1- Write the equation $2x + ky = 7$ and $4x - 9y = 4$ in matrix form. Also find the value of k if the matrix of the coefficients is singular.

Sol:

$$2x + ky = 7$$

$$4x - 9y = 4$$

In matrix form

$$\begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$A \quad X = B$$

$$A = \begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$|A| = 0$$

Now, if A is singular matrix.
Therefore,

$$\begin{bmatrix} 2 & k \\ 4 & -9 \end{bmatrix} = 0$$

$$(2)(-9) - (k)(4) = 0$$

$$-18 - 4k = 0$$

$$-4k = 18$$

$$k = \frac{18}{-4}$$

$$= -\frac{9}{2}$$

2. Solve the simultaneous equations by the matrix inversion method where possible. Where there is no solution, explain why this is so

(i)
$$2x - 5y = 1$$
$$3x - 7y = 2$$

Sol: In matrix form

$$\begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A \quad X = B$$

$$A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and $$A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -5 \\ 3 & -7 \end{vmatrix}$$

$$= (2)(-7) - (-5)(3)$$

$$= -14 + 15 = 1 \neq 0$$

A is non-singular matrix. Therefore, equations can be solve

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}$$

because $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7(1) + (5)(2) \\ -3(1) + (2)(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 + 10 \\ -3 + 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Hence, $x = 3$

$y = 1$

$$\text{S.S} = \{(3,1)\}$$

(ii) $3x + 2y = 10$

$2y - 3x = -4$

Sol: Write equations again

$3x + 2y = 10$

$-3x + 2y = -4$

In matrix form

$$\begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$AX = B$

$$A = \begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 3 & 2 \\ -3 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 2 \\ -3 & 2 \end{vmatrix}$$

$$= (3)(2) - (2)(-3)$$

$$= 6 + 6 = 12 \neq 0$$

A is non-singular matrix, therefore equations can be solve

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}}{12}$$

$$= \frac{1}{12}\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}$$

Now $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 2 & -2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ -4 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 2(10) + (-2)(-4) \\ 3(10) + (3)(-4) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 20 + 8 \\ 30 - 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 28 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{28}{12} \\ \frac{18}{12} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{3} \\ \frac{3}{2} \end{bmatrix}$$

Hence, $x = \frac{7}{3}$

$y = \frac{3}{2}$

$$S.S = \left\{ \left(\frac{7}{3}, \frac{3}{2} \right) \right\}$$

(iii) $4x + 5y = 0$

$2x + 5y = 1$

Sol: $\begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ In matrix form

Let $A \; X = B$

Now $A^{-1}A \; X = A^{-1}B$

$X = A^{-1}B$

Now $A = \begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 5 \\ 2 & 5 \end{vmatrix}$$

$$|A| = (4)(5) - (5)(2)$$

$$= 20 - 10 = 10 \neq 0$$

A is non-singular matrix, therefore equations can be solve.

$$|A| = 10$$

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}}{10} = \frac{1}{10}\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}$$

Now $$A^{-1}B = \frac{1}{10}\begin{bmatrix} 5 & -5 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} (5)(0) + (-5)(1) \\ (-2)(0) + (4)(1) \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 0 - 5 \\ 0 + 4 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{10} \\ \frac{4}{10} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

But $X = A^{-1}B$

Thus
$$X = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

or
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{2}{5} \end{bmatrix}$$

Hence, $x = -\frac{1}{2}$

$y = \frac{2}{5}$

$$S.S = \left\{\left(-\frac{1}{2}, \frac{2}{5}\right)\right\}$$

(iv)
$$5x + 6y = 25$$
$$3x + 4y = 17$$

Sol: $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \\ 17 \end{bmatrix}$ In matrix form

Let $A \quad X = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 6 \\ 3 & 4 \end{vmatrix}$$

$$= (5)(4) - (6)(3)$$

$$= 20 - 18$$

$$|A| = 2 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}}{2}$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}$$

Now $A^{-1}B = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 25 \\ 17 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} (4)(25) + (-6)(17) \\ (-3)(25) + (5)(17) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 100 - 102 \\ -75 + 85 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{2} \\ \frac{10}{2} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

