Exercise 6.4

In Problems 1 to 8 Verify Each Statement, Using

$$A=\begin{bmatrix}4 & 2\\0 & 0\end{bmatrix},\ B=\begin{bmatrix}2 & 1\\-2 & 4\end{bmatrix},\ C=\begin{bmatrix}-1 & 2\\4 & 2\end{bmatrix}$$

1. $(AB)C = A(BC)$

L.H.S = (AB)C

$$(AB)C=\left(\begin{bmatrix}4 & 2\\0 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\-2 & 4\end{bmatrix}\right)\times\begin{bmatrix}-1 & 2\\4 & 2\end{bmatrix}$$

$$=\left(\begin{bmatrix}8-4 & 4+8\\0+0 & 0+0\end{bmatrix}\right)\times\begin{bmatrix}-1 & 2\\4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4 & 12\\0 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 2\\4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-4+48 & 8+24\\0+0 & 0+0\end{bmatrix}=\begin{bmatrix}44 & 32\\0 & 0\end{bmatrix}\ldots\ldots (i)$$

R.H.S = A(BC)

$$A(BC)=\begin{bmatrix}4 & 2\\0 & 0\end{bmatrix}\times\left(\begin{bmatrix}2 & 1\\-2 & 4\end{bmatrix}\times\begin{bmatrix}-1 & 2\\4 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}4 & 2\\0 & 0\end{bmatrix}\times\left(\begin{bmatrix}-2+4 & 4+2\\2+16 & -4+8\end{bmatrix}\right)$$

$$=\begin{bmatrix}4 & 2\\0 & 0\end{bmatrix}\times\begin{bmatrix}2 & 6\\18 & 4\end{bmatrix}$$

$$= \begin{bmatrix} 8+36 & 24+8 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 44 & 32 \\ 0 & 0 \end{bmatrix} \ldots\ldots (ii)$$

$(AB)C = A(BC)$ from (i) and (ii)

2. $AB \neq BA$

Sol: $AB = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 8-4 & 4+8 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} \ldots\ldots (i)$$

$BA = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$ اب

$$= \begin{bmatrix} 8+0 & 4+0 \\ -8+0 & -4+0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ -8 & -4 \end{bmatrix} \ldots\ldots (ii)$$

$AB \neq BA$ from (i) and (ii)

3. $A(B+C) = AB + AC$

Sol: L.H.S = A(B + C)

Putting values of A, B, C

$$A(B+C) = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 12+12 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 8 & 24 \\ 0 & 0 \end{bmatrix} \text{......} (i)$$

R.H.S = AB + AC

Putting values of A, B, C

$$AB + AC = \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4 & 4+8 \\ 0+0 & 0+0 \end{bmatrix} + \begin{bmatrix} -4+8 & 8+4 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 12+12 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 8 & 24 \\ 0 & 0 \end{bmatrix} \text{........} (ii)$$

from (i) and (ii)

$$A(B+C) = AB + AC$$

4. $$(B+C)A = BA + CA$$

Sol: L.H.S = (B + C)A

(B+C)A $$(B+C)A = \left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\right)\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix}\right)\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 2+0 \\ 8+0 & 4+0 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \text{.......} (i)$$

R.H.S = BA + CA

$BA + CA$

$$BA + CA = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 4+0 \\ -8+0 & -4+0 \end{bmatrix} + \begin{bmatrix} -4+0 & -2+0 \\ 16+0 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ -8 & -4 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 16 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8-4 & 4-2 \\ -8+16 & -4+8 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \text{.......} (ii)$$

from (i) and (ii)

$$(B+C)A = BA + CA$$

5. $(B+C)(B-C) \neq B^2 - C^2$

Sol: L.H.S = $(B+C)(B-C)$

Putting values of B, C

$$(B+C)(B-C) = \left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-1 & 1+2 \\ -2+4 & 4+2 \end{bmatrix}\begin{bmatrix} 2+1 & 1-2 \\ -2-4 & 4-2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-18 & -1+6 \\ 6-36 & -2+12 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 5 \\ -30 & 10 \end{bmatrix} \text{........} (i)$$

R.H.S $B^2 - C^2 = B \times B - C \times C$

$$= \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & 2+4 \\ -4-8 & -2+16 \end{bmatrix} - \begin{bmatrix} 1+8 & -2+4 \\ -4+8 & 8+4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -12 & 14 \end{bmatrix} - \begin{bmatrix} 9 & 2 \\ 4 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 2-9 & 6-2 \\ -12-4 & 14-12 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 4 \\ -16 & 2 \end{bmatrix} \text{........} (ii)$$

