Y' = U - Y$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 5, \dots, 17\}$$

$$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$$

$$X \cap Y' = \{1, 3, 7, 9, 15, 18, 20\} \cap$$

$$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20\}$$

$$= \{18, 20\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S,}$$

$$\text{So, } X - Y = X \cap Y'$$

$$(ii) Y - X = Y \cap X'$$

$$\text{L.H.S} = Y - X$$

$$= \{1, 3, 5, \dots, 17\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{5, 11, 13, 17\} \dots \dots \dots (i)$$

$$\text{R.H.S} = Y \cap X'$$

$$X' = U - X$$

$$= \{1, 2, 3, \dots, 20\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16,$$

$$17, 19\}$$

$$Y \cap X' = \{1, 3, 5, 7, 9, 11, 13, 15, 17\} \cap$$

$$\{2, 4, 5, 6, 8, 10, 11, 12, 13, 14,$$

$$16, 17, 19\}$$

$$= \{5, 11, 13, 17\} \dots \dots \dots (ii)$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{So, } Y - X = Y \cap X'$$

## Verification of Fundamental

### Properties of Sets

(a) Commutative Property of Union i.e.,

$$A \cup B = B \cup A$$

For example  $A = \{1, 3, 5, 7\}$  and

$$B = \{2, 3, 5, 7\}$$

Then  $A \cup B = \{1, 3, 5, 7\} \cup \{2, 3, 5, 7\}$

$$= \{1, 2, 3, 5, 7\}$$

and  $B \cup A = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7\}$

$$= \{1, 2, 3, 5, 7\}$$

Hence, verified that  $A \cup B = B \cup A$

(b) Commutative property of intersection

$$\text{i.e., } A \cap B = B \cap A$$

For example  $A = \{1, 3, 5, 7\}$  and

$$B = \{2, 3, 5, 7\}$$

Then  $A \cap B = \{1, 3, 5, 7\} \cap \{2, 3, 5, 7\}$

$$= \{3, 5, 7\}$$

and  $B \cap A = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7\}$

$$= \{3, 5, 7\}$$

Hence, verified that  $A \cap B = B \cap A$

(c) Associative Property of Union

$$\text{i.e., } (A \cup B) \cup C = A \cup (B \cup C).$$

Suppose  $A = \{1, 2, 4, 8\}$ ,  $B = \{2, 4, 6\}$

and  $C = \{3, 4, 5, 6\}$  then

$$\text{L.H.S} = (A \cup B) \cup C$$

$$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cup \{3, 4, 5, 6\}$$

$$= \{1, 2, 4, 6, 8\} \cup \{3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

and  $\text{R.H.S} = A \cup (B \cup C)$

$$= \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cup \{3, 4, 5, 6\})$$

$$= \{1, 2, 4, 8\} \cup \{2, 3, 4, 5, 6\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S}$$

Hence, union of sets is associative

(d) Associative Property of intersection

$$\text{i.e., } (A \cap B) \cap C = A \cap (B \cap C)$$

Suppose  $A = \{1, 2, 4, 8\}$

$$B = \{2, 4, 6\}$$

and  $C = \{3, 4, 5, 6\}$

Then  $\text{L.H.S} = (A \cap B) \cap C$

$$= (\{1, 2, 4, 8\} \cap \{2, 4, 6\}) \cap \{3, 4, 5, 6\}$$

$$= \{2, 4\} \cap \{3, 4, 5, 6\}$$

$$= \{4\}$$

and  $\text{R.H.S} = A \cap (B \cap C)$

$$= \{1, 2, 4, 8\} \cap (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$$

$$= \{1, 2, 4, 8\} \cap \{4, 6\}$$

$$= \{4\}$$

$$\text{L.H.S} = \text{R.H.S}$$

Hence, intersection of sets is associative law.