But $X = A^{-1}B$

Thus $X = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

Hence, $x = -1$

$y = 5$

$S.S = \{(-1, 5)\}$

(v) $x + y = 2$

$y = 2 + x$

Sol: Write equations again

$$x + y = 2$$

$$-x + y = 2$$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Let $A \;\; X = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= (1)(1) - (1)(-1)$$

$$= 1 + 1$$

$$|A| = 2 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}{2} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

and Now $A^{-1}B = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} (1)(2) + (-1)(2) \\ (1)(2) + (1)(2) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 - 2 \\ 2 + 2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}0\\4\end{bmatrix}$$

$$= \begin{bmatrix}\frac{0}{2}\\\frac{4}{2}\end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix}0\\2\end{bmatrix}$$

while $X = A^{-1}B$

$$X = \begin{bmatrix}0\\2\end{bmatrix}$$

or $\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\2\end{bmatrix}$

Hence, $x = 0$

$y = 2$

$\text{S.S} = \{(0,2)\}$

(vi) $\frac{x}{2} + \frac{y}{3} = 1 \quad (i)$

$-4x + y = 14 \quad (ii)$

Sol: Multiply (i) by 6.

$$6\left(\frac{x}{2}\right) + 6\left(\frac{y}{3}\right) = 6$$

$3x + 2y = 6$ (2nd condition)

$-4x + y = '4$

$$\begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$$ (In matrix form)

Let $A\ X = B$

$$A^{-1} A\ X = A^{-1} B$$

$$X = A^{-1} B$$

Now $A = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}$

$$|A| = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}$$

$$= (3)(1) - (2)(-4)$$

$$= 3 + 8$$

$$= 11 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}}{11} = \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

and now $A^{-1} B = \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \end{bmatrix}$

$$= \frac{1}{11} \begin{bmatrix} (1)(6) + (-2)(14) \\ (4)(6) + (3)(14) \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 6-28 \\ 24+42 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} -22 \\ 66 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-22}{11} \\ \frac{66}{11} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

while $X = A^{-1}B$

$$X = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

Hence, $x = -2$

$y = 6$

$$S.S = \{(-2, 6)\}$$

3. Solve, using matrix inversion method

$3x - y = 10$

$2x + 3y = 3$

In matrix form

$$\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

Let $A\ X = B$

and $A^{-1}A\ X = A^{-1}B$

$$X = A^{-1}B$$

Now $A = \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= (3)(3) - (-1)(2)$$

$$= 9 + 2$$

$$= 11 \neq 0$$

A is non-singular matrix. Therefore equations can be solve.

$$A^{-1} = \frac{adj\ A}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}}{11} = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$$

and now $A^{-1}B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 3 \end{bmatrix}$

$$= \frac{1}{11}\begin{bmatrix} (3)(10) + (1)(3) \\ (-2)(10) + (3)(3) \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 30 + 3 \\ -20 + 9 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 33 \\ -11 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{33}{11} \\ \frac{-11}{11} \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

while $X = A^{-1}B$

$$X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Hence, $x = 3$

$y = -1$

$S.S = \{(3, -1)\}$

4. Use Cramer's rule to solve the simultaneous equations. Give the reason where solution is not possible.

(i) $x + 2y = 3$

$x + 3y = 5$

Remember that:

Cramer's Rule

If $a_1x + a_2y = b_1$

$$a_3x + a_4y = b_2$$

then

$$x = \frac{\begin{vmatrix} b_1 & a_2 \\ b_2 & a_4 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \frac{|D_1|}{|A|}$$

$$y = \frac{\begin{vmatrix} a_1 & b_1 \\ a_3 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}} = \frac{|D_2|}{|A|}$$

Sol: $x + 2y = 3$

$$x + 3y = 5$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$ In matrix form

$$AX = B$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = (1)(3) - (1)(2) = 3 - 2 = 1$$

$$|D_1| = \begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} = (3)(3) - (2)(5) = 9 - 10 = -1$$

$$|D_2| = \begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = (1)(5) - (3)(1) = 5 - 3 = 2$$

$$x = \frac{|D_1|}{|A|} = \frac{-1}{1} = -1$$

$$y=\frac{|D_2|}{|A|}=\frac{2}{1}=2$$

$$S.S=\{(-1,2)\}$$

(ii) $2x+y=1$

$5x+3y=2$

Sol: $\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$ In matrix form

$AX=B$

$$A=\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & 1\\5 & 3\end{vmatrix}$$

