EXERCISE 7.2

Q.1. Find θ, when:

(i) $l = 2\text{cm}, \quad r = 3.5 \text{ cm}$

Solution : Using rule

$$l = r\theta,$$
$$2 = 3.5\,\theta$$
$$\frac{2}{3.5} = \theta$$
$$\theta = 0.57 \text{ radian}$$

(ii) $l = 4.5$ m, $r = 2.5$ m

Solution : Using rule

$$l = r\theta,$$
$$\frac{\ }{r} = \theta$$
$$\frac{4.5}{2.5} = \theta$$
$$\theta = 1.8 \text{ radian}$$

Q.2. Find l, when

(i) $\theta = 180°, \; r = 4.9$ cm

Solution: As θ should be in radians so

$$\theta = 180°$$
$$= \cancel{180}\frac{\pi}{\cancel{180}} \text{ radian}$$
$$= \pi \text{ radian}$$

Using rule $l = r\theta$
$$= 4.9 \text{ cm} \times \pi$$
$$= 15.4 \text{ cm}$$

(ii) $\theta = 60°30'$, $r = 15$ mm 07(036)

Solution : As 'θ' should be in radians, so

$$\theta = 60°30'$$
$$= 60° + \frac{30}{60}^{\circ}$$
$$= 60° + 0.5°$$
$$= 60.5°$$
$$= 60.5\frac{\pi}{180} \text{ radian}$$
$$\theta = 1.056 \text{ radian}$$

Using rule $l = r\theta$
$$= 15 \text{ mm} \times 1.056$$
$$= 15.84 \text{ mm}$$

Q.3. Find r, when

(i) $l = 4$ cm, $\theta = \frac{1}{4}$ radian

Solution: Using rule $l = r\theta$

$$4\text{cm} = r\,\frac{1}{4}$$
$$4\text{cm} \times 4 = r$$
$$r = 16 \text{ cm}$$

(ii) $l = 52$ cm , $\theta = 45°$

Solution : As θ should be in radians.

$$\theta = 45°$$
$$= 45\frac{\pi}{180} \text{ radian}$$
$$= \frac{\pi}{4} \text{ radian}$$

Now using rule $l = r\theta$

$$52 \text{ cm} = r\,\frac{\pi}{4}$$
$$\frac{52\text{cm} \times 4}{\pi} = r$$
$$r = 66.21 \text{ cm}$$

Q.4. In a circle of radius 12m, find the length of an arc which subtends a central angle $\theta = 1.5$ radian.

Solution : Radius = $r = 12$m

Arc length = l ? =

Central angle = $\theta = 1.5$ radian

Using rule $l = r\theta$
$$l = 12\text{m} \times 1.5$$
$$l = 18\text{m}$$

Q.5. In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolution.

Solution: Radius = r = 10m

Number of revolutions = 3.5

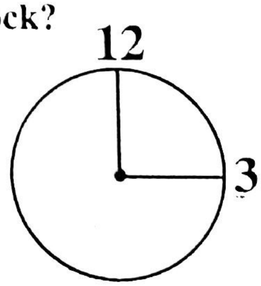
Angle of one revolution = 2π radian

Angle of 3.5 revolution = θ
$$= 3.5 \times 2\pi \text{ radian}$$
$$\theta = 7\pi \text{ radian}$$

Distance travelled = l ? =

Using rule $l = r\theta$
$$l = 10 \text{ m} \times 7\pi$$
$$l = 220 \text{ m}$$

Q.6. What is the circular measure of the angle between the hands of the watch at 3 O' clock?



Solution:

At 3 O' clock the minute hand will be at 12 and hour hand will be at 3 i.e the angle between the hands of watch will be one quarter of the central angle of full circle

i.e $= \frac{1}{4}$ of 360°

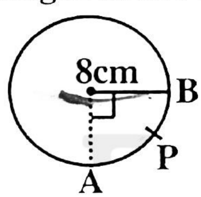
$= \frac{1}{4} \times 360°$

$= 90°$

$= 90 \frac{\pi}{180}$ radian

$= \frac{\pi}{2}$ radian.

Q.7. What is the length of arc APB?



Solution: From the figure we see that

Radius $= r = 8$cm

Central angle $= \theta$

$= 90°$

$= \frac{\pi}{2}$ radian

Arc length $= l$? =

By rule $l = r\,\theta$

$l = 4\not{8}\text{cm} \times \frac{\pi}{\not{2}}$

$l = 4\text{cm} \times \pi$

$l = 12.57$ cm

So, length of arc APB is 12.57 cm

Q.8. In a circle of radius 12 cm, how long an arc subtends a central angle of 84°?