(e) Distributive Property of Union over Intersection

$$\text{i.e., } A \cup (B \cap C) = (A \cup B) \cap (A \cup C):$$

Suppose  $A = \{1, 2, 4, 8\}$

$$B = \{2, 4, 6\} \text{ and}$$

$$C = \{3, 4, 5, 6\}$$

Let  $\text{L.H.S} = A \cup (B \cap C)$

$$= \{1, 2, 4, 8\} \cup (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$$

$$= \{1, 2, 4, 8\} \cup \{4, 6\}$$

$$= \{1, 2, 4, 6, 8\}$$

$\text{R.H.S} = (A \cup B) \cap (A \cup C)$

$$(A \cup B) = (\{1, 2, 4, 8\} \cup \{2, 4, 6\})$$

$$= \{1, 2, 4, 6, 8\}$$

$$(A \cup C) = (\{1, 2, 4, 8\} \cup \{3, 4, 5, 6\})$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$(A \cup B) \cap (A \cup C)$

$$= \{1, 2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$$

$$= \{1, 2, 4, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S}$$

**(f) Distributive Property of Intersection over Union**

i.e.,  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Suppose

$$A = \{1, 2, 3, 4, 5, \dots, 20\}$$

$$B = \{5, 10, 15, 20, 25, 30\}$$

$$C = \{3, 9, 15, 21, 27, 33\}$$

$$\text{L.H.S} = A \cap (B \cup C)$$

$$= A \cap (\{5, 10, 15, 20, 25, 30\} \cup \{3, 9, 15, 21, 27, 33\})$$

$$= \{1, 2, 3, 4, 5, \dots, 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$$

$$\text{L.H.S} = \{3, 5, 9, 10, 15, 20\}$$

$$\text{R.H.S} = (A \cap B) \cup (A \cap C)$$

$$(A \cap B) = \{1, 2, 3, 4, \dots, 20\} \cap \{5, 10, 15, 20, 25, 30\} = \{5, 10, 15, 20\}$$

$$(A \cap C) = \{1, 2, 3, 4, 5, \dots, 20\} \cap \{3, 9, 15, 21, 27, 33\} = \{3, 9, 15\}$$

$$(A \cap B) \cup (A \cap C) = \{5, 10, 15, 20\} \cup \{3, 9, 15\} = \{3, 5, 9, 10, 15, 20\}$$

$$\text{L.H.S} = \text{R.H.S}$$

**(g) De Morgan's laws**

If set A and B are the subsets of universal set U then De Morgan's laws are expressed as.

(i)  $(A \cup B)' = A' \cap B'$

(ii)  $(A \cap B)' = A' \cup B'$

Proof:

(i)  $(A \cup B)' = A' \cap B'$

Suppose

$$U = \{1, 2, 3, 4, \dots, 10\}$$

$$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{1, 3, 5, 7, 9\}$$

$$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$$

$$\text{L.H.S} = (A \cup B)'$$

$$(A \cup B) = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$\text{L.H.S} = (A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, \dots, 10\} - \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$\text{L.H.S} = \{7, 9\} \dots \dots \dots \text{(i)}$$

$$\text{R.H.S} = A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{7, 8, 9, 10\} = \{7, 9\} \dots \dots \dots \text{(ii)}$$

From (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

**(ii)  $(A \cap B)' = A' \cup B'$**

Suppose

$$U = \{1, 2, 3, 4, \dots, 10\}$$

$$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{1, 3, 5, 7, 9\}$$

$$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$$

Let  $\text{L.H.S} = (A \cap B)'$

$$A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6\} = \{2, 4, 6\}$$

$$\text{L.H.S} = (A \cap B)' = U - (A \cap B) = \{1, 2, 3, 4, \dots, 10\} - \{2, 4, 6\} = \{1, 3, 5, 7, 8, 9, 10\} \dots \dots \dots \text{(i)}$$

$$\text{R.H.S} = A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{7, 8, 9, 10\} = \{1, 3, 5, 7, 8, 9, 10\} \dots \dots \dots \text{(ii)}$$

From (i) and (ii)

