EXERCISE 7.4

In problem 1— 6, simply each expression to single trigonometric function:

Q.1. $\dfrac{\sin^2 x}{\cos^2 x}$

Solution: $\dfrac{\sin^2 x}{\cos^2 x} = \tan x^2$

Q.2. **tan$x$ sin$x$ sec$x$**

Solution: $\tan x \sin x \sec x$

$= \dfrac{\sin x}{\cos x} . \sin x . \dfrac{1}{\cos x}$

$= \dfrac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

Q.3. $\dfrac{\tan x}{\sec x}$

Solution: $\dfrac{\tan x}{\sec x} = \tan x \div \sec x$

$= \dfrac{\sin x}{\cos x} \div \dfrac{1}{\cos x}$

$= \dfrac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$

$= \sin x$

Q.4. **1 – cos²x**

Solution: $1 - \cos^2 x$

$= \sin^2 x + \cancel{\cos^2} x - \cancel{\cos^2} x$

$= \sin^2 x$

Q.5. $\sec^2 x - 1$

Solution: $\sec^2 x - 1$

$= 1 + \tan^2 x - 1$

$= \tan^2 x$

Q.6. $\sin^2 x.\cot^2 x$

Solution:

$\sin^2 x.\cot^2 x$

$= \sin^2 x \dfrac{\cos^2 x}{\sin^2 x}$

$= \cos^2 x$

In problem 7 — 24, verify the identities

Q.7. $(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$

Solution:

L.H.S $= (1 - \sin\theta)(1 + \sin\theta)$

$= (1)^2 - (\sin\theta)^2$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

L.H.S = R.H.S

Q.8. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} = 1 + \tan\theta$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\cos\theta}$

$= \dfrac{\cos\theta + \sin\theta}{\cos\theta}$

$= \dfrac{\cos\theta}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= 1 + \tan\theta \qquad \because \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

L.H.S = R.H.S

Q.9. $(\tan\theta + \cot\theta)\tan\theta = \sec^2\theta$

Solution: Let

L.H.S $= (\tan\theta + \cot\theta)\tan\theta$

$= \tan^2\theta + \cot\theta.\tan\theta$

$= \tan^2\theta + \dfrac{1}{\tan\theta}.\tan\theta$

$= 1 + \tan^2\theta$

$= \sec^2\theta \qquad (1 + \tan^2\theta = \sec^2\theta)$

L.H.S = R.H.S

Q.10. $(\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$

Solution: Let

L.H.S $= (\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta)$

$= \dfrac{1}{\tan\theta} + \dfrac{1}{\sin\theta} \quad (\tan\theta - \sin\theta)$

$= \dfrac{\sin\theta + \tan\theta}{\tan\theta.\sin\theta} \quad (\tan\theta - \sin\theta)$

$= \dfrac{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}{\tan\theta.\sin\theta}$

$= \dfrac{(\tan\theta)^2 - (\sin\theta)^2}{\tan\theta.\sin\theta}$

$= \dfrac{\tan^2\theta - \sin^2\theta}{\tan\theta.\sin\theta}$

$= \dfrac{\tan^2\theta}{\tan\theta.\sin\theta} - \dfrac{\sin^2\theta}{\tan\theta.\sin\theta}$

$= \dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta}$

$= (\tan\theta \div \sin\theta) - (\sin\theta \div \tan\theta)$

$= \dfrac{\sin\theta}{\cos\theta} \div \sin\theta - \sin\theta \div \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sin\theta} - \sin\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= \sec\theta - \cos\theta$

L.H.S = R.H.S

Q.11. $\dfrac{\sin\theta + \cos\theta}{\tan^2\theta - 1} = \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

Solution: Let

L.H.S $= \dfrac{\sin\theta + \cos\theta}{\tan^2\theta - 1}$

$= (\sin\theta + \cos\theta) \div (\tan^2\theta - 1)$

$= (\sin\theta + \cos\theta) \div \dfrac{\sin^2\theta}{\cos^2\theta} - 1$

$= (\sin\theta + \cos\theta) \div \dfrac{\sin^2 - \cos^2}{\cos^2}$

$= (\sin\theta + \cos\theta) \times \dfrac{\cos^2}{(\sin^2\theta - \cos^2\theta)}$

$= \dfrac{(\sin\theta + \cos\theta) \times \cos^2\theta}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$

$= \dfrac{\cos^2\theta}{\sin\theta - \cos\theta}$

L.H.S = R.H.S

Q.12. $\dfrac{\cos^2\theta}{\sin\theta} + \sin\theta = \text{cosec}\,\theta$

Solution: Let

$$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta} + \sin\theta$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \text{cosec}\,\theta$$

