EXERCISE 5.4

Q.1 If $A = \{a, b\}$ and $B = \{c, d\}$, then find $A \times B$ and $B \times A$

Solution:

$$A \times B = \{a, b\} \times \{c, d\}$$
$$= \{(a, c), (a, d), (b, c), (b, d)\}$$
$$B \times A = \{c, d\} \times \{a, b\}$$
$$= \{(c, a), (c, b), (d, a), (d, b)\}$$

Q.2 If $A = \{0, 2, 4\}$, $B = \{-1,3\}$, then find $A\times B$, $B \times A$, $A \times A$, $B \times B$

Solution:

$$A \times B = \{0, 2, 4\} \times \{-1, 3\}$$
$$=\{(0,-1),(0,3),(2,-1),(2,3),(4,-1),(4,3)\}$$
$$B \times A = \{-1, 3\} \times \{0, 2, 4\}$$
$$= \{(-1,0),(-1,2),(-1,4),(3,0),(3,2),(3,4)\}$$
$$A\times A = \{0, 2, 4\} \times \{0, 2, 4\}$$
$$=\{(0,0),(0,2),(0,4),(2,0),(2,2),(2,4),(4,0)(4,2),(4,4)\}$$
$$B \times B = \{-1, 3\} \times \{-1, 3\}$$
$$= \{(-1, -1), (-1, 3), (3, -1), (3, 3)\}$$

Q.3 Find a and b if

Solution:

(i) $(a - 4, b - 2) = (2, 1)$

$a - 4 = 2$, $b - 2 = 1$

$a = 2+ 4$, $b = 1 + 2$

$\boxed{a = 6}$, $\boxed{b = 3}$

(ii) $(2a + 5, 3) = (7, b - 4)$

$2a + 5 = 7$, $3 = b - 4$

$2a = 7 - 5$, $3 + 4 = b$

$2a = 2$, $7 = b$

$a = \frac{2}{2} = 1$, $\boxed{b = 7}$

$\boxed{a = 1}$

(iii) (3 – 2a, b – 1) = (a – 7, 2b + 5)

$3 - 2a = a - 7$, $b - 1 = 2b + 5$

$3 + 7 = a + 2a$, $-1 - 5 = 2b - b$

$10 = 3a$, $-6 = b$

$\frac{10}{3} = a$ $\boxed{b = -6}$

$\boxed{a = \frac{10}{3}}$

Q.4 Find the sets X and Y if

$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

Solution:

$X \times Y = \{(a, a), (b, a), (c, a), (d, a)\}$

$X \times Y = \{a, b, c, d\} \times \{a\}$

$X = \{a, b, c, d\}$

$Y = \{a\}$

Q.5 If X = {a, b, c} and Y = {d, e}, then find the number of elements in

(i) $X \times Y$ (ii) $Y \times X$ (iii) $X \times X$

Solution:

No. of elements in X = 3

No. of elements in Y = 2

(i) No. of Elements in $X \times Y = 3 \times 2 = 6$

(ii) No. of Elements in $Y \times X = 2 \times 3 = 6$

(iii) No. of Elements in $X \times X = 3 \times 3 = 9$

Binary Relation:

If A and B are any two non-empty sets, then a subset $R \subseteq A \times B$ is called binary relation from set A into set B, because there exists some relationship between first and second element of each ordered pair in R.

Domain of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

Range of relation denoted by Rang R is the set consisting of all the second elements of each ordered pair in the relation.

Example 1: Suppose

$A = \{2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$

Form a relation

$R: A \to B = \{x\ R\ y \mid y = 2x \text{ for } x \in A, y \in B\}$

$R = \{(2, 4), (3, 6), (4, 8)\}$

Dom $R = \{2, 3, 4\} \subseteq A$ and

Rang $R = \{4, 6, 8\} \subseteq B$

Example 2: Suppose

$A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 5\}$

Form a relation

$R: A \to B = \{x\ R y \mid x + y = 6 \text{ for } x \in A, y \in B\}$

$R = \{(1, 5), (3, 3), (4, 2)\}$

Dom $R = \{1, 3, 4\} \subseteq A$ and

Rang $R = \{2, 3, 5\} \subseteq B$

Function or Mapping:

(i) Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if (i) Dom f = A (ii) every $x \in A$ appears in one and only one ordered pair in f.

Alternate Definition:

Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if (i) Dom f = A (ii) $\forall x \in A$ we can associate some unique image element $y = f(x) \in B$.

Domain, Co-domain and Range of Function:

If $f : A \to B$ is a function then A is called the domain of f and B is called the co-domain of f.

Domain f is the set consisting of all first elements of each ordered pair in f and range f is the set consisting of all second elements of each ordered pair in f.

