Laws of Addition of Matrices

Commutative Law:

For any two matrices A and B of the same order

$A + B = B + A$

This law is called commutative law of matrices with respect to addition.

Associative Law:

For three matrices A, B and C of same order,

$(A + B) + C = A + (B + C)$

This law is called associative law of matrices with respect to addition.

Additive Identity of Matrices

$A + O = O + A = A$

Additive Inverse of a Matrix

If two matrices A and B are such that their sum (A + B) is a zero matrix, then A and B are called additive inverse of each other.

For example: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *and* $B = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$

Exercise 6.3

1- If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$

Find

(i)	A + B	(ii)	A − B	(iii)	B − A
(iv)	2A + 3B	(v)	3A − 4B	(vi)	A − 2B

Solutions:

(i)

$$A + B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 3+1 & 4+5 \\ 1+2 & 5+3 & 5+6 \\ 4+1 & 9+4 & 3-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 9 \\ 3 & 8 & 11 \\ 5 & 13 & 1 \end{bmatrix}$$

(ii)

$$A-B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 3-1 & 4-5 \\ 1-2 & 5-3 & 5-6 \\ 4-1 & 9-4 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & -1 \\ 3 & 5 & 5 \end{bmatrix}$$

(iii)

$$B-A = \begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 1-3 & 5-4 \\ 2-1 & 3-5 & 6-5 \\ 1-4 & 4-9 & -2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 & 1 \\ 1 & -2 & 1 \\ -3 & -5 & -5 \end{bmatrix}$$

(iv)

$$2A + 3B = 2\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} + 3\begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 8 \\ 2 & 10 & 10 \\ 8 & 18 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 15 \\ 6 & 9 & 18 \\ 3 & 12 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 6+3 & 8+15 \\ 2+6 & 10+9 & 10+18 \\ 8+3 & 18+12 & 6-6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 & 23 \\ 8 & 19 & 28 \\ 11 & 30 & 0 \end{bmatrix}$$

(v)

$$3A-4B = 3\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - 4\begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 & 12 \\ 3 & 15 & 15 \\ 12 & 27 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 4 & 20 \\ 8 & 12 & 24 \\ 4 & 16 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 6-0 & 9-4 & 12-20 \\ 3-8 & 15-12 & 15-24 \\ 12-4 & 27-16 & 9+8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 5 & -8 \\ -5 & 3 & -9 \\ 8 & 11 & 17 \end{bmatrix}$$

(vi)

$$A-2B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - 2\begin{bmatrix} 0 & 1 & 5 \\ 2 & 3 & 6 \\ 1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 5 \\ 4 & 9 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 10 \\ 4 & 6 & 12 \\ 2 & 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 3-2 & 4-10 \\ 1-4 & 5-6 & 5-12 \\ 4-2 & 9-8 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & -6 \\ -3 & -1 & -7 \\ 2 & 1 & 7 \end{bmatrix}$$

2- **Find the additive inverse of the following matrices.**

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix}, B = \begin{bmatrix} \sqrt{2} & 3 \\ 4 & \sqrt{3} \end{bmatrix}, C = \begin{bmatrix} 1 \\ -7 \\ 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & 4 \\ 2 & -1 & -3 \end{bmatrix}, E = [2 \quad 5 \quad -3]$$

Sol:

Matrix	**Additive Inverse**
$A = \begin{bmatrix} 4 & 3 \\ 2 & 6 \end{bmatrix}$,	$-A = \begin{bmatrix} -4 & -3 \\ -2 & -6 \end{bmatrix}$,

$$B=\begin{bmatrix}\sqrt{2} & 3\\ 4 & \sqrt{3}\end{bmatrix}, \qquad -B=\begin{bmatrix}-\sqrt{2} & -3\\ -4 & -\sqrt{3}\end{bmatrix},$$

$$C=\begin{bmatrix}1\\ -7\\ 4\end{bmatrix}, \qquad -C=\begin{bmatrix}-1\\ 7\\ -4\end{bmatrix},$$

$$D=\begin{bmatrix}1 & 0 & -2\\ 0 & 3 & 4\\ 2 & -1 & -3\end{bmatrix}, \qquad -D=\begin{bmatrix}-1 & 0 & 2\\ 0 & -3 & -4\\ -2 & 1 & 3\end{bmatrix},$$

$$E=[2 \quad 5 \quad -3] \qquad -E=[-2 \quad -5 \quad 3]$$

3- **If** $A=\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 7\\ 4 & 6\end{bmatrix}$ **then show that**

