(x) *Symmetric Matrix:*

A square matrix A is called symmetric if $A' = A$

For example: $A = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$, *and* $A' = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$

(xi) *Skew-Symmetric Matrix:*

A square matrix A is called skew symmetric (or anti-symmetric) if $A' = -A$

For example: $A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} = -A$$

$A' = -A$ Hence A is skew symmetric.

Exercise 6.2

1- Identify row matrices, column matrices, square matrices, and rectangular matrices in the following matrices.

$$A = \begin{bmatrix} 3 & 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 5+2 & 4 \\ 2 & 6 \end{bmatrix}, C = \begin{bmatrix} a+x \\ b+y \end{bmatrix},$$

$$D = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, E = \begin{bmatrix} x & -2 \\ b & 5 \end{bmatrix}, F = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & -5 & 0 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 7 & 8 \end{bmatrix}, H = [\,0\,]$$

Solution:

$$A=\begin{bmatrix}3 & 1 & 1 & 1\end{bmatrix}$$ Row Matrix

$$C=\begin{bmatrix}(a+x)\\(b+y)\end{bmatrix}$$ Column Matrix

$$B=\begin{bmatrix}(5+2) & 4\\2 & 6\end{bmatrix},\ D=\begin{bmatrix}2 & 4\\1 & 3\end{bmatrix}$$ Square Matrix

$$E=\begin{bmatrix}x & -2\\b & 5\end{bmatrix},\ F=\begin{bmatrix}1 & 3 & 2\\2 & 4 & 5\\1 & -5 & 0\end{bmatrix},\ H=[0]$$

$$C=\begin{bmatrix}(a+x)\\(b+y)\end{bmatrix},\ G=\begin{bmatrix}1 & 2 & 4\\5 & 7 & 8\end{bmatrix}$$ Rectangle Matrix

2. **Identify, diagonal matrices, scalar matrices, identity matrices.**

$$A=\begin{bmatrix}3 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{bmatrix},\ B=\begin{bmatrix}7 & 0 & 0\\0 & 7 & 0\\0 & 0 & 7\end{bmatrix},\ C=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 4\end{bmatrix},$$

$$D=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix},\ E=\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix},\ F=\begin{bmatrix}8 & 0\\0 & 0\end{bmatrix},$$

$$G=\begin{bmatrix}k & 0 & 0\\0 & k & 0\\0 & 0 & k\end{bmatrix}$$

Solution:

$$G=\begin{bmatrix}k & 0 & 0\\0 & k & 0\\0 & 0 & k\end{bmatrix}$$ Diagonal Matrix

$$A=\begin{bmatrix}3&0&0\\0&1&0\\0&0&0\end{bmatrix}, C=\begin{bmatrix}0&0&0\\0&0&0\\0&0&4\end{bmatrix}, F=\begin{bmatrix}8&0\\0&0\end{bmatrix}$$

Scalar Matrix

$$B=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}, E=\begin{bmatrix}5&0\\0&5\end{bmatrix}, G=\begin{bmatrix}k&0&0\\0&k&0\\0&0&k\end{bmatrix}$$

$$D=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Identity Matrix

3. **Find transpose of the following matrices.**

$$A=\begin{bmatrix}3&4\\-1&4\end{bmatrix}, B=\begin{bmatrix}-3&-2\\-1&4\end{bmatrix},$$

$$C=\begin{bmatrix}a&-b\\c&d\end{bmatrix}, D=\begin{bmatrix}l&m&n\\p&q&r\\a&b&c\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}3&4\\-1&4\end{bmatrix}, A'=\begin{bmatrix}3&-1\\4&4\end{bmatrix}$$

$$B=\begin{bmatrix}-3&-2\\-1&4\end{bmatrix}, B'=\begin{bmatrix}-3&-1\\-2&4\end{bmatrix}$$

$$C=\begin{bmatrix}a&-b\\c&d\end{bmatrix}, C'=\begin{bmatrix}a&c\\-b&d\end{bmatrix}$$

$$D=\begin{bmatrix} l & m & n \\ p & q & r \\ a & b & c \end{bmatrix},\ D'=\begin{bmatrix} l & p & a \\ m & q & b \\ n & r & c \end{bmatrix}$$