$(B+C)(B-C) \neq B^2 - C^2$ from (i) and (ii)

6. $(BC)' = C'B'$

Sol: L.H.S = $(BC)'$

$$BC = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4 & 4+2 \\ 2+16 & -4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix}$$

$$(BC)' = \begin{bmatrix} 2 & 18 \\ 6 & 4 \end{bmatrix} \text{........} (i) \quad \text{and}$$

Now R.H.S = $C'B'$

$$C = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$C' = \begin{bmatrix} -1 & 4 \\ 2 & 2 \end{bmatrix}$$

and $$B = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$$

Now $$C'B' = \begin{bmatrix} -1 & 4 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4 & 2+16 \\ 4+2 & -4+8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 \\ 6 & 4 \end{bmatrix} \ldots\ldots. (ii)$$

$(BC)' = C'B'$ from (i) and (ii)

7. $\boldsymbol{BI = B}$

Sol: L.H.S $= BI$

$$BI = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BI = \begin{bmatrix} 2+0 & 0+1 \\ -2+0 & 0+4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} = B$$

8. $BC \neq CB$

Sol: $BC = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2+4 & 4+2 \\ 2+16 & -4+8 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 18 & 4 \end{bmatrix}$ ....(i)

$CB = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2-4 & -1+8 \\ 8-4 & 4+8 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 4 & 12 \end{bmatrix}$ ....(ii)

from (i) and (ii)

$BC \neq CB$

Find the Matrix Products.

9. $\begin{bmatrix} 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Sol: $= \begin{bmatrix} 2(1)+5(2) & 2(-1)+(5)(3) \end{bmatrix}$

$= \begin{bmatrix} 2+10 & -2+15 \end{bmatrix}$

$= \begin{bmatrix} 12 & 13 \end{bmatrix}$

10. $\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Sol: $= \begin{bmatrix} 3(-1)+4(2) \\ -1(-1)+(-2)(2) \end{bmatrix}$

$= \begin{bmatrix} -3+8 \\ +1-4 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}$

Sol: $= \begin{bmatrix} 2(1)+(-3)(0) & 2(-1)+(-3)(-2) \\ 1(1)+2(0) & 1(-1)+(2)(-2) \end{bmatrix}$

$= \begin{bmatrix} 2+0 & -2+6 \\ 1+0 & -1-4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ -1 & 3 \end{bmatrix}$

Sol: $= \begin{bmatrix} -3(-1)+(2)(-1) & -3(5)+(2)(3) \\ 4(-1)+(-1)(-1) & 4(5)+(-1)(3) \end{bmatrix}$

$= \begin{bmatrix} 3-2 & -15+6 \\ -4+1 & 20-3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -9 \\ -3 & 17 \end{bmatrix}$

13. $= \begin{bmatrix} -5 & -2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$

Sol: $= \begin{bmatrix} (-5)(-2)+(-2)(0) & (-5)(1)+(-2)(-3) \\ 1(-2)+(-3)(0) & (1)(1)+(-3)(-3) \end{bmatrix}$

$= \begin{bmatrix} 10+0 & -5+6 \\ -2+0 & 1+9 \end{bmatrix}$

$= \begin{bmatrix} 10 & 1 \\ -2 & 10 \end{bmatrix}$

14. $$\begin{bmatrix}-2 & 4\\ 0 & -3\end{bmatrix}\begin{bmatrix}-5 & -5\\ 1 & -3\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -0\end{bmatrix}$$

Sol:
$$=\left(\begin{bmatrix}-2 & 4\\ 0 & -3\end{bmatrix}\begin{bmatrix}-5 & -5\\ 1 & -3\end{bmatrix}\right)\begin{bmatrix}1 & -1\\ 2 & -0\end{bmatrix}$$

$$=\begin{bmatrix}(-2)(-5)+(4)(1) & (-2)(-5)+(4)(-3)\\ (0)(-5)+(-3)(1) & (0)(-5)+(-3)(-3)\end{bmatrix}$$

$$=\begin{bmatrix}10+4 & 10-12\\ 0-3 & 0+9\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}14 & -2\\ -3 & 9\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}14(1)+(-2)(2) & 14(-1)+(-2)(0)\\ -3(1)+9(2) & -3(-1)+9(0)\end{bmatrix}$$

$$=\begin{bmatrix}14-4 & -14+0\\ -3+18 & 3+0\end{bmatrix}$$

$$=\begin{bmatrix}10 & -14\\ 15 & 3\end{bmatrix}$$

15. If $\begin{bmatrix}1 & 5\\ 3 & a\end{bmatrix}\begin{bmatrix}b\\ 7\end{bmatrix}=\begin{bmatrix}35\\ 10\end{bmatrix}$, then find the values of a and b.

