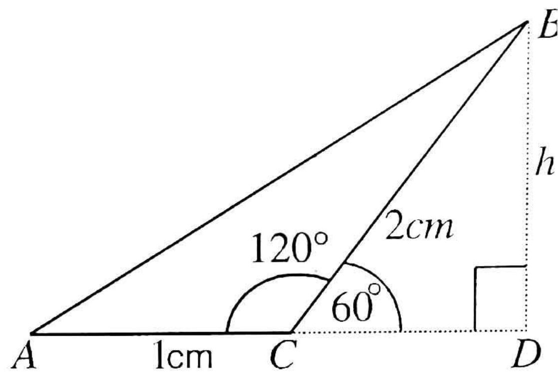
EXERCISE 8.1

Q. 1 Given $m\overline{AC}=1cm,\ m\overline{BC}=2cm,\ m\angle C=120^\circ$

Compute the length AB and the area of ΔABC.

Hint : $(m\overline{AB})^2=(m\overline{AC})^2+(m\overline{BC})^2+2(m\overline{AC})(m\overline{CD})$

where $(m\overline{CD})=(m\overline{BC})\text{Cos}(180^\circ-C)$ (Use theorem I)



Solution:

Given: In a ABC $m\overline{AC}=1cm,\ m\overline{BC}=2cm,\ m\angle C=120^\circ$

To Find: (i) $m\overline{AB}$ **(ii) Area of** ABC

Calculations:

(i) In obtuse angled triangle ABC, by theorem I

$(m\overline{AB})^2 + (m\overline{AC})^2 + (m\overline{BC})^2 = 2(m\overline{AC}).(m\overline{CD})$ .... (i)

In right angled BCD

$$\text{Cos } 60^\circ = \frac{m\overline{CD}}{m\overline{BC}}$$

$$\frac{1}{2} = \frac{m\overline{CD}}{/2}$$

$$\overline{CD} = 1cm \qquad \boxed{\overline{CD} = 1cm}$$

Now putting the corresponding values in (i)

$(m\overline{AB})^2 = (1cm)^2 + (2cm)^2 + 2(1cm)(1cm)$

$= 1cm^2 + 4cm^2 + 2cm^2$

$= 7\ cm^2$

$$\sqrt{(m\overline{AB})^2} = \sqrt{7cm^2} \qquad \boxed{m\overline{AB} = 2.645\ cm}$$

(ii) Area of ΔABC = $\frac{1}{2}$ base × Altitude

$$\text{Area of } \Delta ABC = \frac{1}{2}\ m\overline{AC} \times m\overline{BD}$$

$$= \frac{1}{2} \times 1cm \times h \text{ .... (ii)}$$

In right angled triangle BCD

By Pythagoras theorem

$$(2cm)^2 = (1cm)^2 + (h)^2$$

$$4cm^2 = 1\ cm^2 + h^2$$

$$h^2 = 3\ cm^2 \qquad h = \sqrt{3}\ cm$$

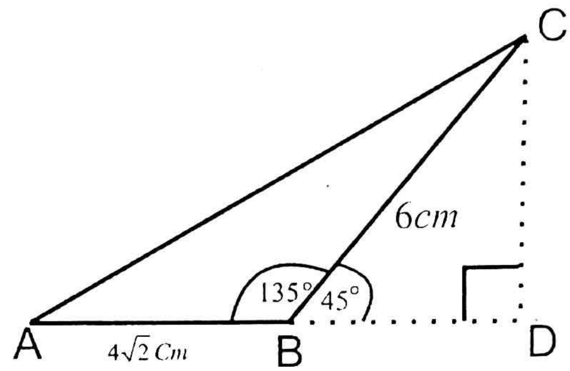
Thus equation (ii) becomes

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 1cm \times \sqrt{3}\,cm$$

$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{2} cm^2$$

Q. 2 Find $m\overline{AC}$ if in ABC, $m\overline{BC} = 6cm$, $m\overline{AB} = 4\sqrt{2}cm$ and $m\angle ABC = 135^o$.

Solution:



Given:

$m\overline{BC} = 6cm$

$m\overline{AB} = 4\sqrt{2}cm$

$m\angle ABC = 135^o$

To Find: $m\overline{AC} = ?$

Calculation:

In obtuse angled triangle ABC, by theorem 1

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 + 2\left(m\overline{AB}\right)\left(m\overline{BD}\right) \text{..........(i)}$$

In right angled BCD

$$\text{Cos } 45^o = \frac{m\overline{BD}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{BD}}{6cm}$$

$$m\overline{BD} = \frac{6}{\sqrt{2}}\ cm$$

Now putting the corresponding values in equation (i) we get

$$\left(m\overline{AC}\right)^2 + \left(4\sqrt{2}\,cm\right)^2 \quad (6cm)^2 = 2\left(4\sqrt{2}\,cm\right)\frac{6}{\sqrt{2}}cm$$

$$= 16(2\ cm^2) + 36cm^2 + 8cm\ (6cm)$$

$$= 32\ cm^2 + 36\ cm^2 + 48\ cm^2$$

$$= 116\ cm^2$$

By taking square root of both sides, we get

$$\sqrt{\left(m\overline{AC}\right)^2} = \sqrt{116cm^2} = \sqrt{4\times 29cm^2}$$

$$m\overline{AC} = 2\sqrt{29}\ cm$$