Solution: Radius $= r = 12$cm

Arc length $= l$? =

Central angle $= \theta = 84°$

$84 \frac{\pi}{180}$ radian

$= 1.466$ radian

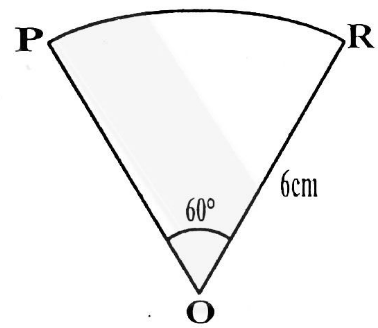
Now by rule $l = r\,\theta$

$= 12\text{cm} \times 1.466$

$= 17.6$ cm

Q.9. Find the area of sector OPR.

(a)



Radius $= r = 6$cm

Central angle $= \theta = 60°$

$= 60 \frac{\pi}{180}$ radian

$= \frac{\pi}{3}$ radian

Area of sector = ?

As Area of sector $= \frac{1}{2} r^2 \theta$

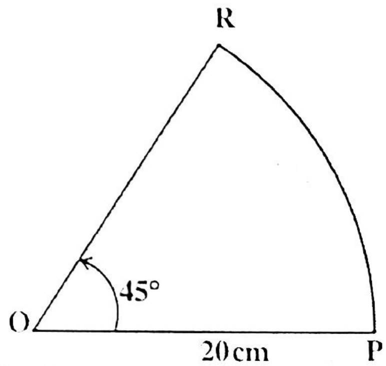
$= \frac{1}{2} \times (6\text{cm})^2 \times \frac{\pi}{3}$

$= \frac{1}{\not{6}} \times {}^{6}\not{36}\text{cm}^2 \times \pi$

$= 6\pi \text{ cm}^2$

$= 18.85 \text{ cm}^2$

(b)



Radius $= r = 20cm$

Central angle $= \theta = 45°$

$$= 45 \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{4} \text{ radian}$$

Area of sector = ?

Area of sector $= \frac{1}{2} r^2 \theta$

$$= \frac{1}{2}(20cm)^2 \times \frac{\pi}{4}$$

$$= \frac{400cm^2}{8} \times \pi$$

$$= 50\pi \ cm^2$$

$$= 157.1 \ cm^2$$

Q.10. Find area of sector inside a central angle of 20° in a circle of radius 7 m.

Solution: Area of sector = ?

Radius $= r = 7m$

Central angle $= \theta = 20°$

$$= 20 \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{9} \text{ radian}$$

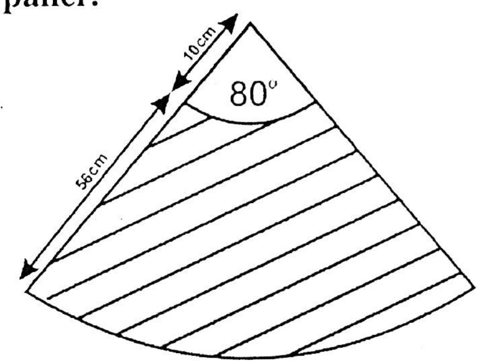
Area of sector $= \frac{1}{2} r^2 \theta$

$$= \frac{1}{2} \times (7m)^2 \times \frac{\pi}{9}$$

$$= \frac{49\pi}{18} m^2$$

$$= 8.55 \ m^2$$

Q.11. Sehar is making skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel?



Solution: Central angle $= \theta = 80°$

$$= 80 \frac{\pi}{180} \text{ radian}$$

$$= \frac{4\pi}{9} \text{ radian}$$

Radius of bigger sector $= R = (56 + 10)cm$

$R = 66 \ cm$

Radius of smaller sector $= r = 10 \ cm$

Shaded area = ?

Area of bigger sector $= \frac{1}{2} R^2 \theta$

$$= \frac{1}{\not{2}} \times (66cm)^2 \times \frac{2\not{4}\pi}{9}$$

$$= 484\,\not{4356}cm^2 \times \frac{2\pi}{\not{9}}$$

$$= 968\pi \ cm^2$$

Area of smaller sector $= \frac{1}{2} r^2 \theta$

$$= \frac{1}{\not{2}}(10cm)^2 \times \frac{2\not{4}\pi}{9}$$

$$= \frac{200}{9} \pi \ cm^2$$

Shaded area $= 968\pi - \frac{200}{9}\pi$

$$= \frac{8712\pi - 200\pi}{9}$$

$$= \frac{8512}{9} \pi \ cm^2$$

$$= 2971.25 \ cm^2$$

Q.12. Find the area of a sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10 cm.

Solution: Area of sector = ?