Example:

Suppose $A = \{0,1,2,3\}$ and $B = \{1,2,3,4,5\}$

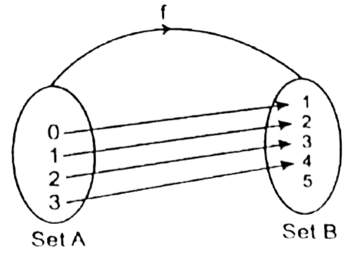
Define a function $f : A \to B$

$f = \{(x, y) \mid y = x + 1\ \forall x \in A, y \in B\}$

$f = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$

Dom $f = \{0, 1, 2, 3\} = A$

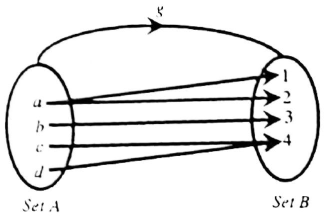
Rang $f = \{1, 2, 3, 4\} \subseteq B$.

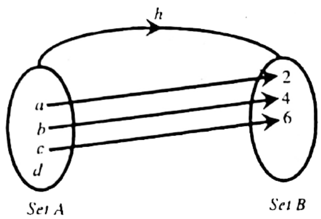


The following are the examples of relations but not functions.

g is not a function, because an element $a \in A$ has two images in set B.

And h is not a function because an element $d \in A$ has no images in set B.

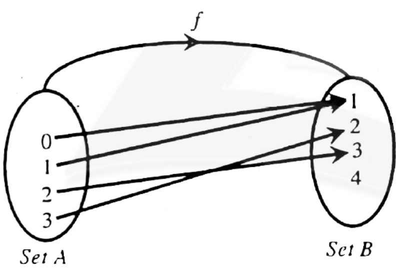




Demonstrate the following:

(a) Into function:

A function $f : A \rightarrow B$ is called an into function, if at least one element in B is not an image of some element of set A i.e, Range of $f \subset$ set B.



For example, we define a function $f: A \rightarrow B$ such that.

$f : A \rightarrow B$ such that

$f = \{(0, 1), (1, 1), (2, 3), (3, 2)$

where $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$

f is an into function.

(b) One- one function:

A function $f : A \rightarrow B$ is called one – one function, if all distinct elements of A have distinct images in B, i.e:

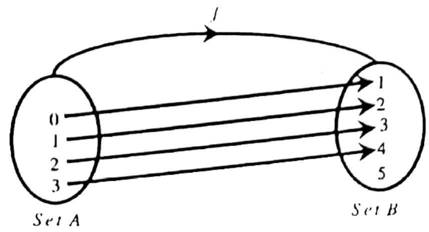
$f(x_1) = f(x_2) \quad x_1 = x_2 \in A$ or

$\forall x_1 \neq x_2 \in A \quad f(x_1) \neq f(x_2)$

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$, then we define a function $f : A \rightarrow B$ such that

$f = \{(x, y) \mid y = x + 1, \forall x \in A, y \in B\}$

$= \{(0, 1), (1, 2), (2, 3), (3, 4)\}$



f is one-one function

(c) Into and one - one function: (Injective function)

The function f discussed in (b) is also an into function. Thus f is an into and one-one function.

(d) An onto or surjective function:

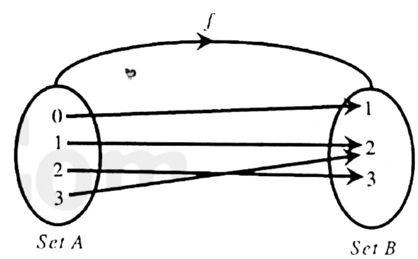
A function $f : A \rightarrow B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e, Range of $f = B$

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \rightarrow B$ such that

$f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}$

Here Rang $f = \{1, 2, 3\} = B$

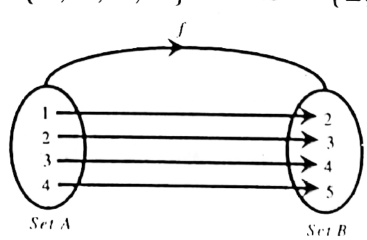
Thus f so defined is an onto function.



(e) Bijective function or one to one correspondence:

A function $f : A \rightarrow B$ is called bijective function if function f is one- one and onto.

e.g., if $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5\}$



We define a function $f : A \rightarrow B$ such that

$f = \{(x, y) \mid y = x + 1, \forall x \in A, y \in B\}$

Then $f = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of B is the image of at least one element of A.

Note: (1) Every function is a relation but converse may not be true.

(2) Every function may not be one – one.

(3) Every function may not be onto.

