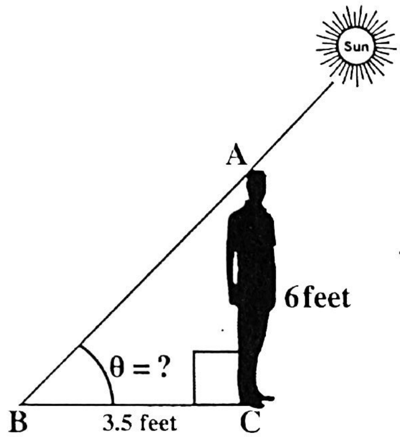
EXERCISE 7.5

Q.1. Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.

Solution:



From the figure we observe that

Height of man $= m\overline{AC} = 6$ feet

Length of shadow $= m\overline{BC} = 3.5$ feet

Angle of elevation $= \theta\ ? =$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan\theta = \frac{6}{3.5}$$

$$\theta = \tan^{-1}\frac{6}{3.5}$$

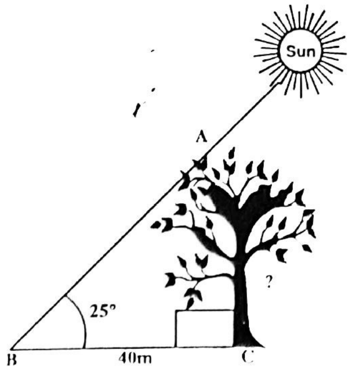
$$\theta = 59.7436$$

$$\theta = 59.74^{\circ}$$

So, the angle of elevation is $59^{\circ}\,44'37''$.

Q.2. A tree casts a 40 meters shadow when the angle of elevation of the sun is 25°. Find the height of the tree.

Solution:



From the figure

Height of tree $= m\overline{AC}\ ? =$

Length of shadow $= m\overline{BC} = 40$m

Angle of elevation $= \theta = 25^{\circ}$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 25^{\circ} = \frac{m\overline{AC}}{40}$$

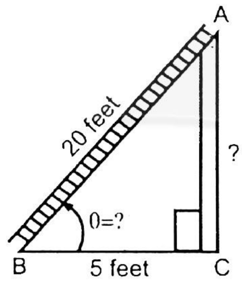
$$m\overline{AC} = 40 \times \tan 25^{\circ}$$

$$m\overline{AC} = 18.65 \text{ m}$$

So, height of tree is 18.65 m

Q.3. A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:



From the figure

Length of ladder $= m\overline{AB} = 20$ feet

Distance of ladder from the wall $= m\overline{BC} = 5$ feet

Angle of elevation $= \theta\ ? =$

Using the fact that

$$\cos\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\cos\theta = \frac{5\text{ ft.}}{20\text{ ft.}}$$

$$\cos\theta = 0.25$$

$$\theta = \cos^{-1}(0.25)$$

$$\theta = 75.5225$$

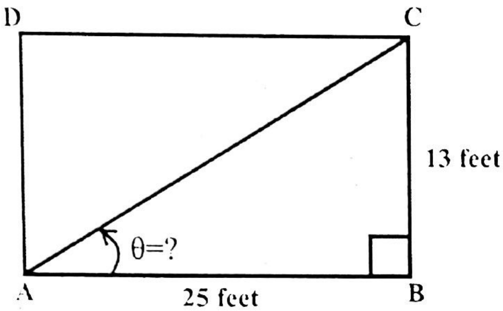
$$\theta = 75.5^{\circ}$$

or $\theta = 75^{\circ}\ 30'$

So, angle of elevation is $75^{\circ}31'21''$

Q.4. The base of rectangle is 25 feet and the height of rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.