Central angle = θ = $\frac{\pi}{5}$ radian

Radius = r = 10cm

Area of sector = $\frac{1}{2}r^2\theta$

$$= \frac{1}{2}(10\text{cm})^2 \times \frac{\pi}{5}$$

$$= \frac{1}{10} \times 100\text{cm}^2 \times \pi$$

$$= 10\pi \text{ cm}^2$$

$$= 31.43 \text{ cm}^2$$

Q.13. The area of sector with central angle θ in a circle of radius 2m is 10 square meter. Find θ in radians.

Solution: Area of sector = 10 m^2

Radius = r = 2m

Central angle = θ ? =

As Area of sector = $\frac{1}{2}r^2\theta$

$$10\text{m}^2 = \frac{1}{2}(2m)^2\theta$$

$$10\text{ m}^2 = \frac{1}{2}(4m^2)\theta$$

$$10\text{m}^2 = 2\theta\text{m}^2$$

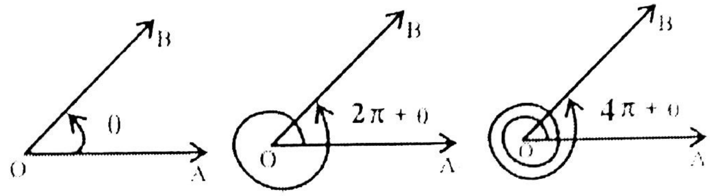
$$\theta = \frac{10\text{m}^2}{2\text{m}^2}$$

$$\theta = 5 \text{ radian}$$

Trigonometric Ratios

General Angle (Coterminal angles)

An angle is indicated by a curved arrow that shows the direction of rotation from initial to the terminal side. Two or more than two angles may have the same initial and terminal sides. Consider an angle ∠AOB with $\overline{OA}$ as initial side and $\overline{OB}$ as terminal side with vertex O. Let m∠AOB = θ radian where $0 \le \theta \le 2\pi$.



If the terminal side $\overline{OB}$ comes to its original position after, one, two or more than two complete revolutions in the anti–clockwise direction, then m∠AOB in above four cases will be

(i) θ rad	After zero revolution
(ii) (2π+ θ) rad.	After one revolution
(iii) (4π+ θ) rad.	After two revolutions.

Coterminal angle: Two or more than two angles with the same initial and terminal sides are called coterminal angles.

It means that terminal side comes to its original position after every revolution of 2π radian in anti clockwise or clockwise direction. In general if θ is in degrees, then 360°k+θ where k ∈ Z, is an angle conterminal with θ, if angle θ is in radian measure, then 2kπ+ θ where k∈Z is an angle coterminal with θ .

Thus, the general angle θ = 2(k) π+ θ, where k ∈ Z.

Example 1: Which of following angles are coterminal with 120°?

$-240°$, $480°$, $\frac{14\pi}{3}$ and $-\frac{14\pi}{3}$

Solution:

- – 240° is coterminal with 120° as their terminal side is same
- 480° = 360° + 120°, the angle 480° terminates at 120° after one complete revolution.
- $\frac{14}{3}\pi = 4\pi + \frac{2\pi}{3} = 720° + 120°$ then angle $\frac{14\pi}{3}$ is coterminal with 120°.

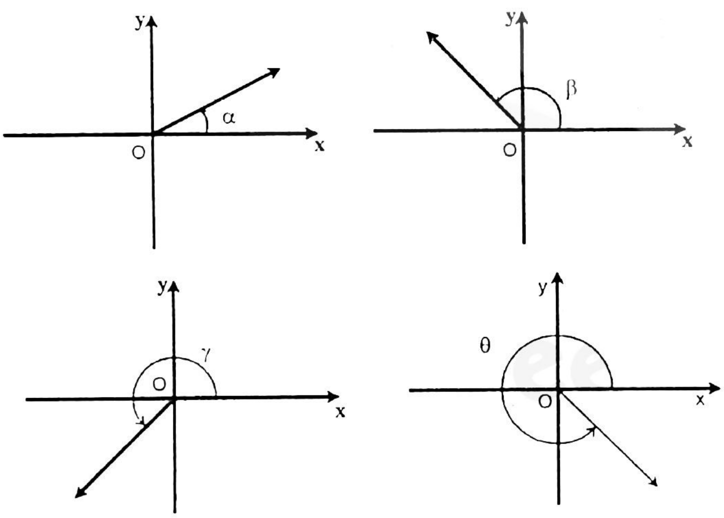
- $\frac{-14\pi}{3} = -4\pi + \frac{-2\pi}{3} = -720° - 120°$ So $\frac{-14\pi}{3}$ is not coterminal with 120°.

Angle in Standard Position:

A general angle is said to be in standard position if its vertex is at the origin and its initial side is directed along the positive direction of the x–axis of a rectangular coordinate system.

The position of the terminal side of a angle in standard position remains the same if measure of the angle is increased or decreased by a multiple of 2π .

