Exercise 4.1

1. Write down the angles marked with letters. Write whether the angles are complimentary or supplementary?

Sol:

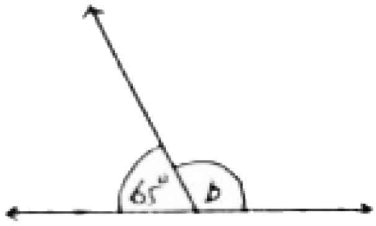
(i)

(i) $m\angle a + 50^\circ = 180^\circ$

$m\angle a = 180^\circ - 50^\circ$

$m\angle a = 130^\circ$

(Supplementary angles)



(ii) $m\angle b + 65^\circ = 180^\circ$

$m\angle b = 180^\circ - 65^\circ$

$m\angle b = 115^\circ$

(Supplementary angles)

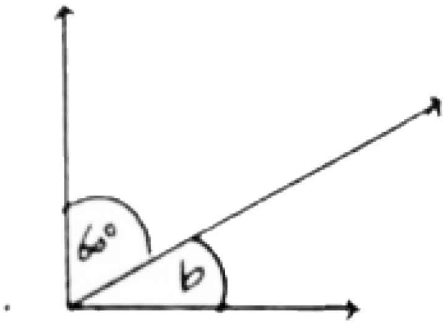
(iii)

(iii) $m\angle c + 138^\circ = 180^\circ$

$m\angle c = 180^\circ - 138^\circ$

$m\angle c = 42^\circ$

(Supplementary angles)

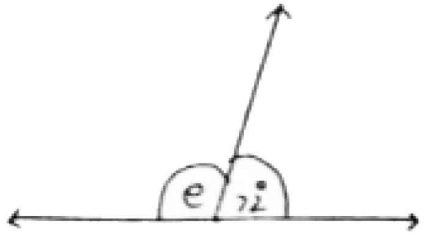


(iv) $m\angle b + 60^\circ = 90^\circ$

$m\angle b = 90^\circ - 60^\circ$

$m\angle b = 30^\circ$

(Complementary angles)

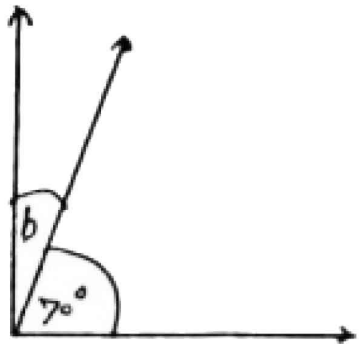


(v) $m\angle e + 72^{\circ} = 180^{\circ}$

$m\angle e = 180^{\circ} - 72^{\circ}$

$m\angle e = 108^{\circ}$

(Supplementary angles)



(vi) $m\angle b + 70^{\circ} = 90^{\circ}$

$m\angle b = 90^{\circ} - 70^{\circ}$

$m\angle b = 20^{\circ}$

(Complementary angles)

2. Two angles are supplementary and the greater exceeds the smaller by 30°. How many degrees are there in each angle?

Sol: The size of small angle $= x$ Let

The size of large angle $= x + 30^{\circ}$

According to statement

$$x + (x + 30^{\circ}) = 180^{\circ}$$

$$x + x + 30^{\circ} = 180^{\circ}$$

$$2x + 30^{\circ} = 180^{\circ}$$

$$2x = 180^{\circ} - 30^{\circ}$$

$$2x = 150^{\circ}$$

$$x = \frac{150^{\circ}}{2}$$

$$x = 75^{\circ}$$

The size of small angle $= 75^{\circ}$

The size of large angle $= x + 30^{\circ} = 75^{\circ} + 30^{\circ} = 105^{\circ}$

angles $= 75^{\circ}, 105^{\circ}$

3. If 40^{O} is added to an angle, the resulting angle is equal to the supplement of the original angle. Find the original angle.