Solution:



From the figure

Base of rectangle = $m\overline{AB}$ = 25 feet

Height of rectangle = $m\overline{BC}$ = 13 feet

Diagonal $\overline{AC}$ is taken

Angle between diagonal and base = θ ? =

Using the fact that

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1}\frac{13}{25}$$

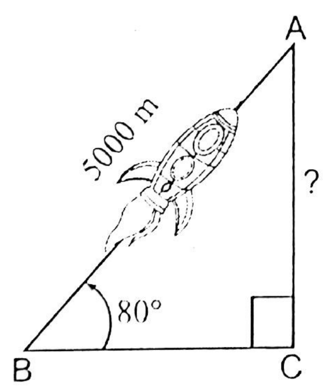
$$\theta = 27.4744$$

$$\theta = 27.47°$$

So, angle between diagonal and base is 27°28′28″.

Q.5. A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meter.

Solution:



From the figure

Distance travelled by rocket = $m\overline{AB}$ = 5000m

Altitude of rocket = $m\overline{AC}$? =

Angle of elevation = θ = 80°

Using $$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\sin 80° = \frac{m\overline{AC}}{5000}$$

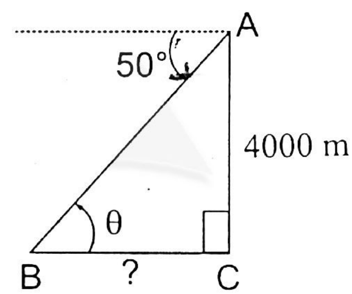
$$m\overline{AC} = 5000 \times \sin 80°$$

$$m\overline{AC} = 4924.04\text{m}$$

So, the altitude of rocket is 4924.04m

Q.6. An aeroplane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descend?

Solution:



From the figure

Altitude of aeroplane = $m\overline{AC}$ = 4000m

Distance of plane from airport = $m\overline{BC}$? =

Angle of depression = 50°

As the alternate angles of parallel lines are equal, so angle

$$\theta = 50°$$

Using the fact that, $$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 50° = \frac{4000\text{m}}{m\overline{BC}}$$

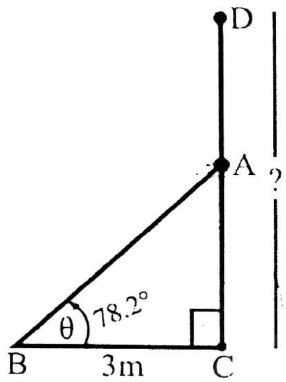
$$m\overline{BC} = \frac{4000\text{m}}{\tan 50°}$$

$$m\overline{BC} = 3356.4\text{ m}$$

So, the distance of aeroplane from airport is 3356.4 m.

Q.7. A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution:



From the figure

Height of pole $= m\overline{CD}$? =

Distance of wire from the base of the pole

$= m\overline{BC} = 3m$

Angle of elevation $= \theta = 78.2°$

As the wire is attached with the pole at its middle point A, so, first we find $m\overline{AC}$

Using the fact that

$$\tan\theta = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 78.2 = \frac{m\overline{AC}}{3}$$

$$m\overline{AC} = 3m \times \tan 78.2°$$

$$m\overline{AC} = 14.36 \text{ m}$$

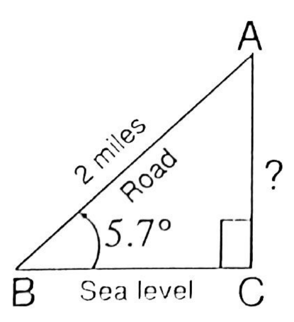
So Height of pole is $= m\overline{DC} = 2(m\overline{AC})$

$= 2 \times 14.36$ m

$= 28.72$ m

Q.8. A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we? 07(121)

Solution:



From the figure

Distance covered on road $= m\overline{AB} = 2$ miles

Angle of inclination $= \theta = 5.7°$

Height from sea level $= m\overline{AC}$? =

Using the fact that,

$$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\sin 5.7° = \frac{m\overline{AC}}{2}$$