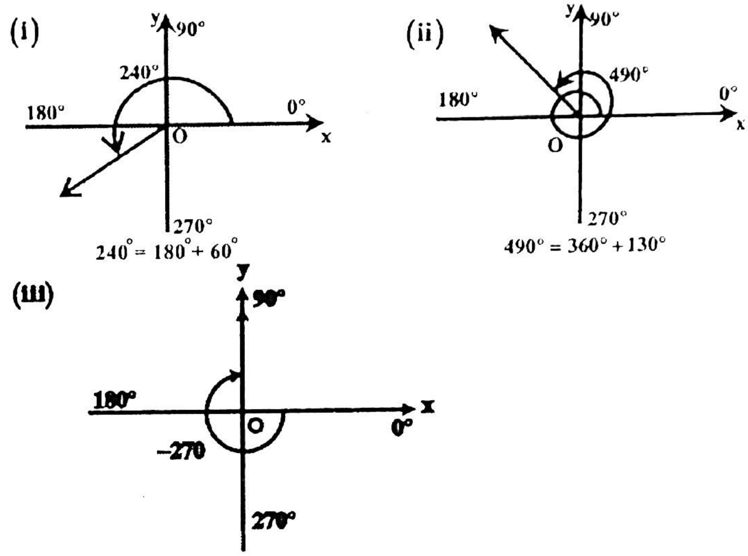
Some standard angles are shown in the following figures:



Example: Locate each angle in standard position.

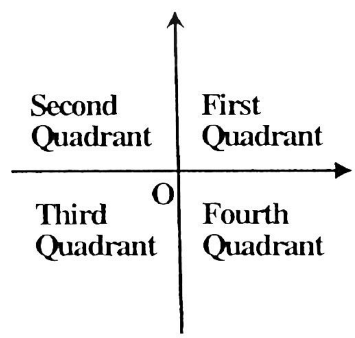
(i) 240° (ii) 490° (iii) – 270°

Solution: The angles are shown in figure.



The Quadrants and Quadrantal Angles:

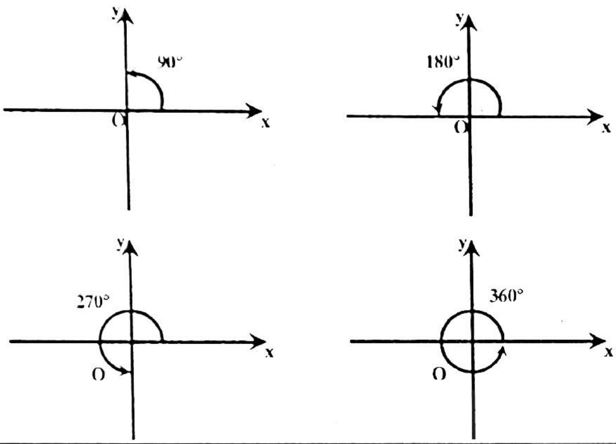
The x–axis and y–axis divides the plane in four regions, called quadrants, when they intersect each other at right angle. The point of intersection is called origin and is denoted by O.



- Angles between 0° and 90° are in the first quadrant.
- Angles between 90° and 180° are in the second quadrant.
- Angles between 180° and 270° are in the third quadrant.
- Angles between 270° and 360° are in the fourth quadrant.
- An angle in standard position is said to lie in a quadrant if its terminal side lies in that quadrant. Angles α, β, γ and θ lie in I, II, III and IV quadrant respectively.

Quadrantal Angles

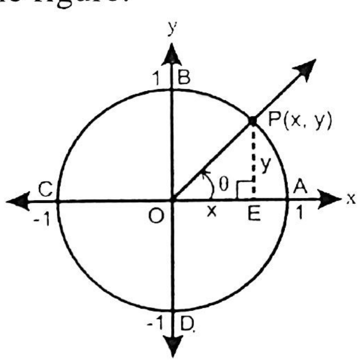
If the terminal side of an angle in standard position falls on x–axis or y–axis, then it is called a quadrantal angle i.e., 90°, 180°, 270° and 360° are quadrantal angles. The quadrantal angles are shown as below:



Trigonometric ratios and their reciprocals with the help of a unit circle:

There are six fundamental trigonometric ratios called sine, cosine, tangent, cotangent, secant and cosecant. To define these functions we use circular approach which involves the unit circle.

Let θ be a real number, which represents the radian measure of an angle in standard position. Let P(x, y) be any point on the unit circle lying on terminal side of θ as shown in the figure.



We define sine of θ, written as sinθ and cosine of θ written as cosθ, as:

$$\sin\theta = \frac{EP}{OP} = \frac{y}{1} \qquad \sin\theta = y$$

$$\cos\theta = \frac{OE}{OP} = \frac{x}{1} \qquad \cos\theta = x$$

i.e., cosθ and sinθ are the x–coordinate and y–coordinate of the point P on the unit circle. The equations x = cosθ and y = sinθ are called circular or trigonometric functions.

