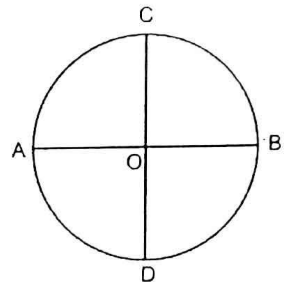
EXERCISE 9.1

Q. 1 Prove that, the diameters of a circle bisect each other.

Given: A circle with centre O. Two diameters $\overline{AB}$ and $\overline{CD}$.

To Prove: Two diameters $\overline{AB}$ and $\overline{CD}$ bisect each other.



Proof:

Statements	Reasons
$\overline{AB}$ and $\overline{CD}$ intersect each other at point O.	$\overline{AB}$ and $\overline{CD}$ are non-parallel.
$\overline{OA} \cong \overline{OB}$ ---------------- (i)	Radii of the same circle
O is the midpoint of $\overline{AB}$ thus $\overline{CD}$ bisects the $\overline{AB}$ at O. Similarly	from (i)
$\overline{OC} \cong \overline{OD}$ ------------------ (ii)	Radii of the same circle
O is the midpoint of $\overline{CD}$ thus $\overline{AB}$ bisects the $\overline{CD}$ at O.	from (ii)
Hence, two diameters $\overline{AB}$ and $\overline{CD}$ bisect each other.	

Q. 2 Two chords of a circle do not pass through the centre. Prove that they cannot bisect each other.

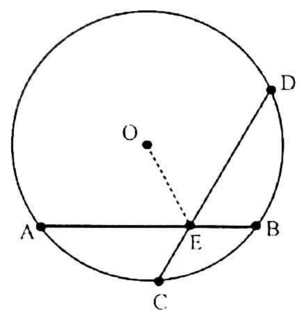
Given:

A Circle with centre "Q". Two different chords $\overline{AB}$ and $\overline{CD}$ not passing through the centre, intersect each other at point E.

To Prove:

$\overline{AB}$ and $\overline{CD}$ do not bisect each other, i.e.

E is not midpoint of $\overline{AB}$ and $\overline{CD}$.



Construction:

Suppose chords $\overline{AB}$ and $\overline{CD}$ bisect each other at point E i.e. E is the common midpoint of $\overline{AB}$ and $\overline{CD}$. Join O to E.

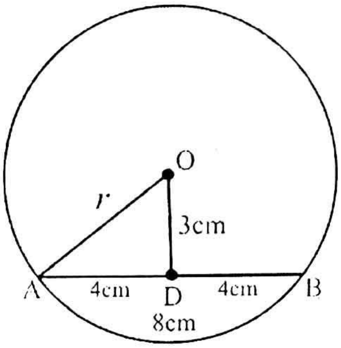
Proof:

Statements	Reasons
As $\overline{OE}$ is perpendicular from "O" to the midpoint E of $\overline{AB}$ and $\overline{CD}$ so ,	A line segment from the centre "O" to the midpoint of a chord is $\perp$ on the chord. (Construction)
$m\angle OEA = 90^\circ$ .... (i)	
$m\angle OED = 90^\circ$ .... (ii)	
$m\angle OEA + m\angle OED = 180^\circ$ .... (iii)	Adding (i) and (ii)
$m\angle AED = 180^\circ$	
It is only possible when A and D are on the same line segment.	
But A and D are not on the same line segment.	Given
So, our supposition, $\overline{AB}$ and $\overline{CD}$ bisect each other, is wrong.	
Thus chords $\overline{AB}$ and $\overline{CD}$ do not bisect each other	

Q. 3 If length of the chord $\overline{AB}$ = 8 cm. Its distance from the centre is 3 cm, then find the diameter of such circle.

Solution:

Given:



Length of chord = $m\overline{AB}$ = 8cm

Distance from centre = $m\overline{OD}$ = 3cm

To Find: Diameter = 2r

Calculations:

$m\overline{AB} = 8cm$ (Given)

Perpendicular from the centre to the chord bisects the chord ($\overline{OD} \perp \overline{AB}$)

$$m\overline{AD} = \frac{1}{2} m\overline{AB} = \frac{1}{2}(8cm) = 4cm$$

In right angled ΔADO by Pythagoras theorem

$$(m\overline{OA})^2 = (m\overline{AD})^2 + (m\overline{OD})^2$$

$$r^2 = (4cm)^2 + (3cm)^2$$

$$r^2 = 16cm^2 + 9cm^2$$

$$r^2 = 25cm^2$$

$$\sqrt{r^2} = \sqrt{25cm^2}$$

r = 5cm

We know that diameter = 2r = 2(5cm) = 10cm

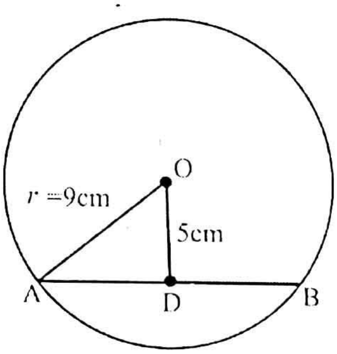
Q.4 Calculate the length of a chord which stands at a distance 5cm from the centre of a circle whose radius is 9cm.

Given:

In a circle with centre O radius = 9cm,

$m\overline{OD} = 5cm$

To find:



Length of chord $\overline{AB}$

Calculations:

In right angled ΔADO, by Pythagoras theorem

$$\left(m\overline{OA}\right)^2 = (mAD)^2 + (mOD)^2$$

$$(9cm)^2 = \left(m\overline{AD}\right) + (5cm)^2$$

$$81cm^2 = \left(m\overline{AD}\right)^2 + 25cm^2$$

$$81cm^2 - 25cm^2 = \left(m\overline{AD}\right)^2$$

$$56cm^2 = \left(m\overline{AD}\right)^2$$

$$\sqrt{\left(m\overline{AD}\right)^2} = \sqrt{56cm^2}$$

$$m\overline{AD} = \sqrt{56}cm$$

We know that

$m\overline{AD} = \frac{1}{2} m\overline{AB}$ (Perpendicular from the centre to the cord bisects the chord)

$$\Rightarrow \quad m\overline{AB} = 2m\overline{AD}$$

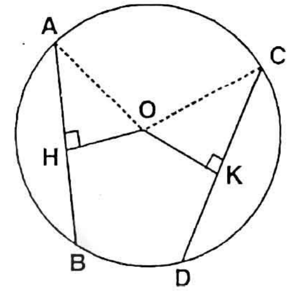
$$= 2 \times \sqrt{56}cm$$

$$m\overline{AB} = 14.966cm$$

or $\quad m\overline{AB} \simeq 14.97cm$

THEOREM 4

If two chords of a circle are congruent then they will be equidistant from the centre.



Given: $\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To Prove: $m\overline{OH} = m\overline{OK}$

Construction: Join O with A and O with C.

So that we have $\angle rt\Delta^s$ OAH and OCK.

Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ .... (i)	$\overline{OH} \perp \overline{AB}$ By Theorem 3
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ .... (ii)	$\overline{OK} \perp \overline{CD}$ By Theorem 3
But $m\overline{AB} = m\overline{CD}$ .... (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ .... (iv)	Using (i) , (ii) and (iii) Both are half of equal segment
Now in $\angle rt\ \Delta^s$ OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp. $\overline{OA}$ = hyp. $\overline{OC}$	Radii of the same circle
$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\Delta OAH \cong \Delta OCK$	H.S $\cong$ H.S
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	Corresponding sides of congruent triangles.