EXERCISE 5.3

Q.1 If $U = \{1, 2, 3, 4, \ldots, 10\}$
$A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$
then verify the following questions:
Solution:

(i) **$A - B = A \cap B'$**

L.H.S $= A - B = \{1,3,5,7,9\} - \{1,4,7,10\}$
$= \{3, 5, 9\}$ .... (i)

R.H.S $= A \cap B'$

$B' = U - B = \{1, 2, 3, \ldots, 10\} - \{1, 4, 7, 10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{3, 5, 9\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

$A - B = A \cap B'$

(ii) **$B - A = B \cap A'$**

L.H.S $= B - A = \{1,4,7,10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$ .... (i)

R.H.S $= B \cap A'$

$A' = U - A = \{1,2,3, \ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B \cap A' = \{1, 4, 7, 10\} \cap \{2, 4, 6, 8, 10\}$
$= \{4, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

$B - A = B \cap A'$

(iii) **$(A \cup B)' = A' \cap B'$**

L.H.S $= (A \cup B)'$

$A \cup B = \{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}$
$= \{1, 3, 4, 5, 7, 9, 10\}$

$(A \cup B)' = U - (A \cup B)$
$= \{1,2,3,\ldots,10\} - \{1,3,4,5,7,9,10\}$
$= \{2, 6, 8\}$ .... (i)

R.H.S $= A' \cap B'$

$A' = U - A = \{1,2,3,\ldots, 10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1,2,3,\ldots, 10\} - \{1,4,7,10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\}$
$= \{2, 6, 8\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iv) **$(A \cap B)' = A' \cup B'$**

L.H.S $= (A \cap B)'$

$A \cap B = \{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}$
$= \{1, 7\}$

$(A \cap B)' = U - (A \cap B)$
$= \{1, 2, 3, \ldots, 10\} - \{1, 7\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$ ...... (i)

R.H.S $= A' \cup B'$

$A' = U - A = \{1,2,3,\ldots,10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$B' = U - B = \{1,2,3,\ldots, 10\} - \{1, 4,7,10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{2, 3, 5, 6, 8, 9\}$
$= \{2, 3, 4, 5, 6, 8, 9, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(v) **$(A - B)' = A' \cup B$**

L.H.S $= (A - B)'$

$A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$
$= \{3, 5, 9\}$

$(A - B)' = U - (A - B)$
$= \{1, 2, 3, \ldots, 10\} - \{3, 5, 9\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$ .... (i)

R.H.S $= A' \cup B$

$A' = U - A = \{1,2,3, \ldots,10\} - \{1,3,5,7,9\}$
$= \{2, 4, 6, 8, 10\}$

$A' \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\}$
$= \{1, 2, 4, 6, 7, 8, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(vi) **$(B - A)' = B' \cup A$**

L.H.S $= (B - A)'$

$B - A = \{1, 4, 7, 10\} - \{1, 3, 5, 7, 9\}$
$= \{4, 10\}$

$(B - A)' = U - (B - A)$
$= \{1, 2, 3, \ldots, 10\} - \{4, 10\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$ .... (i)

R.H.S $= B' \cup A$

$B' = U - B = \{1,2,3,4,\ldots,10\} - \{1, 4, 7, 10\}$
$= \{2, 3, 5, 6, 8, 9\}$

$B' \cup A = \{2,3,5,6,8,9\} \cup \{1, 3, 5, 7, 9\}$
$= \{1, 2, 3, 5, 6, 7, 8, 9\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.2 If $A = \{1, 3, 5, 7, 9\}$ $B = \{1, 4, 7, 10\}$ $C = \{1, 5, 8, 10\}$

Then verify the following:

Solution:

(i) $(A\cup B)\cup C = A\cup(B\cup C)$

L.H.S $= (A\cup B)\cup C$

$= (\{1,3,5,7,9\}\cup\{1,4,7,10\})\cup\{1,5,8,10\}$

$= \{1, 3, 4, 5, 7, 9, 10\}\cup\{1, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ .... (i)