The remaining trigonometric functions tangent, cotangent, secant and cosecant will be denoted by tanθ, cotθ, secθ and cosecθ for any real angle θ .

- $\tan\theta = \frac{EP}{OE} = \frac{y}{x} \qquad \tan\theta = \frac{y}{x}\ (x \neq 0)$

As y = sinθ and x = cosθ $\quad \tan\theta = \frac{\sin\theta}{\cos\theta}$

- $\cot\theta = \frac{x}{y}\ (y \neq 0) \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\sec\theta = \frac{1}{x}(x \neq 0)$ and $\operatorname{cosec}\theta = \frac{1}{y}\ (y \neq 0)$
- $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Reciprocal Identities

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$

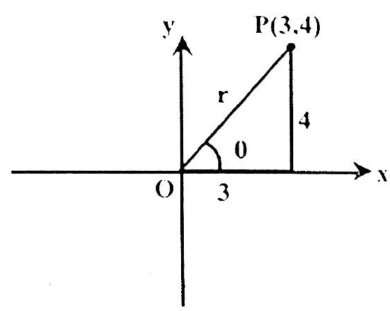
Example 2: Find the value of the trigonometric ratios at θ if point (3, 4) is on the terminal sides of θ .

Solution: We have x = 3 and y = 4

We shall also need value of r, which is found by using the fact that

$r = \sqrt{x^2 + y^2}$; $r = \sqrt{(3)^2 \ (4)^2} = \sqrt{25} + \ 5$

where r = OP



Thus $\sin\theta = \frac{y}{r} = \frac{4}{5}$; $\operatorname{cosec}\theta = \frac{5}{4}$

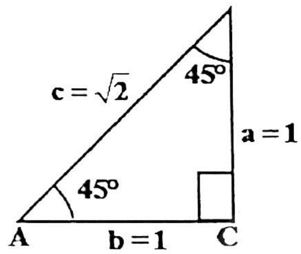
$\cos\theta = \frac{x}{r} = \frac{3}{5}$; $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{4}{3}$; $\cot\theta = \frac{3}{4}$

The values of trigonometric ratios for 45°, 30°, 60°

Consider a right triangle ABC with $m\angle C = 90°$. The sides opposite to the vertices A, B and C are denoted by a, b and c respectively.

Case I When $m\angle A = 45°$, where $45° = \frac{\pi}{4}$ radian. Since the sum of angles in a triangle is 180°, So $m\angle B = 45°$.



As values of trigonometric functions depends on the size of the angle only and not on the size of triangle. For convenience, we take $a = b = 1$. In this case the triangle is isosceles right triangle.

By Pythagorean theorem

$c^2 = a^2 + b^2 \qquad c^2 = (1)^2 + (1)^2 = 2$

$$c^2 = 2$$

$$c = \sqrt{2}$$

From this triangle we have

$$\sin 45° = \sin\frac{\pi}{4} = \frac{a}{c} = \frac{1}{\sqrt{2}}$$

$$\operatorname{cosec} 45° = \frac{1}{\sin 45°} = \sqrt{2}$$

$$\cos 45° = \cos\frac{\pi}{4} = \frac{b}{c} = \frac{1}{\sqrt{2}}$$

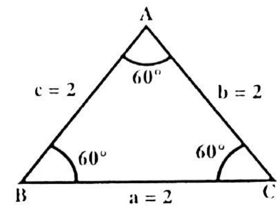
$$\sec 45° = \frac{1}{\cos 45°} = 2$$

$$\tan 45° = \tan\frac{\pi}{4} = \frac{a}{b} = \frac{1}{1} = 1$$

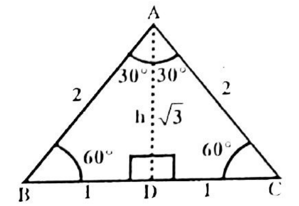
$$\cot 45° = \frac{1}{\tan 45°} = 1$$

Case II when $m\angle A = 30°$ or $m\angle A = 60°$

Consider an equilateral triangle with sides $a = b = c = 2$ for convenience. Since the angles in an equilateral triangle are equal and their sum is 180°, each angle has measure 60°. Bisecting an angle in the triangle, we obtain two right triangles with 30° and 60° angles. The height $|AD|$ of these triangles may be found by Pythagorean theorem, i.e.,



(i)