(i) $4A-3A=A$ **(ii)** $3B-3A=3(B-A)$

Sol:

(i)

$$4A-3A = 4\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}-3\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}$$

$$= \begin{bmatrix}8 & 12\\ 4 & 20\end{bmatrix}-\begin{bmatrix}6 & 9\\ 3 & 15\end{bmatrix}$$

$$= \begin{bmatrix}8-6 & 12-9\\ 4-3 & 20-15\end{bmatrix}$$

$$4A-3A = \begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}=A \qquad \text{(Proved)}$$

(ii)

$$3B-3A = 3(B-A)$$

$$\text{L.H.S. } 3B - 3A = 3\begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 21 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3-6 & 21-9 \\ 12-3 & 18-15 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 12 \\ 9 & 3 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots(i)$$

$$\text{R.H.S} = 3(B - A) = 3\left(\begin{bmatrix} 1 & 7 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\right)$$

$$= 3\left(\begin{bmatrix} 1-2 & 7-3 \\ 4-1 & 6-5 \end{bmatrix}\right)$$

$$= 3\begin{bmatrix} -1 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 12 \\ 9 & 3 \end{bmatrix} \quad \textbf{(ii)}$$

$$3B - 3A = 3(B - A) \text{ From (i) and (ii)}$$

4- Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

Sol:

$$x + 3 = 2$$

$$x = 2 - 3$$

$$\boxed{x = -1}$$

and $3y - 4 = 2$

$$3y = 2 + 4$$

$$3y = 6$$

$$y = \frac{6}{3}$$

$$\boxed{y = 2}$$

5. If $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$ then

prove that,

(i) $A + B = B + A$ (ii) $A + (B + C) = (A + B) + C$

(i) $\mathbf{A + B = B + A}$

Sol:

$$A + B = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 3+7 \\ 4+6 & 5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} \text{........} \quad (i)$$

and $$B + A = \begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 7+3 \\ 6+4 & 5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 10 & 10 \end{bmatrix} \text{........} \quad (ii)$$

From (i) and (ii) $A + B = B + A$

(ii) $\mathbf{A + (B + C) = (A + B) + C}$

$$A+(B+C)=\begin{bmatrix}1 & 3\\ 4 & 5\end{bmatrix}+\left(\begin{bmatrix}4 & 7\\ 6 & 5\end{bmatrix}+\begin{bmatrix}2 & 6\\ 3 & -2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 3\\ 4 & 5\end{bmatrix}+\begin{bmatrix}4+2 & 7+6\\ 6+3 & 5-2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 3\\ 4 & 5\end{bmatrix}+\begin{bmatrix}6 & 13\\ 9 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+6 & 3+13\\ 4+9 & 5+3\end{bmatrix}=\begin{bmatrix}7 & 16\\ 13 & 8\end{bmatrix}\ldots\ldots (i)$$

and

$$(A+B)+C=\left(\begin{bmatrix}1 & 3\\ 4 & 5\end{bmatrix}+\begin{bmatrix}4 & 7\\ 6 & 5\end{bmatrix}\right)+\begin{bmatrix}2 & 6\\ 3 & -2\end{bmatrix}$$

$$=\left(\begin{bmatrix}1+4 & 3+7\\ 4+6 & 5+5\end{bmatrix}\right)+\begin{bmatrix}2 & 6\\ 3 & -2\end{bmatrix}$$

$$=\begin{bmatrix}5 & 10\\ 10 & 10\end{bmatrix}+\begin{bmatrix}2 & 6\\ 3 & -2\end{bmatrix}$$

$$=\begin{bmatrix}5+2 & 10+6\\ 10+3 & 10-2\end{bmatrix}=\begin{bmatrix}7 & 16\\ 13 & 8\end{bmatrix}\ldots\ldots (ii)$$

From (i) and (ii)

$$A+(B+C)=(A+B)+C$$

6- Solve the matrix equation for X.