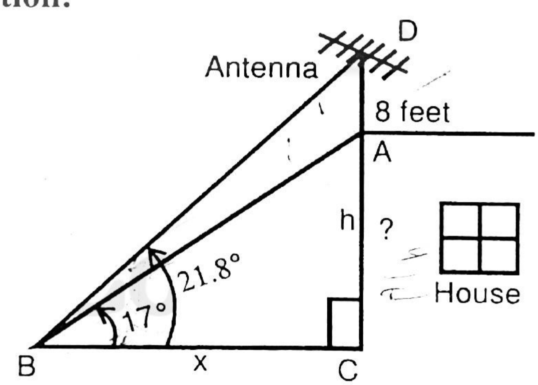
$$m\overline{AC} = 2 \times \sin 5.7°$$

$$m\overline{AC} = 0.199 \text{ mile}$$

Hence, we are at the height of 0.199 mile from the sea level.

Q.9. A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of antenna is 21.8°. Find the height of the house.

Solution:



From the figure

Distance of point from house $= m\overline{BC} = x$

Height of house $= m\overline{AC} = h = ?$

Height of antenna $= m\overline{AD} = 8$ feet

Angle of elevation of top of house $= 17°$

Angle of elevation of top of antenna $= 21.8°$

In right angled ΔABC

$$\tan 17° = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 17° = \frac{h}{x}$$

$$x = \frac{1}{\tan 17°} \times h$$

$x = 3.271 \times h$ ....(i)

Now in right angle ΔDBC

$$\tan 21.8 = \frac{m\overline{CD}}{m\overline{BC}}$$

$$\tan 21.8 = \frac{m\overline{AD} + m\overline{AC}}{m\overline{BC}}$$

$$\tan 21.8 = \frac{8+h}{x}$$

$$0.40 = \frac{8+h}{3.271h} \qquad \text{[From (i)]}$$

$$0.40 \times 3.271h = 8 + h$$

$$1.3084\ h - h = 8$$

$$(1.3084 - 1)\ h = 8$$

$$0.3084\ h = 8$$

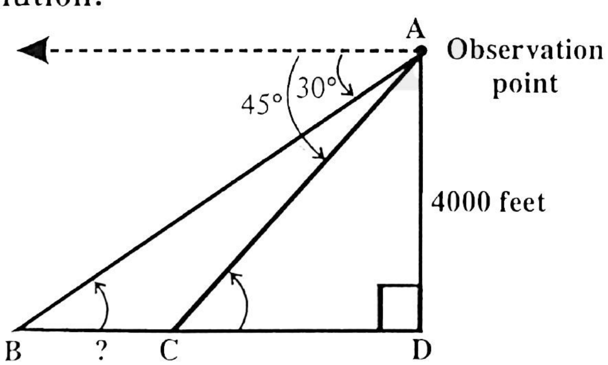
$$h = \frac{8}{0.3084}$$

$$h = 25.94 \text{ feet}$$

So, the height of the house is 25.94 feet

Q.10. From an observation point, the angles of depression of two boats in line with this point are found to 30° and 45°. Find the distance between the two boats if the point of observation is 4000 feet high.

Solution:



From the figure

Height of observation point= $m\overline{AD}$ =4000 feet

Distance between boats = $m\overline{BC}$? =

Angles of depression of points B and C are 30° and 45° respectively from point A.

As the alternate angles of parallel lines are equal, so

$m\angle B = 30°$ and $m\angle C = 45°$

Now in right angled ΔACD

$$\tan 45° = \frac{m\overline{AD}}{m\overline{CD}}$$

$$1 = \frac{4000}{m\overline{CD}}$$

$$m\overline{CD} = 4000 \text{ feet}$$

Now in right angled ΔBCD

$$\tan 30° = \frac{m\overline{AD}}{m\overline{BD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + m\overline{CD}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{m\overline{BC} + 4000}$$