(ii)

$$(m\overline{AD})^2 = (m\overline{BD})^2 + (m\overline{AB})^2$$

$$(m\overline{AD})^2 = (m\overline{AB})^2 - (m\overline{BD})^2$$

$$h^2 = (2)^2 - (1)^2 = 3$$

$$h = \sqrt{3}$$

∴ Using triangle ADB with $m\angle A = 30°$, we have

$$\sin 30° = \sin\frac{\pi}{6} = \frac{m\overline{BD}}{m\overline{AB}} = \frac{1}{2}$$

$$\operatorname{cosec} 30° = \frac{1}{\sin 30°} = 2$$

$$\cos 30° = \cos\frac{\pi}{6} = \frac{m\overline{AD}}{m\overline{AB}} = \frac{\sqrt{3}}{2}$$

$$\sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}}$$

$$\tan 30° = \tan\frac{\pi}{6} = \frac{m\overline{BD}}{m\overline{AD}} = \frac{1}{\sqrt{3}}$$

$$\cot 30° = \frac{1}{\tan 30°} = \sqrt{3}$$

Now using triangle ABD with $m\angle B = 60°$

$$\text{Sin} 60° = \frac{m\overline{AD}}{m\overline{AB}} = \frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} 60° = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}}$$

$$\cos 60° = \frac{m\overline{BD}}{m\overline{AB}} = \frac{1}{2}$$

$$\sec 60° = \frac{1}{\cos 60°} = 2$$

$$\tan 60° = \frac{m\overline{AD}}{m\overline{BD}} = \frac{\sqrt{3}}{1}$$

$$\cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}}$$

Signs of trigonometric ratios in different quadrants

In case of trigonometric ratios like sinθ, cosθ and tanθ if θ is not a quadrantal angle, then θ will lie in a particular quadrant. Since

$r = \sqrt{x^2 + y^2}$ is always +ve, the signs of ratios can be found if the quadrant of θ is known.

(i) If θ lies in first quadrant then a point P(x, y) on its terminal side has x and y co–ordinate positive.

Therefore, all trigonometric functions are positive in quadrant I.

(ii) If θ lies in second quadrant then a point P(x, y) on its terminal side has negative x–coordinate and positively y–coordinate i.e.,

- $\sin\theta = \dfrac{y}{r}$ is +ve or > 0,
- $\cos\theta = \dfrac{x}{r}$ is –ve or < 0
- $\tan\theta = \dfrac{y}{x}$ is –ve or < 0

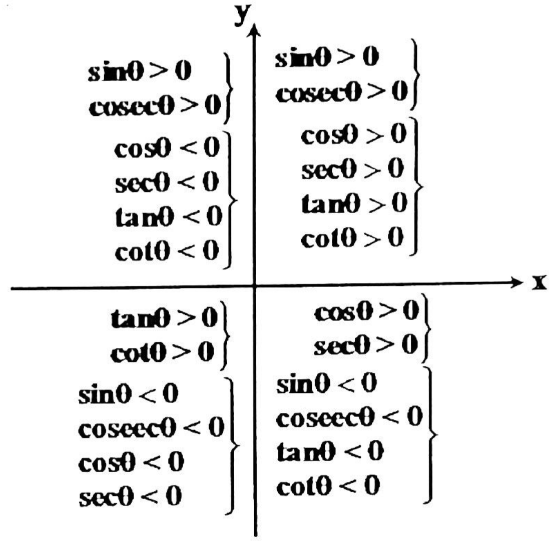
(iii) When θ lies in third quadrant, then a point P(x, y) on its terminal side has negative x–coordinate and negative y–coordinate.

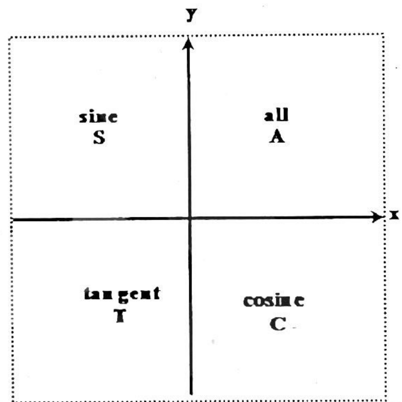
- $\sin\theta = \dfrac{y}{r}$ is –ve or < 0,
- $\cos\theta = \dfrac{x}{r}$ is –ve or < 0 and
- $\tan\theta = \dfrac{y}{x}$ is +ve or > 0

(iv) When θ lies in fourth quadrant, then the point P(x, y) on the terminal side of θ has positive x–coordinate and negative y–coordinate.

- $\sin\theta = \dfrac{y}{r}$ is –ve or < 0,
- $\cos\theta = \dfrac{x}{r}$ is +ve or > 0 and
- $\tan\theta = \dfrac{y}{x}$ is –ve or < 0

The sign of all trigonometric functions are summarized as below.