4. **Identify all row matrices, if:**

$$A=\begin{bmatrix} 3 & 4 & 5 \end{bmatrix},\ B=\begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix},\ C=\begin{bmatrix} e & f & g \end{bmatrix},$$

$$D=\begin{bmatrix} 3 & 7 & 5 \\ 4 & 6 & 2 \\ 1 & 9 & 8 \end{bmatrix},\ F=\begin{bmatrix} 1 & 4 & 6 \\ 3 & 7 & 3 \end{bmatrix}$$

Solution: Row Matrix

$$A=\begin{bmatrix} 3 & 4 & 5 \end{bmatrix},\ C=\begin{bmatrix} e & f & g \end{bmatrix}$$

5. **Identify all column matrices, if:**

$$A=\begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix},\ B=\begin{bmatrix} 2 & 3 \\ 6 & 5 \\ 4 & 7 \end{bmatrix},\ C=\begin{bmatrix} a \\ b \\ c \end{bmatrix},$$

$$D=\begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 5 \\ -2 & 3 & 4 \end{bmatrix},\ E=\begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix},\ F=\begin{bmatrix} 9 & 7 & 1 \end{bmatrix}$$

Solution: Column Matrix

$$A=\begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix},\ C=\begin{bmatrix} a \\ b \\ c \end{bmatrix},\ E=\begin{bmatrix} 5 \\ 7 \\ -4 \end{bmatrix}$$

6. **Identify all column matrices, if:**

$$A=\begin{bmatrix}1 & 3\\ 2 & 2\end{bmatrix}, B=\begin{bmatrix}3 & -2 & 4\\ 1 & 6 & 5\\ 7 & 3 & 4\end{bmatrix}, C=\begin{bmatrix}1\\ 2\\ 9\end{bmatrix},$$

$$D=\begin{bmatrix}7 & 8\\ 6 & 5\end{bmatrix}, E=\begin{bmatrix}3 & 7 & 5\end{bmatrix}, F=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

Solution: Column Matrix

$$C=\begin{bmatrix}1\\ 2\\ 9\end{bmatrix}$$

7. **Identify all 3-by-3 square matrices, if:**

$$A=\begin{bmatrix}2 & -3 & 6\\ 1 & 5 & 4\\ 3 & 6 & -3\end{bmatrix}, B=\begin{bmatrix}2\\ 4\\ 7\end{bmatrix}, C=\begin{bmatrix}7 & 3 & 4\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}2 & -3 & 6\\ 1 & 5 & 4\\ 3 & 6 & -3\end{bmatrix}$$

A Scalar Multiplication

Any element from the set of real numbers is also called a scalar. We define the product of a matrix A and a scalar k, denoted by kA, to be the matrix formed by multiplying each element of A by k.

For example: $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, *then* $kA=\begin{bmatrix}ka & kb\\ kc & kd\end{bmatrix}$

Laws of Addition of Matrices

Commutative Law:

For any two matrices A and B of the same order

$A + B = B + A$

This law is called commutative law of matrices with respect to addition.

Associative Law:

For three matrices A, B and C of same order,

$(A + B) + C = A + (B + C)$

This law is called associative law of matrices with respect to addition.

Additive Identity of Matrices

$A + O = O + A = A$

Additive Inverse of a Matrix

If two matrices A and B are such that their sum (A + B) is a zero matrix, then A and B are called additive inverse of each other.

For example: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *and* $B = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

Exercise 6.3

1- If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$

Find

(i) $A + B$ (ii) $A - B$ (iii) $B - A$

(iv) $2A + 3B$ (v) $3A - 4B$ (vi) $A - 2B$

Solutions:

(i)

$$A + B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$