$3X - 2A = B$ if $A=\begin{bmatrix}2 & 3\\ -4 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & -3\\ 4 & 4\end{bmatrix}$

Solution:

$$3X - 2A = B$$

$$3X = B + 2A$$

$$3X = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} + 2\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & -3+6 \\ 4-8 & 4+2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 6 & 3 \\ -4 & 6 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} 6 & 3 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \times \frac{1}{3} & 3 \times \frac{1}{3} \\ -4 \times \frac{1}{3} & 6 \times \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -\frac{4}{3} & 2 \end{bmatrix}$$

7- Find a, b, c, d, e and f such that

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 \\ 5 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

Sol: $$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 \\ 5 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a-3 & b+2 & c-1 \\ d-5 & e-0 & f+4 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \end{bmatrix}$$

Hence

$a-3=-1,\quad b+2=-2,\quad c-1=3$

$a=-1+3\quad b=-2-2\quad c=3+1$

$\boxed{a=2}\quad \boxed{b=-4}\quad \boxed{c=4}$

and

$d-5=-2,\quad e-0=4,\quad f+4=6$

$d=-2+5\quad \boxed{e=4}\quad f=6-4$

$\boxed{d=3}\quad \boxed{f=2}$

8- Find w, x, y, z such that

$$\begin{bmatrix} w & x \\ y & z \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & -3 \end{bmatrix}$$

Sol: $$\begin{bmatrix} w+3 & x+0 \\ y-1 & z+5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 6 & -3 \end{bmatrix}$$

Hence $w+3=2,\quad x+0=1$

$w=2-3\quad \boxed{x=1},$

$\boxed{w=-1}$

$y-1=6\quad z+5=-3$

$y=6+1\quad z=-3-5$

$\boxed{y=7}$ $\boxed{z=-8}$

9- If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then what is the additive inverse of A?

Sol: $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$-A=-\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ **(Additive Inverse of A)**

$$=\begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

10- **Given that** $A=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ verify that $A^2-4A+5I=0$.

Sol: $A=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$$\text{L.H.S}=A^2-4A+5I$$

$$=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}-4\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}+5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1-2 & -1-3 \\ 2+6 & -2+9 \end{bmatrix}-\begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix}+\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}-\begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix}+\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$=\begin{bmatrix} -1-4 & -4+4 \\ 8-8 & 7-12 \end{bmatrix}+\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -5+5 & 0+0 \\ 0+0 & -5+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence proved

$$A^2 - 4A + 5I = 0$$

11- If $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$, then verify that

$$(A+B)^t = A^t + B^t$$

Sol: $$A + B = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & 4-2 \\ 1+4 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 5 & 11 \end{bmatrix}$$

$$(A+B)^t = \begin{bmatrix} 5 & 5 \\ 2 & 11 \end{bmatrix} \dots\dots (i)$$

Now $$A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \dots\dots (ii)$$

and $$B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$$

$$B' = \begin{bmatrix} 3 & 4 \\ -2 & 6 \end{bmatrix} \ldots\ldots\ (iii)$$

$$A' + B' = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -2 & 6 \end{bmatrix} \quad \text{From (i) and (ii)}$$

$$= \begin{bmatrix} 2+3 & 1+4 \\ 4-2 & 5+6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 5 \\ 2 & 11 \end{bmatrix} \ldots\ldots\ (iv)$$

$$(A+B)' = A' + B' \quad \text{From (i) and (iv)}$$

12- **If** $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -7 \\ 5 & 8 \end{bmatrix}, C = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$ **then**

show that $A + B - C = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$

Solution:

$$A + B - C = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 2 & -7 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2-7 \\ 3+5 & -4+8 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-1 & -5-5 \\ 8-0 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$$

Hence proved

$$A + B - C = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix}$$

MULTIPLICATION OF MATRICES

Two matrices A and B are said to be conformable for the product AB, if the number of columns in A is equal to the number of rows in B.

> ***Remember that:***
>
> For multiplication AB of two matrices A and B the following points should be kept in mind.
>
> (i) The number of columns in A = number of rows in B.
>
> (ii) The product of matrices A and B is denoted by $A \times B$ or AB.
>
> (iii) If A is a m-by-p matrix and B is a p-by-n matrix then AB is m-by-n matrix.

Associative Law of Matrices with respect to Multiplication

If three matrices A, B and C are conformable for multiplication, then

$$A(BC) = (AB)C$$

is called associative law with respect to multiplication.

Distributive Laws:

If the matrices A, B and C are conformable for addition and multiplication, then

(i) $A(B + C) = AB + AC$ (left distributive law for matrices)

(ii) $(A + B)C = AC + BC$ (right distributive law for matrices)

(i) and (ii) are called distributive laws.