Values of remaining trigonometric ratios if one trigonometric ratio is given

Example 1:

If $\sin\theta = \dfrac{-3}{4}$ and $\cos\theta = \dfrac{\sqrt{7}}{4}$, then find the values of tanθ, cotθ, secθ and cosecθ .

Solution: Applying the identities that express the remaining trigonometric functions in terms of sine and cosine, we have

$$\sin\theta = \frac{-3}{4}$$

$$\therefore \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-3}{4}} = \frac{-4}{3}$$

$$\operatorname{cosec}\theta = \frac{-4}{3}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

$$\therefore \quad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{7}}{4}}$$

$$\sec\theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$$

$$\text{Now } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-3}{4}}{\frac{\sqrt{7}}{4}} = \frac{-3}{\sqrt{7}}$$

$$\tan\theta = \frac{-3}{\sqrt{7}}$$

$$\text{And} \quad \cot\theta = \frac{1}{\tan\theta} = \frac{-\sqrt{7}}{3}$$

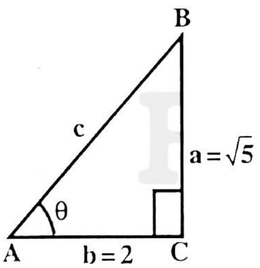
Example 2:

If $\tan\theta = \frac{\sqrt{5}}{2}$, then find the values of other trigonometric ratios at θ.

Solution: In any right triangle ABC

$$\tan\theta = \frac{\sqrt{5}}{2} = \frac{a}{b}$$

$$a = \sqrt{5}, \quad b = 2$$



Now by Pythagorean theorem

$$a^2 + b^2 = c^2$$

$$(\sqrt{5})^2 \quad (2)^2 + \ c^2$$

$$c^2 = 5 + 4 = 9$$

$$c = \pm 3 \quad \text{or} \quad c = 3$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{1}{\frac{\sqrt{5}}{2}} \qquad \cot\theta = \frac{2}{\sqrt{5}}$$

$$\sin\theta = \frac{a}{c} = \frac{\sqrt{5}}{3},$$

$$\text{cosec}\theta = \frac{1}{\sin\theta}$$

$$\text{cosec}\theta = \frac{1}{\frac{\sqrt{5}}{3}}$$

$$\therefore \quad \text{cosec}\theta = \frac{3}{\sqrt{5}}$$

$$\text{Also} \quad \cos\theta = \frac{b}{c} = \frac{2}{3},$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec\theta = \frac{1}{\frac{2}{3}}$$

$$\therefore \quad \sec\theta = \frac{3}{2}$$

Calculate the values of trigonometric ratios for 0°, 90°, 180°, 270°, 360°

An angle θ is called a quadrantal angle if its terminal side lies on the x–axis on the y–axis.

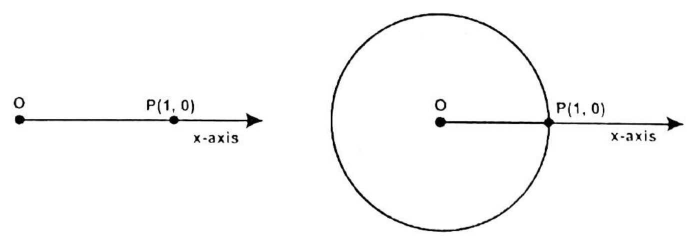
Case I when $\theta = 0°$

The point (1, 0) lies on the terminal side of angle $\theta°$. We may consider the point on the unit circle on the terminal side of the angle.

P (1, 0)

$x = 1$ and $y = 0$

$$\text{so} \quad r \quad \sqrt{x^2 \ y^2} = \sqrt{1+0} = 1$$



$$\sin 0° = \frac{y}{r} = \frac{0}{1} = 0,$$

$$\text{cosec}\,0° = \frac{1}{\sin 0°} = \frac{1}{0} = \infty \text{ (Undefined)}$$

$$\cos 0° = \frac{x}{r} \ \frac{1}{1} = 1,$$

$$\sec 0^\circ = \frac{1}{\cos 0^\circ} = 1$$

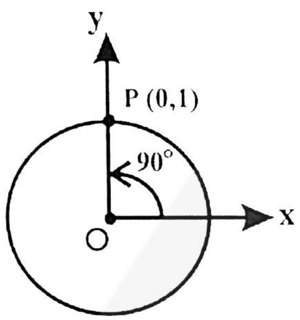
$$\tan 0^\circ = \frac{y}{x} = \frac{0}{1} = 0,$$

$$\cot 0^\circ = \frac{1}{\tan 0^\circ} = \frac{1}{0} = \infty \quad \text{(Undefined)}$$

Case II when θ = 90°

The point P(0, 1) lies on the terminal side of angle 90°. $\quad r \quad \sqrt{x^2 \quad y^2}$