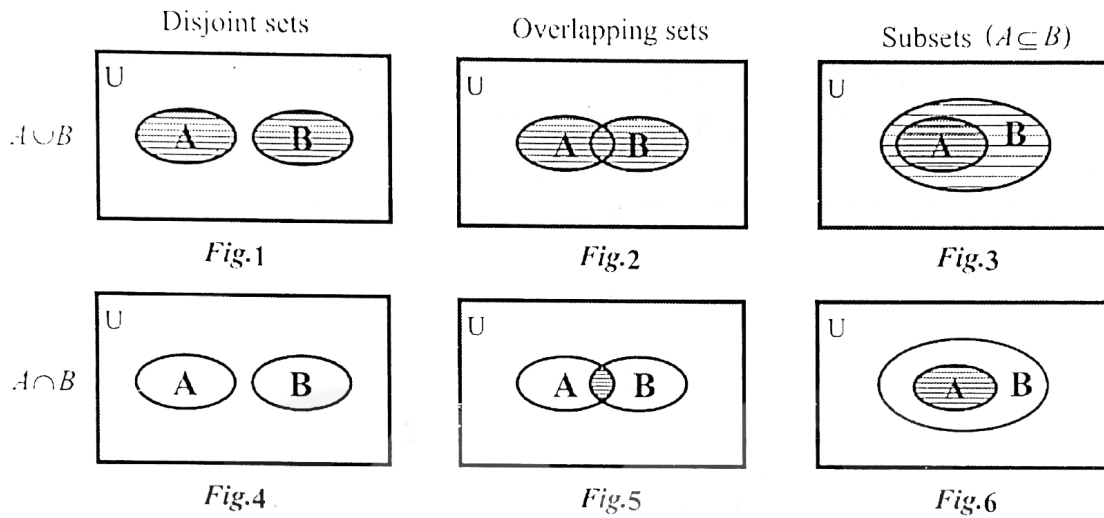
$$\text{L.H.S} = \text{R.H.S}$$

# Venn Diagram

British mathematician John Venn (1834–1923) introduced rectangle for a universal set  $U$  and its subsets  $A$  and  $B$  as closed figures inside this rectangle.

## Use Venn diagrams to represent:

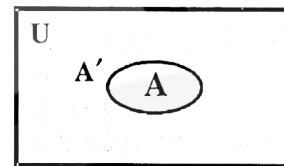
(a) Union and intersection of sets



Regions shown by horizontal line segments in figures 1 to 6 shows  $A \cup B$  and  $A \cap B$

(b) Complement of a set

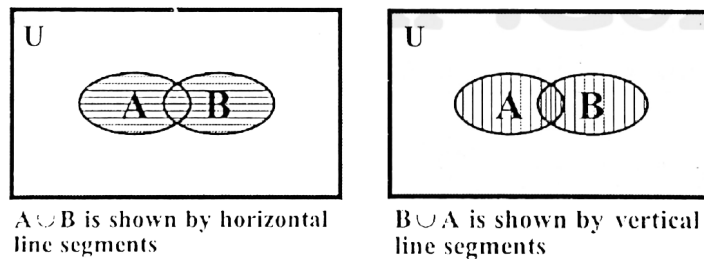
$U - A = A'$  is shown by Shaded area.



Use Venn diagram to verify:

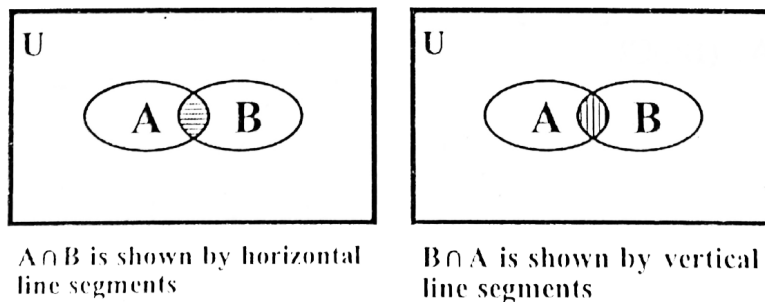
Commutative law for union and intersection of sets.

• Commutative Law for Union:



The region shown in both cases are equal. Thus  $A \cup B = B \cup A$ .

