

THEOREM 2

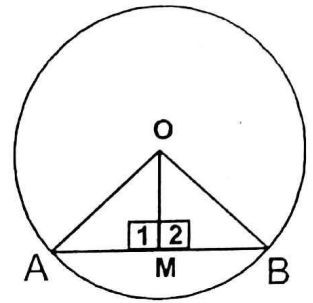
A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

Given: M is the midpoint of any chord \overline{AB} of a circle with centre at O.

Where chord \overline{AB} is not the diameter of the circle.

To prove: $\overline{OM} \perp$ the chord \overline{AB} .

Construction: Join A and B with centre O. write $\angle 1$ and $\angle 2$ as shown in the figure.



Proof:

Statements	Reasons
In $\triangle OAM \leftrightarrow \triangle OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of the same circle
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\triangle OAM \cong \triangle OBM$	S.S.S \cong S.S.S
$\Rightarrow m\angle 1 = m\angle 2 \dots \dots \dots (i)$	Corresponding angles of congruent Δ 's
$i.e., m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ \dots \dots \dots (ii)$	Adjacent supplementary angles
$m\angle 1 = m\angle 2 = 90^\circ$	From (i) and (ii)
$i.e. \overline{OM} \perp \overline{AB}$	

THEOREM 3

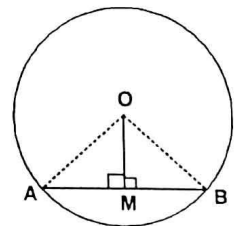
Perpendicular from the centre of a circle on a chord bisects it.

Given: \overline{AB} is the chord of a circle with centre at O so that $\overline{OM} \perp$ chord \overline{AB} .

To prove: M is the mid point of chord \overline{AB} i.e. $m\overline{AM} = m\overline{BM}$

Construction: Join A and B with centre O.

Proof:



Statements	Reasons
In $\angle rt \triangle OAM \leftrightarrow \angle rt \triangle OBM$	
$m\angle OMA = m\angle OMB = 90^\circ$	Given
hyp. $\overline{OA} = \text{hyp. } \overline{OB}$	Radii of the same circle
$m\overline{OM} = m\overline{OM}$	Common
$\triangle OAM \cong \triangle OBM$	In $\angle rt \Delta^s$ H.S \cong H.S
Hence, $m\overline{AM} = m\overline{BM}$	Corresponding sides of congruent triangles.
$\Rightarrow \overline{OM}$ bisects the chord \overline{AB} .	

Corollary 1: \perp bisector of the chord of a circle passes through the centre of a circle.

Corollary 2: The diameter of a circle passes through the mid points of two parallel chords of a circle.

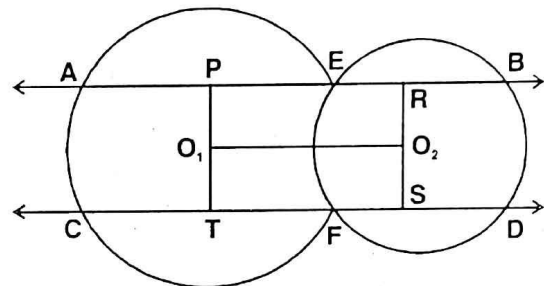
Example:

Parallel lines passing through the points of intersection of two circles and intercepted by them are equal.

Given:

Two circles have centres O_1 and O_2 . They intersect each other at points E and F.

Line segment $\overline{AB} \parallel$ Line segment \overline{CD}



To Prove:

$$m\overline{AB} = m\overline{CD}$$

Construction:

Draw \overline{PT} and $\overline{RS} \perp$ both \overline{AB} and \overline{CD} and join the centres O_1 and O_2 .

Proof:

Statements	Reasons
PRST is rectangle	Construction
$m\overline{PR} = m\overline{TS}$(i)	
Now $m\overline{PR} = m\overline{PE} + m\overline{ER}$	
$= \frac{1}{2} m\overline{AE} + \frac{1}{2} m\overline{EB}$	By theorem 3
$= \frac{1}{2} (m\overline{AE} + m\overline{EB})$	
$m\overline{PR} = \frac{1}{2} m\overline{AB}$(ii)	$m\overline{AE} + m\overline{EB} = m\overline{AB}$
Similarly	
$m\overline{TS} = \frac{1}{2} m\overline{CD}$(iii)	
$\Rightarrow \frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (i), (ii) and (iii)
$\Rightarrow m\overline{AB} = m\overline{CD}$	