Here $x = 0$ and $y = 1$ $\quad r = \sqrt{0^2 \quad (1)^2} \quad 1$



$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

i.e.

$$\sin 90^\circ = 1 \text{ and } \operatorname{cosec} 90^\circ = \frac{r}{y} = 1$$

Using reciprocal identities, we have

$$\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

$$\sec 90^\circ = \frac{r}{x} = \frac{1}{0} = \infty \quad \text{(Undefined)}$$

$$\tan 90^\circ = \frac{y}{x} = \frac{1}{0} = \infty \quad \text{(Undefined)},$$

$$\cot 90^\circ = \frac{x}{y} = \frac{0}{1} = 0$$

Case III when θ = 180°

When θ = 180° and the point P (−1, 0) lies on x′–axis or on terminal side of angle 180°

Here $\quad x = -1$ and $y = 0$

$$r = \sqrt{x^2 + y^2} = 1$$

$$\sin 180^\circ = \frac{y}{r} = \frac{0}{1} = 0$$

$$\operatorname{cosec} 180^\circ = \frac{r}{y} = \frac{1}{0} = \infty \text{ (undefined)}$$

$$\cos 180^\circ = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\sec 180^\circ = \frac{r}{x} = \frac{1}{-1} = -1$$

$$\tan 180^\circ = \frac{y}{x} = \frac{0}{-1} = 0$$

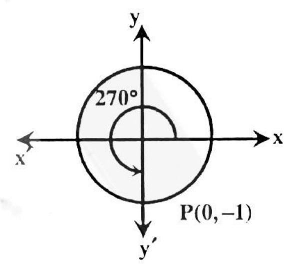
$$\cot 180^\circ = \frac{x}{y} = \frac{-1}{0} = \infty \text{ (Undefined)}$$

Case IV when θ = 270°

When θ = 270° and the point P(0, −1) lis on y′–axis or on the terminal side of angle 270°.

The point P(0, −1) shows that $x=0$ and $y=-1$

So $\quad r = \sqrt{(0)^2+(-1)^2} = 1$



$$\text{Sin } 270^\circ = \frac{y}{r} = \frac{-1}{1} = -1$$

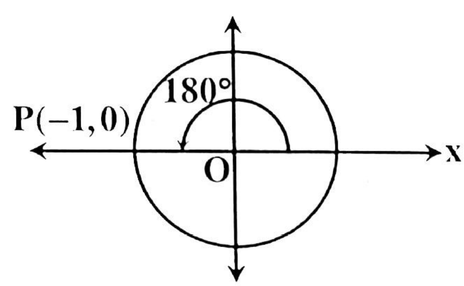
$$\operatorname{cosec} 270^\circ = \frac{r}{y} = \frac{1}{-1} = -1$$

$$\text{Cos} 270^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

$$\sec 270^\circ = \frac{r}{x} = \frac{1}{0} = \infty$$

$$\tan 270^\circ = \frac{y}{x} = \frac{-1}{0} = -\infty$$

$$\cot 270 = \frac{x}{y} = \frac{0}{-1} = 0$$



Case V When θ° = 360°

Now the point P (1, 0) lies once again on x–axis

We know that $\theta+2k\pi=\theta$ where $k \in Z$.

Now $\theta = 360^\circ = 0^\circ + (360^\circ)1 = 0^\circ$ where $k = 1$

So $\quad \sin 360^\circ = \sin 0^\circ = 0$

$$\operatorname{cosec}360^\circ = \frac{1}{\sin 360^\circ} = \frac{1}{\sin 0^\circ} = \frac{1}{0} = \infty \text{ (undefined)}$$

$$\cos 360^\circ = \cos 0^\circ = 1$$

$$\sec 360^\circ = \frac{1}{\cos 0^\circ} = \frac{1}{1} = 1,$$

$$\tan 360^\circ = \tan 0^\circ = 0$$

$$\cot 360^\circ = \frac{1}{\tan 0^\circ} = \frac{1}{0} = \infty \text{ (undefined)}$$

Example: **Find each of the following without using table or calculator:**

(i) Cos540° (ii) sin315° (iii) sec(–300)°

Solution:

We know that $2k\pi + \theta = \theta$, where $k \in Z$.

(i) $540^\circ = (360^\circ + 180^\circ) = 2(1)\pi + 180^\circ$

$\text{Cos}540^\circ = \cos(2\pi + \pi) = \cos\pi = -1$

(ii) $\sin 315^\circ = \sin(360^\circ - 45^\circ) = \sin\ 2\pi - \frac{\pi}{4}$

$$= \sin\frac{-\pi}{4} = -\sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$$

(iii) $\sec(-300^\circ) = \sec(-360^\circ + 60^\circ)$

$= \sec[2(-1)\pi + 60]$

$$= \sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2$$