• Commutative Law for Intersection:



The regions shown in both cases are equal. Thus  $A \cap B = B \cap A$

(c) De Morgan's laws

(i)  $(A \cup B)' = A' \cap B'$

(ii)  $(A \cap B)' = A' \cup B'$

(i)  $(A \cup B)' = A' \cap B'$

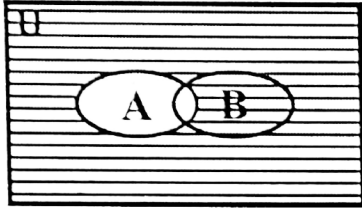


Fig. 1: A is shown by horizontal line segments

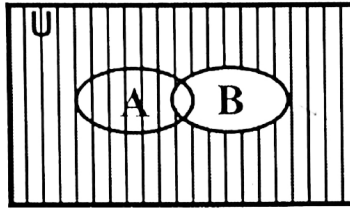


Fig. 2: B is shown by vertical line segments

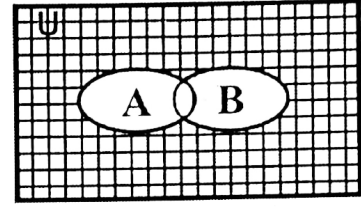


Fig. 3:  $A' \cap B'$  is shown by squares

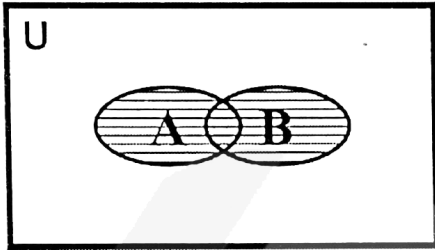


Fig. 4:  $A \cup B$  is shown by horizontal line segments

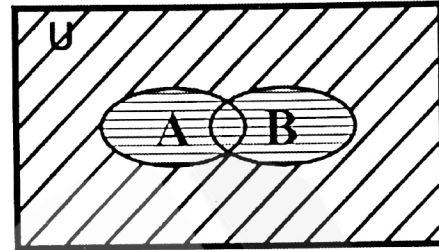
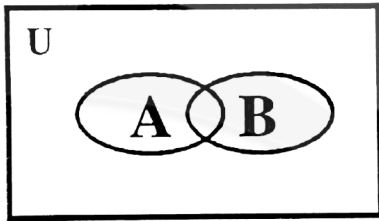


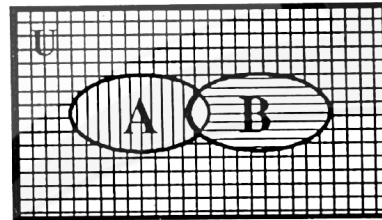
Fig. 5:  $(A \cup B)'$  is shown by slanting line segments

Regions shown in figure 3 and 5 are equal thus  $(A \cup B)' = A' \cap B'$ .

(ii)  $(A \cap B)' = A' \cup B'$



$U - (A \cap B) (= A \cap B)'$  is shown by shading



$A' \cup B'$  is shown by squares, horizontal and vertical line segments.

Regions shown in fig. 6 and fig. 7 are equal.

Thus  $(A \cap B)' = A' \cup B'$

(d) Associative law of Union and Intersection:

(i) Associative law of Union:  $(A \cup B) \cup C = A \cup (B \cup C)$

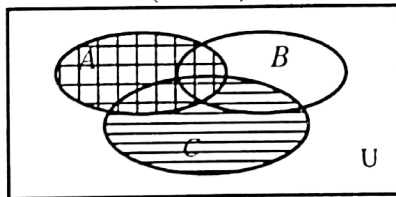


Fig-1

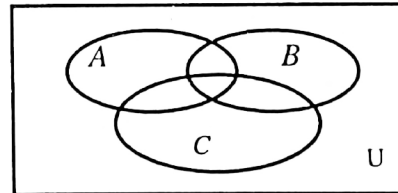


Fig-2

$(A \cup B) \cup C$  is shown in the above Fig-1.  $A \cup (B \cup C)$  is shown in the above Fig-2.