Sol: Let the required angle $= x$

size of the angle after adding $= x + 40^{O}$

The supplement of 1st angle $= 180^{O} - x$

According to statement

$$x + 40^{O} = 180^{O} - x$$

$$x + x = 180^{O} - 40^{O}$$

$$2x = 140^{O}$$

$$x = \frac{140^{O}}{2}$$

$$x = 70^{O}$$

4. The sum of two angles is 100^{O}, and the difference between their supplements is 100^{O}. Find the angles.

Let the size of 1st angle $= x^{O}$

The size of 2nd angle $= 100^{O} - x^{O}$

The supplement of $x^{O} = 180^{O} - x^{O}$

The supplement of $100^{O} - x^{O} = 180^{O} - (100^{O} - x^{O})$

$$= 180^{O} - 100^{O} + x^{O}$$

According to the statement

$$(180^{O} - 100^{O} + x^{O}) - (180^{O} - x^{O}) = 100^{O}$$

$$180^{O} - 100^{O} + x^{O} - 180^{O} + x^{O} = 100^{O}$$

$$2x^{O} - 100^{O} = 100^{O}$$

$$2x = 100^{O} + 100^{O}$$

$$2x = 200^{O}$$

$$x = \frac{200^{O}}{2}$$

$$x = 100^{O}$$

The size of 1st angle $= 100^{O}$

The size of 2nd angle $= 100^{O} - 100^{O}$

$$= 0^{O}$$

5. The sum of two angles is 100^{O}, the supplement of the first angle exceeds the supplement of the second angle 40^{O}. Find the angles.

Sol: Let the size of 1st angle $= x^{O}$

The size of 2nd angle $= 100^{O} - x^{O}$

The supplement of 1st angle $= 180^{O} - x^{O}$

The supplement of 2nd angle $= 180^{O} - (100^{O} - x)$

$$= 180^{O} - 100^{O} + x^{O}$$

$$= 80^{O} + x^{O}$$

According to the statement

$$180^{O} - x^{O} - 40^{O} = 80^{O} + x^{O}$$

$$-x^{O} - x^{O} = 80^{O} + 40^{O} - 180^{O}$$

$$-2x^{O} = -60^{O}$$

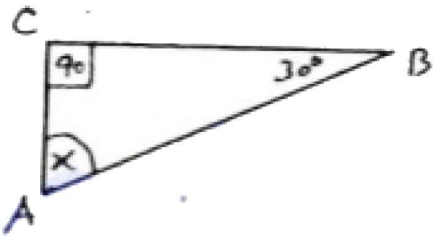
$$x^O = 30^O$$

$$1st\ angle = 30^O$$

$$2nd\ angle = 100^O - 30^O$$

$$= 70^O$$

6. Write the equation for the given triangle and solve it.



Sol: The sum of angles of any triangles $= 180^O$

The sum of angles of $\Delta ABC = x + 90^O + 30^O$

$$= x + 120^O$$

According to the statement

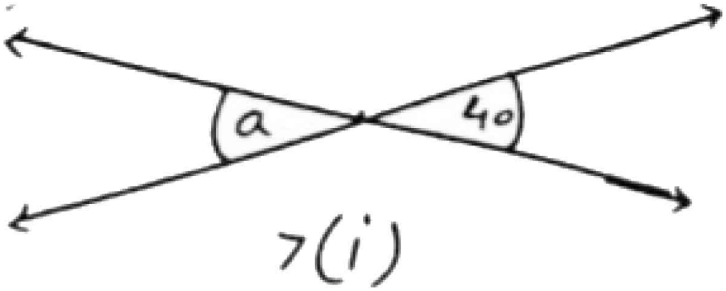
$$x + 120^O = 180^O$$

$$x = 180^O - 120^O$$

$$x = 60^O$$

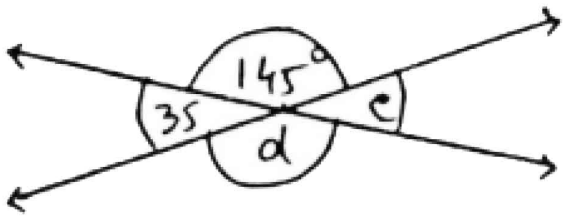
7. Write down the angles marked with letters.

7(i) $m\angle a = 40^{()}$ (vertically opp. angles)



7(ii) $m\angle c = 35^{0}$ (vertically opp. angles)

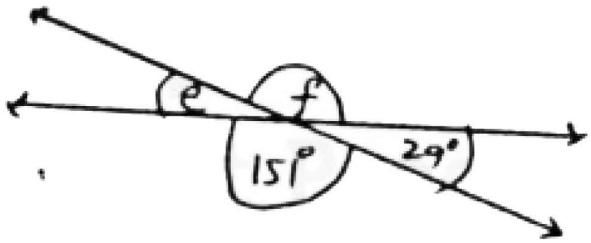
$m\angle d = 145^{0}$ (vertically opp. angles)



7 (ii)

