$$x = \frac{16 \pm \sqrt{196}}{6}$$

$$x = \frac{16 \pm 14}{6}$$

$$x = \frac{16+14}{6}, \frac{16-14}{6}$$

$$x = \frac{30}{6}, \frac{2}{6}$$

$$x = 5, \frac{1}{3}$$

Solution set $= \left\{5, \frac{1}{3}\right\}$.

Exercise 5.3

Q.1. Find two consecutive positive odd numbers such that the sum of their squares is 74.

Sol. Let 1^{st} odd number $= 2x + 1$

2^{nd} odd number $= 2x + 3$

According to statement

$$(2x+1)^2 + (2x+3)^2 = 74$$

$$4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 74$$

$$8x^2 + 16x + 10 - 74 = 0$$

$$8x^2 + 16x - 64 = 0$$

$$x^2 + 2x - 8 = 0 \text{ (Divided by 8)}$$

$$x^2 + 4x - 2x - 8 = 0$$

$$(x^2 + 4x) - (2x + 8) = 0$$

$$x(x + 4) - 2(x + 4) = 0$$

$$(x + 4)(x - 2) = 0$$

If $x - 2 = 0$

then $\boxed{x = 2}$

and if $x + 4 = 0$

then $\boxed{x = -4}$

when $x = 2$ then 1st number $= 2x + 1$

$= 2(2) + 1$

$= 4 + 1$

$= 5$

and 2nd number $= 2x + 3$

$= 2(2) + 3$

$= 4 + 3$

$= 7$

Required odd number = 5, 7

Q.2. Find two consecutive positive even numbers such that the sum of their squares is 164.

Sol Let 1st number $= 2x$

2nd number $= 2x + 2$

According to the statement

$$(2x)^2 + (2x + 2)^2 = 164$$

$$4x^2 + 4x^2 + 8x + 4 = 164$$

$$8x^2 + 8x + 4 - 164 = 0$$

$$8x^2 + 8x - 160 = 0$$

$$x^2 + x - 20 = 0 \quad \text{(Divided by 8)}$$

$$x^2 + 5x - 4x - 20 = 0$$

$$(x^2 + 5x) - (4x + 20) = 0$$

$$x(x + 5) - 4(x + 5) = 0$$

$$(x + 5)(x - 4) = 0$$

If $x - 4 = 0$

then $\boxed{x = 4}$

and if $x + 5 = 0$

then $\boxed{x = -5}$

when $x = 4$, then 1st number $= 2x$

$= 2(4) = 8$

And 2nd number $= 2x + 2$

$= 2(4) + 2 = 8 + 2 = 10$

Required even number = 8, 10

Q.3. The difference of two numbers is 9 and the product of the numbers is 162. Find the two numbers.

Sol. Let 1st number $= x$

2nd number $= x + 9$

According to the statement

$$x^2 + 9x = 162$$

$$x^2 + 9x = 162$$

$$x^2 + 9x - 162 = 0$$

$$x^2 + 18x - 9x - 162 = 0$$

$$(x^2 + 18x) - (9x + 162) = 0$$

$$x(x + 18) - 9(x + 18) = 0$$

$$(x - 9)(x + 18) = 0$$

If $x - 9 = 0$

then $\boxed{x = 9}$

and if $x + 18 = 0$

$\boxed{x = -18}$

If $x = 9$, then 1st number $= x = 9$

2nd number $= x + 9$

$= 9 + 9 = 18$

Required numbers = $\boxed{9, 18}$

If $x = -18$

1st number $= x = -18$

2nd number $= x + 9$

$= -18 + 9 = -9$

Required numbers = $\boxed{-18, -9}$

Q.4. **The base and height of a triangle are $(x + 3)cm$ and $(2x - 5)cm$ respectively. If the area of the triangle is $20cm^2$, find x.**

Sol.

$$\text{Length of base} = x + 3\text{cm}$$

$$\text{Length of altitude} = 2x - 5\text{cm}$$

$$\text{Area of } \Delta = 20\text{cm}^2$$

According to the statement

$$\frac{(x+3)(2x-5)}{2} = 20$$

$$\text{Area of } \Delta = \frac{B \times A}{2}$$

$$\frac{2x^2 + x - 15}{2} = 20$$

$$2x^2 + x - 15 = 2 \times 20$$

$$2x^2 + x - 15 = 40$$

$$2x^2 + x - 15 - 40 = 0$$

$$2x^2 + x - 55 = 0$$

$$2x^2 + 11x - 10x - 55 = 0$$

$$(2x^2 + 11x) - (10x + 55) = 0$$

$$x(2x + 11) - 5(2x + 11) = 0$$

$$(2x + 11)(x - 5) = 0$$

If $x - 5 = 0$

then $\boxed{x = 5}$

Therefore, Length of base $= x + 3$

$= 5 + 3$

$= 8\text{cm}$

Length of altitude $= 2x - 5$

$= 2(5) - 5$

$= 10 - 5$

$= 5\text{cm}$

and if $2x + 11 = 0$

$2x = -11$

then $\boxed{x = -\frac{11}{2}}$

Cancelling due to negativity.

Q.5. The perimeter and area of a rectangle are 22cm and $30cm^2$ respectively. Find the length and breadth of the rectangle.