R.H.S $= A\cup(B\cup C)$

$= \{1,3,5,7,9\}\cup(\{1,4,7,10\}\cup\{1,5,8,10\})$

$= \{1,3,5,7,9\}\cup\{1,4,5,7,8,10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) $(A\cap B)\cap C = A\cap(B\cap C)$

L.H.S $= (A\cap B)\cap C$

$= (\{1,3,5,7,9\}\cap\{1,4,7,10\})\cap\{1,5,8,10\}$

$= \{1, 7\}\cap\{1, 5, 8, 10\}$

$= \{1\}$ .... (i)

R.H.S $= A\cap(B\cap C)$

$= \{1,3,5,7,9\}\cap(\{1,4,7,10\}\cap\{1,5,8,10\})$

$= \{1, 3, 5, 7, 9\}\cap\{1, 10\}$

$= \{1\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iii) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

L.H.S $= A\cup(B\cap C)$

$= \{1,3,5,7,9\}\cup(\{1,4,7,10\}\cap\{1,5,8,10\})$

$= \{1, 3, 5, 7, 9\}\cup\{1, 10\}$

$= \{1, 3, 5, 7, 9, 10\}$ .... (i)

R.H.S $= (A\cup B)\cap(A\cup C)$

$A\cup B = \{1, 3, 5, 7, 9\}\cup\{1, 4, 7, 10\}$

$= \{1, 3, 4, 5, 7, 9, 10\}$

$A\cup C = \{1, 3, 5, 7, 9\}\cup\{1, 5, 8, 10\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

Now $(A\cup B)\cap(A\cup C)$

$= \{1,3,4,5,7,9,10\}\cap\{1,3,5,7,8,9,10\}$

$= \{1, 3, 5, 7, 9, 10\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(iv) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

L.H.S $= A\cap(B\cup C)$

$= \{1,3,5,7,9\}\cap(\{1,4,7,10\}\cup\{1, 5, 8, 10\})$

$= \{1, 3, 5, 7, 9\}\cap\{1, 4, 5, 7, 8, 10\}$

$= \{1, 5, 7\}$ .... (i)

R.H.S $= (A\cap B)\cup(A\cap C)$

$A\cap B = \{1,3,5,7,9\}\cap\{1, 4, 7, 10\}$

$= \{1, 7\}$

$A\cap C = \{1,3, 5, 7, 9\}\cap\{1, 5, 8, 10\}$

$= \{1, 5\}$

Now $(A\cap B)\cup(A\cap C) = \{1,7\}\cup\{1,5\}$

$= \{1,5,7\}$....... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.3 If $U = \mathbb{N}$, then verify De-Morgan's laws by using:

$A = \phi$, $B = \mathbb{P}$

Solution:

$A = \{\ \}$

$B = \{2, 3, 5, 7, \ldots\}$

$U = \{1, 2, 3, 4, 5, 6, 7\ldots\}$

(i) $(A\cup B)' = A'\cap B'$

L.H.S $= (A\cup B)'$

$A\cup B = \{\ \}\cup\{2, 3, 5, 7, \ldots\}$

$= \{2, 3, 5, 7, \ldots\}$

$(A\cup B)' = U - (A\cup B)$

$= \{1, 2, 3, 4,5,6,7 \ldots\} - \{2, 3, 5, 7, \ldots$

$= \{1, 4, 6, \ldots\}$ .... (i)

R.H.S $= A'\cap B'$

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7\ldots\} - \{\ \}$

$= \{1, 2, 3, 4, 5, 6, 7\ldots\}$

$B' = U - B = \{1,2,3,4,5,6,7\ldots\} - \{2,3,5,7,\ldots\}$

$= \{1, 4, 6, \ldots\}$

$A'\cap B' = \{1,2,3,4,5,6,7 \ldots\}\cap\{1, 4, 6, \ldots\}$

$= \{1, 4, 6, \ldots\}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) $(A \cap B)' = A' \cup B'$

L.H.S $= (A \cap B)'$

$(A \cap B) = \{\ \} \cap \{2, 3, 5, 7, \ldots\}$

$= \{\ \}$

$(A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7 \ldots\} - \{\ \}$

$= \{1, 2, 3, 4, 5, 6, 7 \ldots\}$....... (i)