Example: Suppose A = {1, 2, 3}

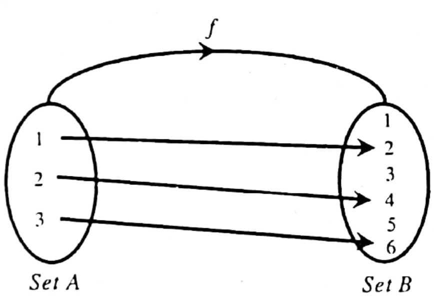
B = {1, 2, 3, 4, 5, 6}

We define a function

$f : A \to B = \{(x, y) \mid y=2x, \forall\ x \in A,\ y \in B\}$

Then f = {(1, 2), (2, 4), (3, 6)}

Evidently this function is one-one but not an onto.



Examine whether a given relation is a function:

A relation in which each x ∈ its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one-one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. The domain of f is A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n(A) = n(B).

EXERCISE 5.5

Q.1 If L = {a, b, c}, M = {3, 4}, then find two binary relations of L×M and M×L.

Solution: L = {a, b, c}, M = {3, 4}

L×M = {a, b, c} × {3, 4}

= {(a, 3),(a, 4), (b, 3), (b, 4), (c, 3), (c, 4)}

Two binary Relations:

R_1 = {(a, 3), (a, 4)}

R_2 = {(b, 4), (c, 3), (c, 4)}

M×L = {3, 4} × {a, b, c}

={(3, a),(3, b),(3, c),(4, a),(4, b),(4, c)}

Two binary Relations:

R_1 = {(3, a), (3, b)}

R_2 = {(4, b), (4, c)}

Q.2 If Y = {-2, 1, 2}, then make two binary relations for Y × Y. Also find their domain and range.

Solution: Y = {-2, 1, 2}

Y×Y = {-2, 1, 2}×{-2, 1, 2}

={(-2,-2),(-2,1),(-2,2),(1,-2),(1,1),(1,2),
(2,-2,) (2,1),(2,2)}

R_1 = {(-2, 1), (-2, 2), (1, -2)}

Domain R_1 = {-2, 1}

Range R_1 = {1, 2, -2}

R_2 = {(1, 1), (2, -2), (2, 2)}

Domain R_2 = {1, 2}

Range R_2 = {1, -2, 2}

Q.3 If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each.

(i) L×L (ii) L×M (iii) M×M

Solution: L = {a, b, c} , M = {d, e, f, g}

(i) $L \times L = \{a, b, c\} \times \{a, b, c\}$

$L \times L = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),$
$(c,a), (c, b),(c, c)\}$

Two binary Relations:

$R_1 = \{(a, a), (a, b), (a, c)\}$

$R_2 = \{(b, c), (c, a), (c, b)\}$

(ii) $L \times M$

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$= \{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),(b,f),$
$(b,g), (c, d), (c, e), (c, f), (c, g)\}$

Two binary Relations:

$R_1 = \{(a, d), (a, e), (a, f)\}$

$R_2 = \{(b, d), (b, e), (b, f)\}$

(iii) $M \times M$

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$=\{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),(e,f),(e,g),$
$(f,d),(f,e),(f,f),(f,g),(g,d),(g,e),(g,f),(g,g)\}$

Two binary Relations:

$R_1= \{(d, e), (d, f), (d, g)\}$

$R_2 = \{(f, d), (g, d)\}$

Q.4 If set M has 5 elements then find the number of binary relations in M.

No. of binary relations in $M=2^{5\times5}=2^{25}$

Q.5 If $L = \{x|x \in N \wedge x \leq 5\}$, $M = \{y|y \in P \wedge y < 10\}$, then make the following relations from L to M. Also write Domain and Range of each Relation.

(i) $R_1 = \{(x, y) \mid y < x\}$,

(ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Solution

$L = \{1, 2, 3, 4, 5\}$,

$M = \{2, 3, 5, 7\}$

$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),$
$(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),$
$(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) $R_1 = \{(x, y) \mid y < x\}$

$R_1 = \{(3, 2), (4, 2), (4, 3), (5, 2), (5, 3)\}$

Domain $R_1 = \{3, 4, 5\}$

Range $R_1 = \{2, 3\}$

(ii) $R_2 = (x, y) \mid y = x\}$

$R_2 = \{(2, 2), (3, 3), (5, 5)\}$

Domain $R_2 = \{2, 3, 5\}$

Range $R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$

$R_3 = \{(1, 5), (3, 3), (4, 2)\}$

Domain $R_3 = \{1, 3, 4\}$

Range $R_3 = \{5, 3, 2\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

$R_4 = \{(1, 3), (3, 5), (5, 7)\}$

Domain $R_4 = \{1, 3, 5\}$

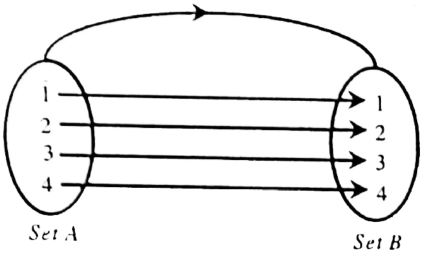
Range $R_4 = \{3, 5, 7\}$

Q.6 Indicate relations, into function, one-one function, onto function, and bijective function from the following. Also find their domain and the range.