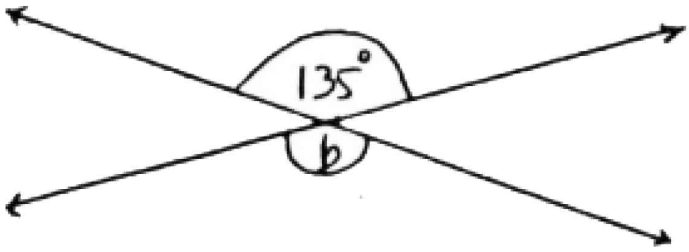
7(iii) $m\angle e = 29^{0}$ (vertically opp. angles)

$m\angle f = 151^{0}$ (vertically opp. angles)



7(iii)

7(iv) $m\angle b = 135^{0}$ (vertically opp. angles)

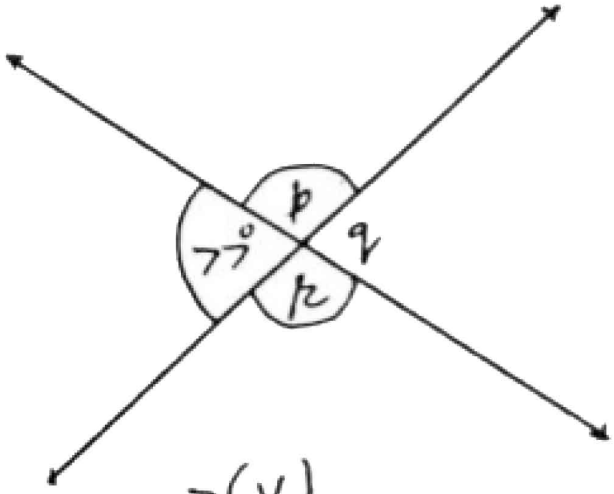


7 (iv)

7(v) $m\angle q = 70^{0}$ (vertically opp. angles)

$m\angle p = 180 - 77^{\circ}$ (supplementary angles)

$= 103^{o}$



7(v)

(vertically opp. angles) $m\angle r = m\angle p$

$= 103^{\circ}$

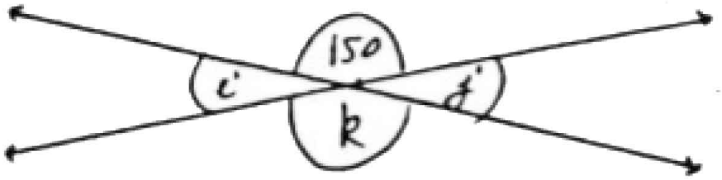
7(i)

7(vi)

(vertically opp. angles) $m\angle k = 150^{\circ}$

(supplementary angles) $m\angle i + 150^{\circ} = 180^{\circ}$

$m\angle i = 180^{\circ} - 150^{\circ}$



7(vi)

$m\angle i = 30^{\circ}$ (i)

(vertically opp. angles) $m\angle j = m\angle i$ and

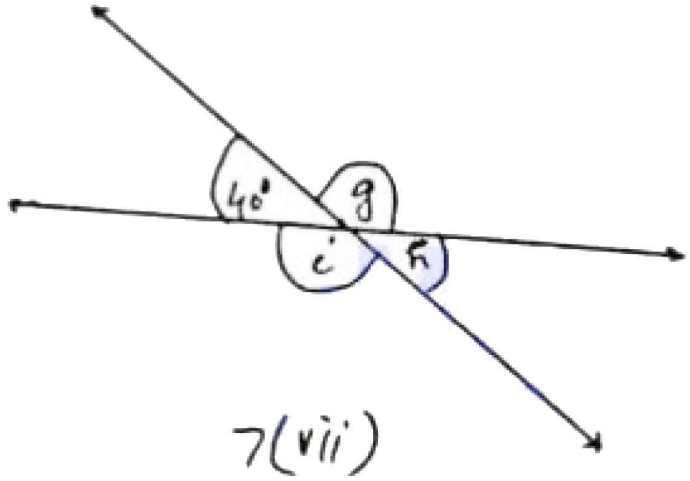
from (i) $m\angle j = 30^{\circ}$

7(vii)

(vertically opp. angles) $m\angle h = 40^{\circ}$

(supplementary angles) $m\angle 40 + \angle g = 180^{\circ}$

$m\angle g = 180^{\circ} - 40$



7(vii)

$m\angle g = 140^{\circ}$ (i)