R.H.S $= A' \cup B'$

$A' = U - A = \{1, 2, 3, 4, \ldots\} - \{\ \}$

$= \{1, 2, 3, 4, \ldots\}$

$B' = U - B = \{1,2,3,4,5,6,7 \ldots\} - \{2,3,5,7,\ldots\}$

$= \{1, 4, 6, \ldots\}$

$A' \cup B' = \{1,2,3,4,5,6,7 \ldots\} \cup \{1, 4, 6, \ldots\}$

$= \{1, 2, 3, 4, \ldots\}$....... (ii)

From (i) and (ii)

L.H.S = R.H.S

Q.4 If $\cup = \{1, 2, 3, 4, \ldots 10\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 4, 5, 8\}$ then prove the following questions by Venn Diagram.

Solution: $A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 4, 5, 8\} = \{3, 5\}$

So given sets A and B are overlapping sets

(i) **$A - B = A \cap B'$**

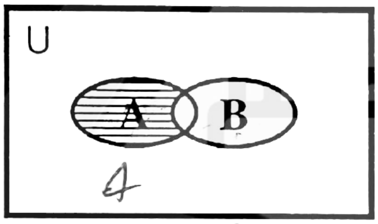

Fig: 1 (A-B)

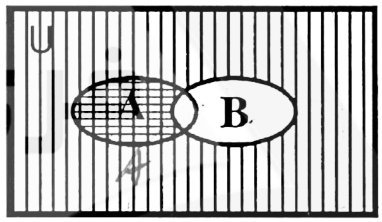

Fig: 2)$A \cap B'$(

- $A - B$ is shown by horizontal line segments in fig. 1.
- B' is shown by vertical line segments and squares in fig. 2.
- $A \cap B'$ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus $A - B = A \cap B'$

(ii) **$B - A = B \cap A'$**

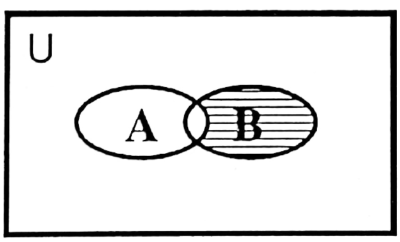

Fig. 1 (B - A)

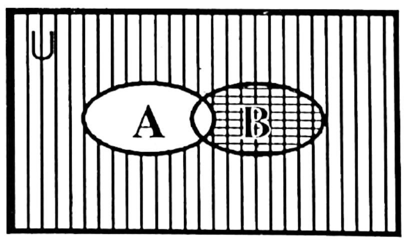

Fig: 2 ($B \cap A'$)

- $B - A$ is shown by horizontal line segments in fig. 1.
- A' is shown by vertical line segments and squares in fig. 2.
- $B \cap A'$ is shown by squares in fig. 2.

Regions shown in fig. 1 and fig. 2 are equal, thus $B - A = B \cap A'$.

(iii) **$(A \cup B)' = A' \cap B'$ (De-Morgan's Law)**

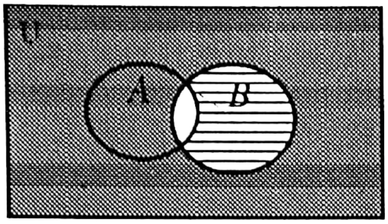

Fig.1

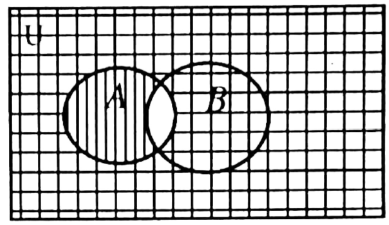

Fig.2

- $A \cup B$ is shown by horizontal line segments in Fig. 1.
- $(A \cup B)'$ is shown by shaded area in Fig. 1.
- $A'$is shown by horizontal line segments and squares in Fig. 2.
- B' is shown by vertical line segments and squares in Fig. 2.
- $A' \cap B'$ is shown by squares in Fig. 2.