Regions shown in Fig. 1 and Fig. 2 by different ways are equal.

Thus  $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) Associative law of Intersection:  $(A \cap B) \cap C = A \cap (B \cap C)$

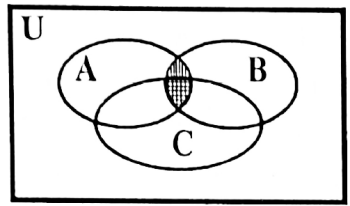


Fig. 3

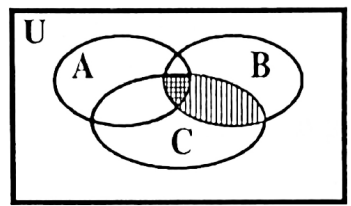


Fig. 4

$(A \cap B) \cap C$  is shown in figure 3 by double crossing line segments.

$A \cap (B \cap C)$  is shown in figure 4 by double crossing line segments.

Regions shown in Fig 3 and Fig. 4 are equal.

Thus  $(A \cap B) \cap C = A \cap (B \cap C)$

(e) Distributive law:

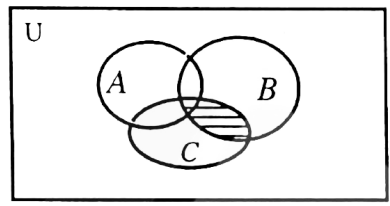


Fig. 1

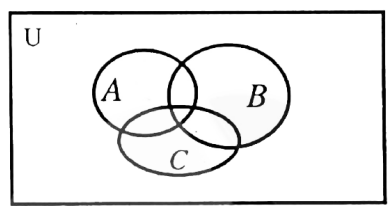


Fig. 2

Fig. 1:  $A \cup (B \cap C)$  is shown by horizontal line segments in the above figure 1.

Fig. 2:  $A \cap (B \cup C)$  is shown by horizontal line segments in the fig. 2.

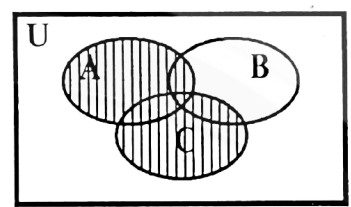


Fig. 3

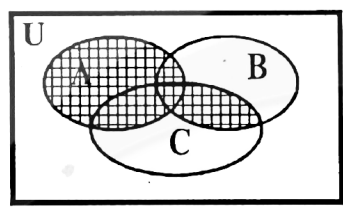


Fig. 4

Fig. 3:  $A \cup C$  is shown by vertical line segments in fig.3

Fig. 4:  $(A \cup B) \cap (A \cup C)$  is shown by double crossing line segments in fig. 4.

Regions shown in fig 1 and Fig.4 are equal. Thus  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

**Distributive Law of Intersection over Union:**

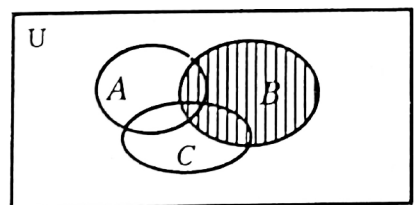


Fig.5

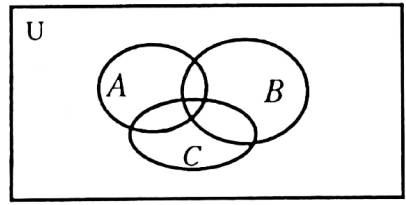


Fig.6

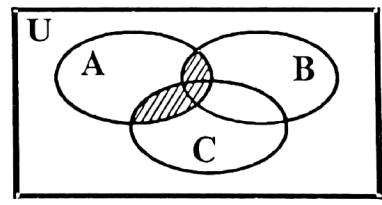


Fig. 7

Fig. 5:  $B \cup C$  is shown by vertical line segments in Fig 5.

Fig. 6:  $A \cap (B \cup C)$  is shown in Fig.6 by vertical line segments.

Fig. 7:  $(A \cap B) \cup (A \cap C)$  is shown in fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig. 7 are equal.

Thus  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$