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Domain $R_1 = \{1, 2, 3, 4\}$

Range $R_1 = \{1, 2, 3, 4\}$

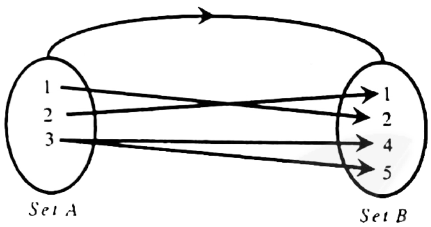


It is a bijective function.

(ii) $R_2 = \{(1, 2), (2, 1), (3, 4), (3, 5)\}$

Domain $R_2 = \{1, 2, 3\}$

Range $R_2 = \{1, 2, 4, 5\}$

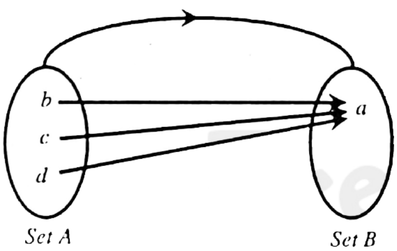


It is a relation. As 3 has no distinct image.

(iii) $R_3 = \{(b, a), (c, a), (d, a)\}$

Domain $R_3 = \{b, c, d\}$

Range $R_3 = \{a\}$

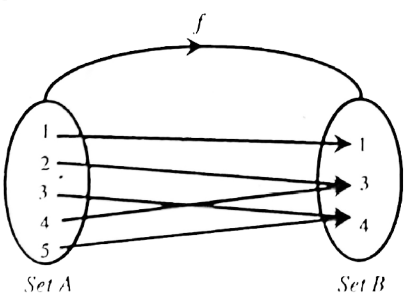


It is an onto function.

(iv) $R_4 = \{(1, 1), (2, 3), (3, 4), (4, 3), (5, 4)\}$

Domain $R_4 = \{1, 2, 3, 4, 5\}$

Range $R_4 = \{1, 3, 4\}$



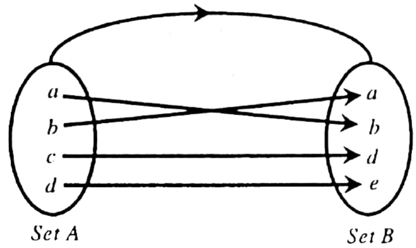
It is an onto function.

(v) $R_5 = \{(a, b), (b, a), (c, d), (d, e)\}$

Domain $R_5 = \{a, b, c, d\}$

Range $R_5 = \{a, b, d, e\}$

It is a bijective function.

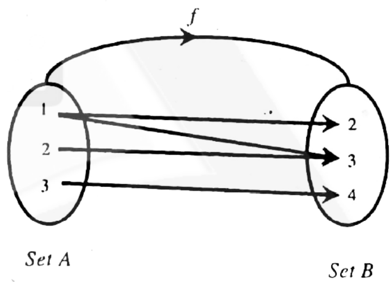


(vi) $R_6 = \{(1, 2), (2, 3), (1, 3), (3, 4)\}$

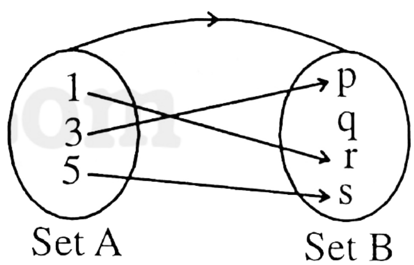
Domain $R_6 = \{1, 2, 3\}$

Range $R_6 = \{2, 3, 4\}$

It is a relation. As 1 has no distinct image.



(vii) $R_7 =$

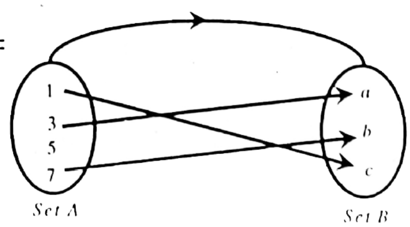


Domain $R_7 = \{1, 3, 5\}$

Range $R_7 = \{p, r, s\}$

R_7 is one-one function.

(viii) $R_8 =$



Domain $R_8 = \{1, 3, 7\}$

Range $R_8 = \{a, b, c\}$

It is a relation. As 5 has no distinct image.