$$x = \frac{11}{2} \pm \frac{15}{2}$$

$$x = \frac{11 \pm 15}{2}$$

$$x = \frac{11+15}{2}, \frac{11-15}{2}$$

$$x = \frac{26}{2}, \frac{-4}{2}$$

$$x = 13, -2$$

$$\text{S.S.} = \{13, -2\}$$

5.3 The Quadratic Formula

If $ax^2 + bx + c = 0 \qquad a \neq 0$

then $x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve using quadratic formula:

Q.1. $x^2 - 5x + 6 = 0$

Sol. Here $a = 1$

$b = -5$

$c = 6$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Putting values of a, b, c

$$x = \frac{-(-5) + \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

$$x = \frac{5+1}{2}, \frac{5-1}{2}$$

$$x = \frac{6}{2}, \frac{4}{2}$$

$$x = 3, 2$$

Solutin set $= \{3, 2\}$

Q.2. $(3 - 4x) = (4x - 3)^2$

Sol. $3 - 4x = 16x^2 - 24x + 9$

$0 = 16x^2 - 24x + 9 - 3 + 4x$

$0 = 16x^2 - 20x + 6$

or $16x^2 - 20x + 6 = 0$

$8x^2 - 10x + 3 = 0$ (Divided by 2)

Here $a = 8$

$b = -10$

$c = 3$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Putting the values of a, b, c

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(8)(3)}}{2(8)}$$

$$x = \frac{10 \pm \sqrt{100 - 96}}{16}$$

$$x = \frac{10 \pm \sqrt{4}}{16}$$

$$x = \frac{10 \pm 2}{16}$$

$$x = \frac{10+2}{16}, \frac{10-2}{16}$$

$$x = \frac{12}{16}, \frac{8}{16}$$

$$x = \frac{3}{4}, \frac{1}{2}$$

Solution set $= \left\{\frac{3}{4}, \frac{1}{2}\right\}$

Q.3. $3x^2 + x - 2 = 0$

Sol. Here $a = 3$

$b = 1$

$c = -2$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{1 + 24}}{6}$$

$$x = \frac{-1 \pm \sqrt{25}}{6}$$

$$x = \frac{-1 \pm 5}{6}$$

$$x = \frac{-1+5}{6}, \frac{-1-5}{6}$$

$$x = \frac{4}{6}, \frac{-6}{6}$$

$$x = \frac{2}{3}, -1$$

Solution set $= \left\{\frac{2}{3}, -1\right\}$

Q.4. $10x^2 - 5x = 15$

Sol. $10x^2 - 5x - 15 = 0$

$2x^2 - x - 3 = 0$ (Divided by 5)

Here $a = 2$

$b = -1$

$c = -3$

Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1+24}}{4}$$

$$x = \frac{1 \pm \sqrt{25}}{4}$$

$$x = \frac{1 \pm 5}{4}$$

$$x = \frac{1+5}{4}, \frac{1-5}{4}$$

$$x = \frac{6}{4}, \frac{-4}{4}$$

$$x = \frac{3}{2}, -1$$

$$\text{Solution set} = \left\{\frac{3}{2}, -1\right\}$$

Q.5. $(x-1)(x+3) - 12 = 0$

Sol. $x^2 + 2x - 3 - 12 = 0$

$$x^2 + 2x - 15 = 0$$

Here $a = 1$

$b = 2$

$c = -15$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-15)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+60}}{2}$$

$$x = \frac{-2 \pm \sqrt{64}}{2}$$

$$x = \frac{-2 \pm 8}{2}$$

$$x = \frac{-2+8}{2}, \frac{-2-8}{2}$$

$$x = \frac{6}{2}, \frac{-10}{2}$$

$$x = 3, -5$$

Solution set $= \{3, -5\}$

Q.6. $\mathbf{x(2x + 7) - 3(2x + 7) = 0}$

Sol. $2x^2 + 7x - 6x - 21 = 0$

$$2x^2 + x - 21 = 0$$

Here $a = 2$

$b = 1$

$c = -21$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-21)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1 + 168}}{4}$$

$$x = \frac{-1 \pm \sqrt{169}}{4}$$

$$x = \frac{-1 \pm 13}{4}$$

$$x = \frac{-1 + 13}{4}, \frac{-1 - 13}{4}$$

$$x = \frac{12}{4}, \frac{-14}{4}$$

$$x = 3, -\frac{7}{2}$$

Solution set $= \left\{3, -\frac{7}{2}\right\}$

Q.7. $\frac{x+1}{x+4} = \frac{2x-1}{x+6}$, **where** $x \neq -4, -6$

Sol. $(x+1)(x+6) = (2x-1)(x+4)$

$x^2 + 7x + 6 = 2x^2 + 7x - 4$

$x^2 - 2x^2 + 7x - 7x + 6 + 4 = 0$

$-x^2 + 10 = 0$

$x^2 - 10 = 0$

Here $a = 1$

$b = 0$

$c = -10$

(Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-10)}}{2(1)}$

$x = \frac{0 \pm \sqrt{0 + 40}}{2}$

$x = \frac{0 \pm \sqrt{40}}{2}$

$x = \frac{0 \pm 2\sqrt{10}}{2}$

$x = \pm\frac{2\sqrt{10}}{2}$

$x = \pm\sqrt{10}$

Solution set $= \{\pm\sqrt{10}\}$

Q.8. $\frac{x}{6}+\frac{6}{x}=\frac{4}{x}+\frac{x}{4}$, where $x \neq 0$

Sol. Multiplying by L.C.M. $12x$

$$12x\left(\frac{x}{6}\right)+12x\left(\frac{6}{x}\right)=12x\left(\frac{4}{x}\right)+12x\left(\frac{x}{4}\right)$$