Sol. Rectangle's perimeter $= 22\text{cm}$

Rectangle's area $= 30\text{cm}^2$

Let, Length of rectangle $= x$ cm

Breadth of rectangle $= \frac{30}{x}$ cm

$\therefore$ Perimeter $=$ (length + breadth) $\times 2$

$22 = \left(\frac{30}{x} + x\right) \times 2$

$11 = x + \frac{30}{x}$ (Divided by 2)

$11x = 30 + x^2$ (Multiplying by x)

$0 = 30 + x^2 - 11x$

$$x^2 - 11x + 30 = 0$$

$$x^2 - 5x - 6x + 30 = 0$$

$$(x^2 - 5x) - (6x - 30) = 0$$

$$x(x - 5) - 6(x - 5) = 0$$

$$(x - 5)(x - 6) = 0$$

If $x - 5 = 0$

then $\boxed{x = 5}$

Length of rectangle $= x$

$= 5\text{cm}$

and Breadth of rectangle $= \frac{30}{x}$

$= \frac{30}{5}$

$= 6\text{ cm}$

Length of rectangle $= 5\text{cm}$

Breadth of rectangle $= 6\text{cm}$

and if $x - 6 = 0$

then $\boxed{x = 6}$

Length of rectangle $= x$

$= 6\text{cm}$

and Breadth of rectangle $= \frac{30}{x}$

$= \frac{30}{6}$

$= 5\text{cm}$

Length of rectangle = 6cm

So, Breadth of rectangle = 5cm

Q.6. The product of two consecutive positive numbers is 156. Find the numbers.

Sol. Let, 1^{st} number $= x$

2^{nd} number $= x + 1$

According to the statement

$$(x)(x+1) = 156$$

$$x^2 + x = 156$$

$$x^2 + x - 156 = 0$$

$$x^2 + 13x - 12x - 156 = 0$$

$$(x^2 + 13x) - (12x + 156) = 0$$

$$x(x+13) - 12(x+13) = 0$$

$$(x-12)(x+13) = 0$$

If $x - 12 = 0$

then $\boxed{x = 12}$

Required numbers $= x, x+1$

$= 12, 12+1$

$= \boxed{12, 13}$

and if $x + 13 = 0$

then $\boxed{x = -13}$

Required numbers $= x, x+1$

$$= -13, -13 + 1$$

$$= \boxed{-13, -12}$$

Q.7. Find two consecutive positive odd numbers given that the difference between their reciprocals is $\frac{2}{63}$.

Sol. Suppose that 1^{st} number $= 2x + 1$

2^{nd} number $= 2x + 3$

According to given condition

$$\frac{1}{2x+1} - \frac{1}{2x+3} = \frac{2}{63}$$

Multiplying by $(2x + 1)(2x + 3)(63)$

$$63\cancel{(2x+1)}(2x+3)\times\frac{1}{\cancel{(2x+1)}} - 63(2x+1)\cancel{(2x+3)}\times\frac{1}{\cancel{(2x+3)}}$$

$$= \frac{2}{63}\times 63(2x+1)(2x+3)$$

$$63(2x + 3) - 63(2x + 1) = 2(2x + 1)(2x + 3)$$

$$\cancel{126x} + 189 - \cancel{126x} - 63 = 2(4x^2 + 8x + 3)$$

$$126 = 8x^2 + 16x + 6$$

$$8x^2 + 16x + 6 - 126 = 0$$

$$8x^2 + 16x - 120 = 0$$

$$x^2 + 2x - 15 = 0 \quad \text{(Dividing by 8)}$$

$$x^2 + 5x - 3x - 15 = 0$$

$$(x^2 + 5x) - (3x + 15) = 0$$

$$x(x + 5) - 3(x + 5) = 0$$

$$(x + 5)(x - 3) = 0$$

If $x - 3 = 0$

then $\boxed{x = 3}$

Required numbers: 1st number $= 2x + 1$ required numbers

$= 2(3) + 1$

$= 6 + 1$

$$\boxed{1st\ number = 7}$$

2nd number $= 2x + 3$

$= 2(3) + 3$

$= 6 + 3$

$$\boxed{2nd\ number = 9}$$

and if $x + 5 = 0$

then $\boxed{x = -5}$

Cancelling due to negativity.

Q.8. The sum of the two positive number is 12 and the sum of whose squares is 80. Find the numbers.

Sol. Suppose that 1st number $= x$

2nd number $= 12 - x$

According to given condition

$$(x)^2 + (12 - x)^2 = 80$$

$$x^2 + 144 + x^2 - 24x = 80$$

$$2x^2 - 24x + 144 - 80 = 0$$

$$2x^2 - 24x + 64 = 0$$

$$x^2 - 12x + 32 = 0 \quad \text{(Dividing by 2)}$$

$$x^2 - 4x - 8x + 32 = 0$$

$$x(x-4) - 8(x-4) = 0$$

$$(x-4)(x-8) = 0$$

If $x - 4 = 0$

then $\boxed{x = 4}$

Required numbers: 1st number = $\boxed{x = 4}$

2nd number = $12 - x$

$$\boxed{2nd\ number = 12 - 4 = 8}$$

and if $x - 8 = 0$

$$x = 8$$

Hence $\boxed{1st\ number = 8}$

2nd number = $12 - x$

$$= 12 - 8$$

$$\boxed{2nd\ number = 4}$$

Required numbers = 8, 4

or Required numbers = 4, 8