$$=(2)(3)-(1)(5)$$

$$=6-5=1$$

$$|D_1|=\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}$$

$$=(1)(3)-(1)(2)$$

$$=3-2=1$$

$$|D_2|=\begin{vmatrix}2 & 1\\5 & 2\end{vmatrix}=(2)(2)-(1)(5)$$

$$=4-5$$

$$=-1$$

$$x = \frac{|D_1|}{|A|} = \frac{1}{1} = 1$$

$$y = \frac{|D_2|}{|A|} = \frac{-1}{1} = -1$$

S.S= $\{(1,-1)\}$

(iii) $x + 3y = 1$

$2x + 8y = 0$

Sol: $\begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ In matrix form

$AX = B$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 8 \end{vmatrix} = (1)(8) - (3)(2)$$

$$= 8 - 6 = 2$$

$$|D_1| = \begin{vmatrix} 1 & 3 \\ 0 & 8 \end{vmatrix} = (1)(8) - (3)(0)$$

$$= 8 - 0$$

$$= 8$$

$$|D_2| = \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = (1)(0) - (1)(2)$$

$$= 0 - 2$$

$$= -2$$

$$x = \frac{|D_1|}{|A|} = \frac{8}{2} = 4$$

$$y = \frac{|D_2|}{|A|} = \frac{-2}{2} = -1$$

$$S.S = \{(4, -1)\}$$

(iv) $-2x + 6y = 5$

$x - 3y = -7$

Sol: $\begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$ In matrix form

$AX = B$

$$A = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -2 & 6 \\ 1 & -3 \end{vmatrix}$$

$$= (-2)(-3) - (6)(1)$$

$$= 6 - 6 = 0$$

$A = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}$ is singular matrix, therefore S.S is not possible.

(v) $x - 3y = 5$

$2x - 5y = 9$

Sol: $\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$ In matrix form

$AX = B$

$$A = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix}$$

$= (1)(-5) - (-3)(2)$

$= -5 + 6 = 1$

$$|D_1| = \begin{vmatrix} 5 & -3 \\ 9 & -5 \end{vmatrix} = (5)(-5) - (-3)(9)$$

$= -25 + 27$

$= 2$

$$|D_2| = \begin{vmatrix} 1 & 5 \\ 2 & 9 \end{vmatrix}$$

$= (1)(9) - (5)(2)$

$= 9 - 10 = -1$

$$x = \frac{|D_1|}{|A|} = \frac{2}{1} = 2$$

$$y = \frac{|D_2|}{|A|} = \frac{-1}{1} = -1$$

$\text{s.s} = \{(2, -1)\}$

(vi) $5x+2y=13$

$2x+5y=17$

Sol: $\begin{bmatrix}5 & 2\\2 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}13\\17\end{bmatrix}$ In matrix form

$AX=B$

$A=\begin{bmatrix}5 & 2\\2 & 5\end{bmatrix}$

$|A|=\begin{vmatrix}5 & 2\\2 & 5\end{vmatrix}=(5)(5)-(2)(2)$

$=25-4$

$=21$

$|D_1|=\begin{vmatrix}13 & 2\\17 & 5\end{vmatrix}=(13)(5)-(2)(17)$

$=65-34$

$=31$

$|D_2|=\begin{vmatrix}5 & 13\\2 & 17\end{vmatrix}=(5)(17)-(13)(2)$

$=85-26=59$

$x=\frac{|D_1|}{|A|}=\frac{31}{21}$

$y=\frac{|D_2|}{|A|}=\frac{59}{21}$

$\text{S.S}=\left\{\left(\frac{31}{21},\frac{59}{21}\right)\right\}$

5. Write the following matrices in the form of linear equations.

(i) $$\begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Sol: $$\begin{bmatrix} (2)(x)+(-1)(y) \\ (5)(x)+(2)(y) \end{bmatrix}=\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

then $$\begin{bmatrix} 2x-y \\ 5x+2y \end{bmatrix}=\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

and $2x-y=2$

$5x+2y=4$

(ii) $$\begin{bmatrix} -5 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Sol: $$\begin{bmatrix} (-5)(x)+(2)(y) \\ (2)(x)+(-3)(y) \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -5x+2y \\ 2x-3y \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

thus, $-5x+2y=2$

and $2x-3y=-1$

(iii) $$\begin{bmatrix} -4 & 1 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Sol: $$\begin{bmatrix} -4(x)+(1)(y) \\ 5(x)+(4)(y) \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -4x+y \\ 5x+4y \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

thus, $-4x + y = 1$

and $5x + 4y = -1$

(iv) $$\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Sol: $$\begin{bmatrix} (0.8)(x) + (-0.6)(y) \\ (0.6)(x) + (0.8)(y) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 0.8x - 0.6y \\ 0.6x + 0.8y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

thus, $0.8x - 0.6y = 1$

and $0.6x + 0.8y = 2$