Sol:
$$\begin{bmatrix}1 & 5\\ 3 & a\end{bmatrix}\begin{bmatrix}b\\ 7\end{bmatrix}=\begin{bmatrix}35\\ 10\end{bmatrix}$$

$$\begin{bmatrix}1(b)+5(7)\\ 3(b)+(a)(7)\end{bmatrix}=\begin{bmatrix}35\\ 10\end{bmatrix}$$

$$\begin{bmatrix} b+35 \\ 3b+7a \end{bmatrix} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$$

Now $b + 35 = 35$

Hence $\boxed{b = 0}$

and $3b + 7a = 10$

$3(0) + 7a = 10$ (Putting values of b)

$7a = 10$

$$\boxed{a = \frac{10}{7}}$$

16. If $A = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}, B = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$, then verify $(AB)' = B'A'$.

Sol:

L.H.S = $(AB)'$

$$AB = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-1)+6(2) & 2(-3)+6(0) \\ 7(-1)+8(2) & 7(-3)+8(0) \end{bmatrix}$$

$$= \begin{bmatrix} -2+12 & -6+0 \\ -7+16 & -21+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 \\ 9 & -21 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 10 & 9 \\ -6 & -21 \end{bmatrix} \text{........} (i)$$

R.H.S = $B'A'$

$$B = \begin{bmatrix} -1 & -3 \\ 2 & 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix}$$

and $$A = \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix}$$

Now $$B'A' = \begin{bmatrix} -1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -1(2)+2(6) & -1(7)+2(8) \\ -3(2)+(0)(6) & -3(7)+0(8) \end{bmatrix}$$

$$= \begin{bmatrix} -2+12 & -7+16 \\ -6+0 & -21+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 9 \\ -6 & -21 \end{bmatrix} \text{........} (ii)$$

$(AB)' = B'A'$ from (i) and (ii)

MULTIPLICATIVE INVERSE OF A MATRIX

Determinant Function

If A is a square matrix, then det A or $|A|$ read "The determinant of A" is used to denote the unique real number.

Singular Matrix:

A square matrix A is called a singular matrix. If *det* A = 0

$$Example: A = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix}$$

$$\det \quad A = \begin{bmatrix} 12 & 6 \\ 6 & 3 \end{bmatrix} = 36 - 36$$

$$\det \quad A = 0. Hence\ matrix\ A\ is\ singular.$$

Non-Singular Matrix:

A square matrix A is called non-singular matrix, if *det* $A \neq 0$.

Example

$$If \quad A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix}$$

$$\det \quad A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix} = 16 - 30$$

$$\det \quad A = -14. Hence\ matrix\ A\ is\ non-singular.$$

Adjoint of a Matrix:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix of order 2-by-2.

Then the matrix obtained by interchanging the elements of the diagonal (i.e a and d) and by changing the signs of the other elements b and c is called the adjoint of the matrix A.

Multiplicative Inverse

In the set of real numbers, we know that for each real number a (except zero) there exists a real number a^{-1} such that $aa^{-1} = 1$.

The number a^{-1} is called the multiplicative inverse of a.

$$\boxed{A^{-1} = \frac{adj\ A}{|A|}},\ |A| \neq 0$$

If A is a singular matrix then the multiplicative inverse of A does not exist.

Remember that:

(i) Inverse of square matrix A is denoted by A^{-1}.
(ii) Only non-singular matrices have inverses.
(iii) Inverse of square matrix A is always unique.
(iv) Non-square matrices cannot possess inverses.
(v) $A^{-1} = \dfrac{adj\ A}{|A|}$

Exercise 6.5

1- Find the determinants of the following matrices.

(i) $\begin{bmatrix} u & v \\ x & y \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 5 \\ 1 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} -8 & -4 \\ -4 & -2 \end{bmatrix}$ (iv) $\begin{bmatrix} \frac{1}{1} & \frac{3}{8} \\ \frac{1}{8} & \frac{1}{4} \end{bmatrix}$