L.H.S = R.H.S

Q.13. $\sec\theta - \cos\theta = \tan\theta\,.\sin\theta$

Solution: Let

$$\text{L.H.S} = \sec\theta - \cos\theta$$

$$= \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1 - \cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\sin\theta$$

$$= \tan\theta\,.\,\sin\theta$$

L.H.S = R.H.S

Q.14. $\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

Solution: Let

$$\text{L.H.S} = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

L.H.S = R.H.S

Q.15. $\tan\theta + \cot\theta = \sec\theta.\,\text{cosec}\theta$

Solution: Let

$$\text{L.H.S} = \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\sin\theta}$$

$$= \frac{1}{\cos\theta.\sin\theta} \qquad (\sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{1}{\cos\theta}.\frac{1}{\sin\theta}$$

$$= \sec\theta.\text{cosec}\theta$$

L.H.S = R.H.S

Q.16. $(\tan\theta+\cot\theta)(\cos\theta+\sin\theta) = \sec\theta+\text{cosec}\theta$

Solution: Let

$$\text{L.H.S} = (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\cos\theta + \sin\theta)$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\sin\theta}(\cos\theta + \sin\theta)$$

$$= \frac{1}{\cos\theta.\sin\theta}(\cos\theta + \sin\theta)$$

$$= \frac{\cos\theta + \sin\theta}{\cos\theta.\sin\theta}$$

$$= \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}.\sin\theta} + \frac{\cancel{\sin\theta}}{\cos\theta.\cancel{\sin\theta}}$$

$$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$$

$$= \text{cosec}\theta + \sec\theta$$

$$= \sec\theta + \text{cosec}\theta$$

L.H.S = R.H.S

Q.17. $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

Solution: Let

$$\text{L.H.S} = \sin\theta(\tan\theta + \cot\theta)$$

$$= \sin\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \cancel{\sin\theta}\,\frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\cancel{\sin\theta}}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

L.H.S = R.H.S

Q.18. $\dfrac{1+\cos}{\sin} + \dfrac{\sin}{1+\cos} = 2\text{cosec}$

Solution: Let

$$\text{L.H.S} = \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2 + (\sin\theta)^2}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{(1)^2 + 2(1)(\cos\theta) + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 \quad 2\cos\theta + 1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\text{cosec}\theta$$

L.H.S = R.H.S

Q.19. $\dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\cos\theta} = 2\text{cosec}^2\theta$

Solution:

$$\text{L.H.S} = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$$

$$= \frac{1+\cancel{\cos\theta}+1-\cancel{\cos\theta}}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{2}{(1)^2-(\cos^2\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$$= 2\text{cosec}^2\theta$$

L.H.S = R.H.S

Q.20. $\dfrac{1+\sin}{1-\sin} - \dfrac{1-\sin}{1+\sin} = 4\tan\ \sec$

Solution: Let

$$\text{L.H.S} = \frac{1+\sin\theta}{1-\sin\theta} = \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin^2\theta+2\sin\theta)-(1+\sin^2\theta-2\sin\theta)}{(1)^2-(\sin\theta)^2}$$

$$= \frac{1+\sin^2\theta+2\sin\theta-1-\sin^2\theta+2\sin\theta}{1-\sin^2\theta}$$

$$= \frac{4\sin\theta}{\cos^2\theta}$$

$$= \frac{4\sin\theta}{\cos\theta.\cos\theta}$$

$$= 4\frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\theta}$$

$$= 4\tan\theta.\ \sec\theta$$

L.H.S = R.H.S

Q.21. $\sin^3\theta = \sin\theta - \sin\theta.\cos^2\theta$

Solution: Let

$\text{L.H.S} = \sin^3\theta$

$= \sin\theta\ .\ \sin^2\theta$

$= \sin\theta(1-\cos^2\theta)$

$= \sin\theta - \sin\theta.\cos^2\theta$

L.H.S = R.H.S

Q.22. $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

Solution: Let

$\text{L.H.S} = \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= (1)(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

L.H.S = R.H.S

Q.23. $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}} = \dfrac{\sin\theta}{1-\cos\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}\times\frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{(1)^2-(\cos\theta)^2}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\sin\theta}{1-\cos\theta}$$

L.H.S = R.H.S

Q.24. $\sqrt{\dfrac{\sec\theta+1}{\sec\theta-1}} = \dfrac{\sec\theta+1}{\tan\theta}$

Solution: Let

$$\text{L.H.S} = \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$$

$$= \sqrt{\frac{\sec\theta+1}{\sec\theta-1}\times\frac{\sec\theta+1}{\sec\theta+1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{(\sec\theta)^2-(1)^2}}$$

