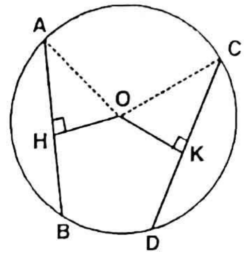
THEOREM 5

Two chords of a circle which are equidistant from the centre, are congruent.



Given: $\overline{AB}$ and $\overline{CD}$ are two chords of a circle with centre at O.

$\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$

To Prove: $m\overline{AB} = m\overline{CD}$

Construction: Join A and C with O. So that we can form $\angle rt\Delta^s$ OAH and OCK.

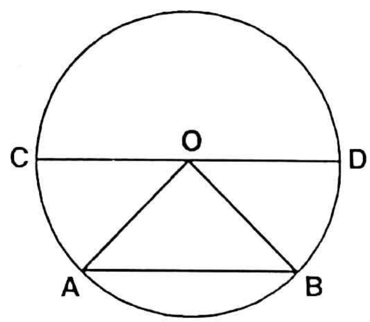
Proof:

Statements	Reasons
In $\angle rt\ \Delta^s$ OAH $\leftrightarrow$ OCK.	
$\because$ hyp. $\overline{OA} =$ hyp. $\overline{OC}$	Radii of the same circle.
$m\overline{OH} = m\overline{OK}$	Given
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S $\cong$ H.S
So $m\overline{AH} = m\overline{CK}$ .... (i)	
But $m\overline{AH} = \frac{1}{2} m\overline{AB}$ .... (ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly $m\overline{CK} = \frac{1}{2} m\overline{CD}$	$\overline{OK} \perp$ chord $\overline{CD}$ (Given)
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\therefore$ $\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (ii) & (iii)
or $m\overline{AB} = m\overline{CD}$	

Example: **Prove that the largest chord in a circle is the diameter.**

Given: $\overline{AB}$ is a chord and $\overline{CD}$ is the diameter of a circle with centre point O.

Prove: If $\overline{AB}$ and $\overline{CD}$ are distinct, then $m\overline{CD} > m\overline{AB}$.



Construction: Join O with A and B to form a Δ OAB.

Proof: Sum of two sides of a triangle is greater than its third side.

$\therefore$ In $\Delta OAB \Rightarrow m\overline{OA} + m\overline{OB} > m\overline{AB}$ ---------- (i)

But $\overline{OA}$ and $\overline{OB}$ are the radii of the same circle with centre O.

So that $m\overline{OA} + m\overline{OB} = m\overline{CD}$ ---------- (ii)

$m\overline{CD} > m\overline{AB}$ [From (i) and (ii)]

$\Rightarrow$ Diameter CD> chord $\overline{AB}$.

Hence, diameter CD is greater than any other chord drawn in the circle.