$$2x^2+72=48+3x^2$$

$$2x^2-3x^2+72-48=0$$

$$-x^2+24=0$$

$$x^2-24=0$$

Here $a=1$

$b=0$

$c=-24$

(Formula) $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

By putting the values of a, b, c

$$x=\frac{-(0)\pm\sqrt{(0)^2-4(1)(-24)}}{2(1)}$$

$$x=\frac{\pm\sqrt{0+96}}{2}$$

$$x=\frac{\pm\sqrt{96}}{2}$$

$$x=\frac{\pm\sqrt{16\times 6}}{2}$$

$$x=\frac{\pm 4\sqrt{6}}{2}$$

$$x=\pm 2\sqrt{6}$$

Solution set $=\left\{\pm 2\sqrt{6}\right\}$

Q.9. $\dfrac{x+4}{x-4}+\dfrac{x-4}{x+4}=\dfrac{10}{3}$ **where** $x \neq -4$

Sol Multiplying by $3(x-4)(x+4)$

$$3(x+4)(x+4)+3(x-4)(x-4) = 10(x-4)(x+4)$$

$$3(x^2+8x+16)+3(x^2-8x+16) = 10(x^2-16)$$

$$3x^2+24x+48+3x^2-24x+48 = 10x^2-160$$

$$3x^2+3x^2-10x^2+24x-24x+48+48+160=0$$

$$-4x^2+256 = 0$$

$$4x^2-256 = 0$$

$$x^2-64 = 0 \quad \text{(Divided by 4)}$$

Here $a = 1$

$b = 0$

$c = -64$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-0 \pm \sqrt{(0)^2-4(1)(-64)}}{2(1)}$$

$$x = \frac{\pm\sqrt{0+256}}{2}$$

$$x = \frac{\pm\sqrt{256}}{2}$$

$$x = \frac{\pm 16}{2}$$

$$x = \pm 8$$

Solution set $= \{8, -8\}$

Q.10. $\dfrac{1}{x-1}+\dfrac{1}{x-2}=\dfrac{2}{x-3}$ **where** $x \neq 1, 2, 3$

Sol. Multiplying by $(x-1)(x-2)(x-3)$

$$(x-1)(x-2)(x-3)\cdot\frac{1}{(x-1)}+(x-1)(x-2)(x-3)\cdot\frac{1}{(x-2)}=(x-1)(x-2)(x-3)\cdot\frac{2}{(x-3)}$$

$$(x-2)(x-3)+(x-1)(x-3)=(x-1)(x-2)(2)$$

$$x^2-5x+6+x^2-4x+3=2(x^2-3x+2)$$

$$2x^2-9x+9=2x^2-6x+4$$

$$-9x+6x=4-9$$

$$-3x=-5$$

$$x=\frac{-5}{-3}$$

$$x=\frac{5}{3}$$

Q.11. $(x+4)(x-1)+(x+5)(x+2)=6$

Sol. $x^2+3x-4+x^2+7x+10=6$

$$2x^2+10x-4+10-6=0$$

$$2x^2+10x=0$$

Here $a=2$

$b=10$

$c=0$

(Formula) $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

By putting the values of a, b, c

$$x=\frac{-10\pm\sqrt{(10)^2-4(2)(0)}}{2(2)}$$

$$x = \frac{-10 \pm \sqrt{100}}{4}$$

$$x = \frac{-10 \pm 10}{4}$$

$$x = \frac{-10+10}{4}, \frac{-10-10}{4}$$

$$x = \frac{0}{4}, \frac{-20}{4}$$

$$x = 0, -5$$

Solution set $= \{0, -5\}$

Q.12 $(2x+4)^2 - (4x-6)^2 = 0$

Sol.

$(4x^2 + 16x + 16) - (16x^2 - 48x + 36) = 0$

$$-12x^2 + 64x - 20 = 0$$

(Dividing by -4)

$$3x^2 - 16x + 5 = 0$$

Here $a = 3$

$b = -16$

$c = 5$

(Formula) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

By putting the values of a, b, c

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{16 \pm \sqrt{256 - 60}}{6}$$

$$x = \frac{16 \pm \sqrt{196}}{6}$$

$$x = \frac{16 \pm 14}{6}$$

$$x = \frac{16+14}{6}, \frac{16-14}{6}$$

$$x = \frac{30}{6}, \frac{2}{6}$$

$$x = 5, \frac{1}{3}$$

Solution set $= \left\{5, \frac{1}{3}\right\}$.

Exercise 5.3

Q.1. Find two consecutive positive odd numbers such that the sum of their squares is 74.

Sol. Let 1^{st} odd number $= 2x + 1$

2^{nd} odd number $= 2x + 3$

According to statement

$$(2x+1)^2 + (2x+3)^2 = 74$$

$$4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 74$$

$$8x^2 + 16x + 10 - 74 = 0$$

$$8x^2 + 16x - 64 = 0$$

$$x^2 + 2x - 8 = 0 \text{ (Divided by 8)}$$

$$x^2 + 4x - 2x - 8 = 0$$

$$(x^2 + 4x) - (2x + 8) = 0$$