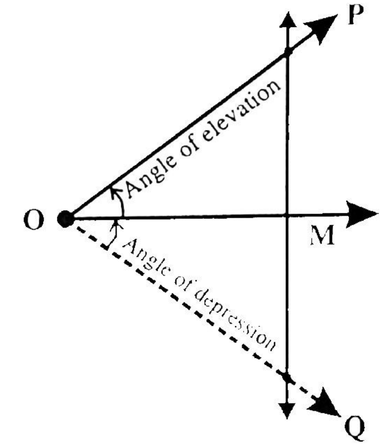
$$= \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$$

$$= \sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$$

$$= \frac{\sec\theta+1}{\tan\theta}$$

L.H.S = R.H.S

Angle of Elevation and Angle of Depression

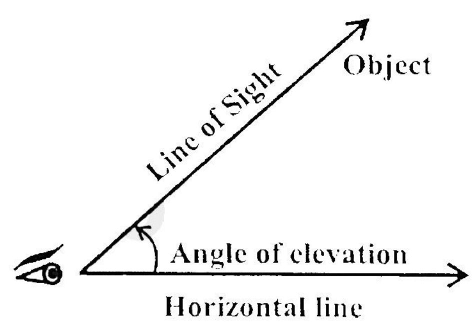


Suppose O, P and Q are three points, P being at a higher level of O and Q being at lower level than O. Let a horizontal line drawn through O meet in M, the vertical line drawn through P and Q.

The angle MOP is called the angle of elevation of point P as seen from O. For looking at Q below the horizontal line we have to lower our eyes and ∠ MOQ is called the angle of depression.

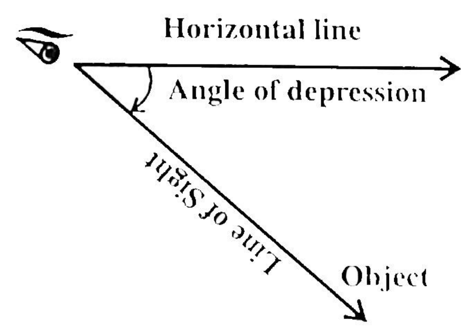
Angle of Elevation:

The angle between the horizontal line through eye and a line from eye to the object, above the horizontal line is called angle of elevation.



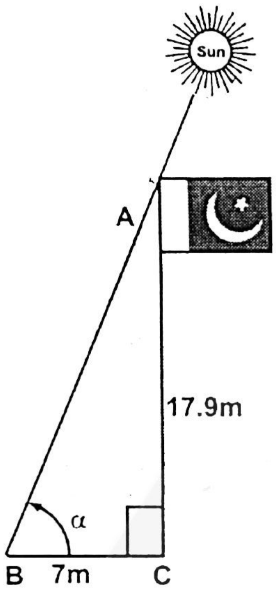
Angle of depression:

The angle between the horizontal line through eye and a line from eye to the object, below the horizontal line is called angle of depression.



Example 1: A flagpole 17.9 meter high casts a 7 meter shadow. Find the angle of elevation of the sun.

Solution:



From the figure, we observe that α is the angle of elevation.

Using the fact that

$$\tan\alpha = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\alpha = \frac{17.9m}{7m}$$

$$\tan\alpha \quad 2.55714$$

Solving for α gives us

$$\alpha \quad \tan^{-1}(2.55714)$$

$$\alpha \quad (68.6666)^\circ$$

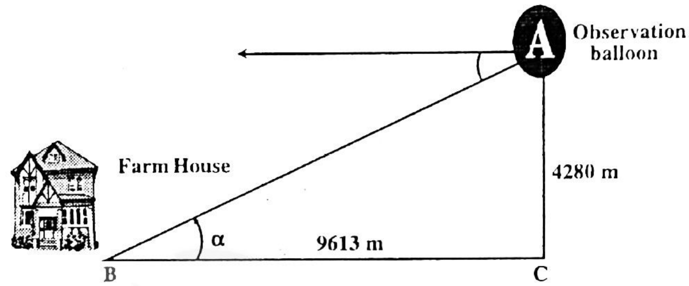
$$68^\circ 40'$$

$$\alpha \quad 68^\circ 40'$$

So angle of elevation is $68^\circ 40'$.

Example 2: An observation balloon is 4280 meter above the ground and 9613 meter away from a farmhouse. Find the angle of depression of the farmhouse as observed from the observation balloon.

Solution:



For problems of this type the angle of elevation of A from B is considered equal to the angle of depression of B from A, as shown in the diagram.

$$\tan\alpha = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\alpha = \frac{4280m}{9613m}$$

$$\tan\alpha = 0.44523$$

$$\alpha = \tan^{-1}(0.144523)$$

$$\alpha = 24^\circ$$

So, angle of depression is 24°.