Shaded area shown in Fig. 1 and square area shown in Fig. 2 are equal.

thus $(A \cup B)' = A' \cap B'$

(iv) **$(A \cap B)' = A' \cup B'$**

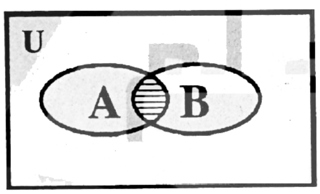

Fig. 1

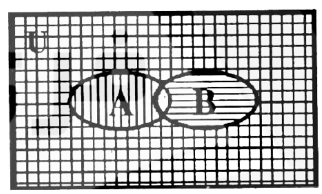

Fig. 2

- $A \cap B$ is shown by horizontal line segments in Fig. 1.
- $(A \cap B)'$ is shown by shaded area in Fig. 1.
- A' is shown by horizontal line segments and squares in Fig. 2.
- B' is shown by vertical line segments and squares in Fig. 2.
- $A' \cup B'$ is shown by squares, horizontal and vertical line segments in Fig. 2.

Shaded area shown in Fig. 1 and area of squares, vertical and horizontal line segments shown in Fig. 2 are equal. thus $(A \cap B)' = A' \cup B'$

(v) **$(A - B)' = A' \cup B$**

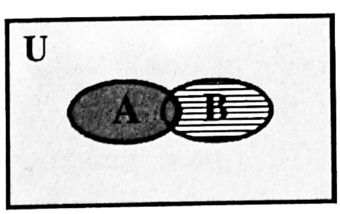

Fig. 1

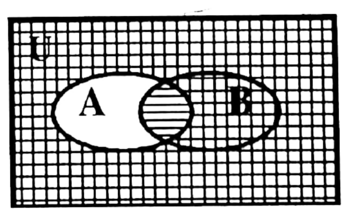

Fig. 2

- $A - B$ is shown by horizontal line segments in Fig. 1.
- $(A - B)'$ is shown by shaded area in Fig.1.
- A' is shown by squares Fig. 2.
- $A' \cup B$ is shown by squares and horizontal line segments in Fig. 2.

Shaded area in Fig. 1 and area of squares & horizontal line segments in Fig. 2 are equal.

thus $(A - B)' = A' \cup B$

(vi) $(B-A)' = B' \cup A$

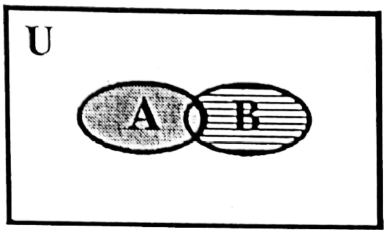

Fig. 1

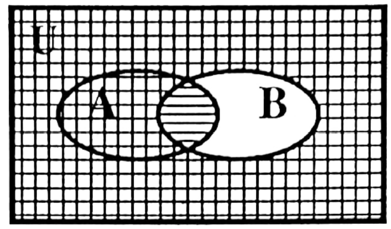

Fig. 2

- $B-A$ is shown by horizontal line segments in Fig. 1.
- $(B-A)'$ is shown by shaded area in Fig.1.
- B' is shown by squares in Fig. 2.
- $B' \cup A$ is shown by squares and horizontal line segments in Fig. 2.

  Shaded area in Fig. 1 and area of squares & vertical line segments in Fig. 2 are equal.
  thus $(B-A)' = B' \cup A$

Ordered Pairs and Cartesian Product:

(a) Ordered pairs

Any two numbers x and y, written in the form (x, y) is called an ordered pair. In an ordered pair (x, y) the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, iff $x=s$ and $y = t$

(b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Example: If A = {1, 2, 3} and B = {2, 5}, then find A × B and B × A

Solution:

$A \times B = \{1,2,3\} \times \{2,5\}$

$A \times B = \{(1,2),(1,5),(2,2),(2,5),(3,2)(3, 5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A \times B$ has $3 \times 2 = 6$ ordered pairs. We note that

$B \times A = \{(2,1),(2,2),(2,3),(5,1),(5,2), (5,3)\}$

Evidently $A \times B \neq B \times A$