$$m\overline{BC} + 4000 = 4000\sqrt{3}$$

$$m\overline{BC} = 4000\sqrt{3} - 4000$$

$$m\overline{BC} = 6928.20 - 4000$$

$$m\overline{BC} = 2928.20 \text{ feet}$$

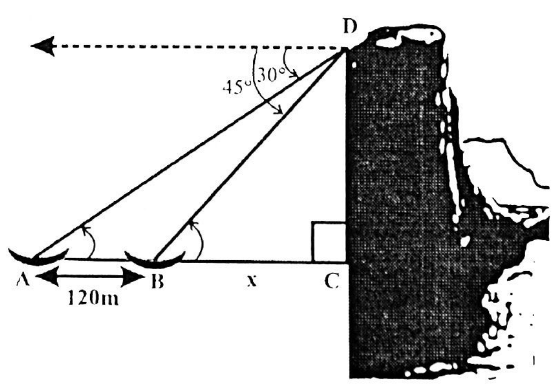
So, the distance between boats is 2928.2 feet.

Q.11. Two ships, which are in line with the base of a vertical cliff are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°, as shown in the diagram.

(a) Calculate the distance BC

(b) Calculate the height CD of the cliff.

Solution:



From the figure

Height of cliff = $\overline{CD}$ = h = ?

Distance = $\overline{BC}$ = x = ?

Distance between boats = $\overline{AB}$ = 120 m

Angles of depression from point D to points A and B are 30° and 45° respectively.

As the alternate angles of parallel lines are equal, so $m\angle A = 30°$ and $m\angle B = 45°$

In right angled ΔBCD

$$\tan 45^\circ = \frac{m\overline{CD}}{m\overline{BC}}$$

$$1 = \frac{h}{x}$$

$$x = h \qquad \ldots\ldots (i)$$

Now in right angled ΔACD

$$\tan 30^\circ = \frac{m\overline{CD}}{m\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{m\overline{AB} + m\overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{120 + x}$$

$$120 + x = \sqrt{3}\,h$$

$$120 + h = \sqrt{3}\,h \qquad (\ x = h)$$

$$120 = \sqrt{3}h - h$$

$$120 = \left(\sqrt{3} - 1\right)h$$

$$120 = (1.7321 - 1)\,h$$

$$120 = 0.7321\ h$$

$$\frac{120}{0.7321} = h$$

$$h = 163.91\ m$$

As x= h, so

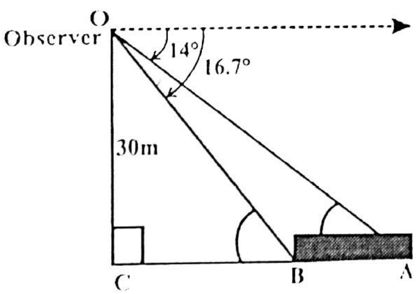
x = 163.91 m or 164m

Thus Distance $m\overline{BC} = 164m$

Height of cliff $= m\overline{CD} = 164m$

Q.12. Suppose that we are standing on a bridge 30 meter above a river watching a log (piece of wood) floating towards us. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°, how long is the log?

Solution:



From the figure

Height of observer's position $= m\overline{OC} = 30m$

Length of log of wood $= m\overline{AB} = x = ?$

Angles of depression from point O of the points A and B are 14° and 16.7° respectively.

In right angled ΔOBC

$$\tan 16.7^\circ = \frac{m\overline{OC}}{m\overline{BC}}$$

$$0.30 = \frac{30}{m\overline{BC}}$$

$$m\overline{BC} = \frac{30}{0.30}$$

$$m\overline{BC} = 100m$$

Now in right angled ΔOAC

$$\tan 14^\circ = \frac{m\overline{OC}}{m\overline{AC}}$$

$$0.249 = \frac{30}{m\overline{AB} + m\overline{BC}}$$

$$0.249 = \frac{30}{(x + 100)}$$

$$0.249(x + 100) = 30$$

$$x + 100 = \frac{30}{0.249}$$

$$x + 100 = 120.482$$

$$x = 120.482 - 100$$

$$x = 20.482\ m$$

So the length of log is 20.482 m.