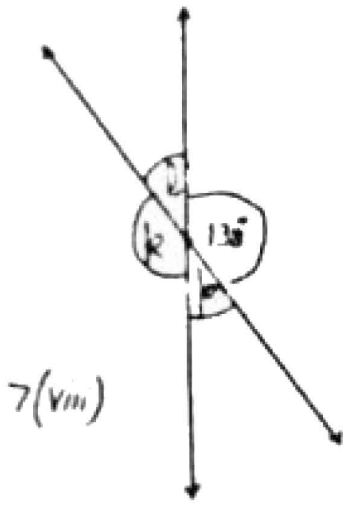
(vertically opp. angles) $m\angle i = m\angle g$ and

from (i) $= 140^{\circ}$

(vertically opp. angles) $m\angle k = 138^{\circ}$ **7(viii)**

(supplementary angles) $m\angle p + 138^{\circ} = 180^{\circ}$

$m\angle p = 180^{\circ} - 138^{\circ}$



7(viii)

$m\angle p = 42^{\circ}$ (i)

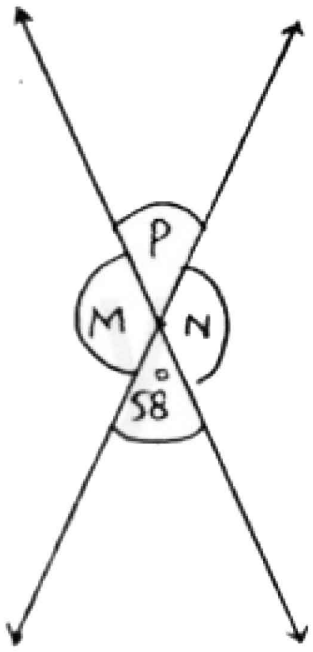
(vertically opp. angles) $m\angle l = m\angle p$ and

from (i) $m\angle l = 42^{\circ}$

(vertically opp. angles) $m\angle P = 58^{\circ}$ **7(ix)**

(supplementary angles) $m\angle N + 58^{\circ} = 180^{\circ}$

$m\angle N = 180^{\circ} - 58$

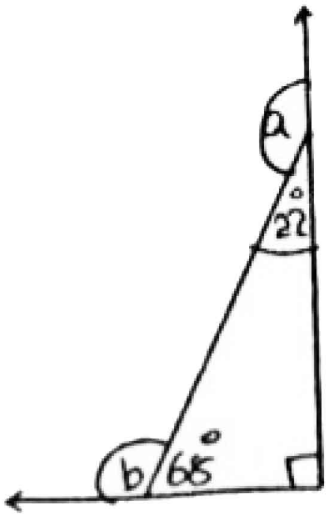


$m\angle N = 122^{\circ}$ (i)

(vertically opp. angles) $m\angle M = m\angle N$ and

from (i) $= 122^{\circ}$

7(x)



(supplementary angles) $m\angle a + 22^{\circ} = 180^{\circ}$

$m\angle a = 180^{\circ} - 22^{\circ}$

$m\angle a = 158^{\circ}$

(supplementary angles) $m\angle b + 68^{\circ} = 180^{\circ}$

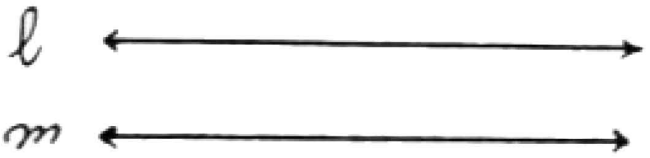
$m\angle b = 180^{\circ} - 68$

$m\angle b = 112^{\circ}$

(i) ***PARALLEL LINES***

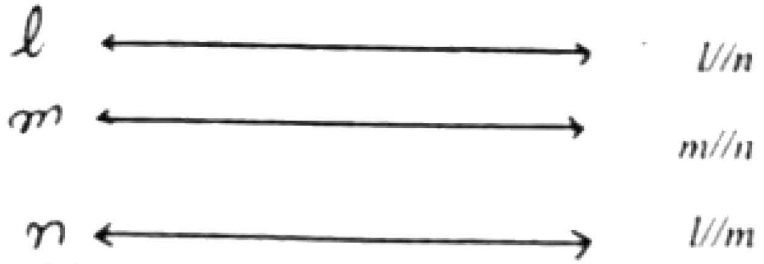
Parallel lines are two straight lines in the same plane which never meet.

The lines a and b are parallel, we write $a \parallel b$.



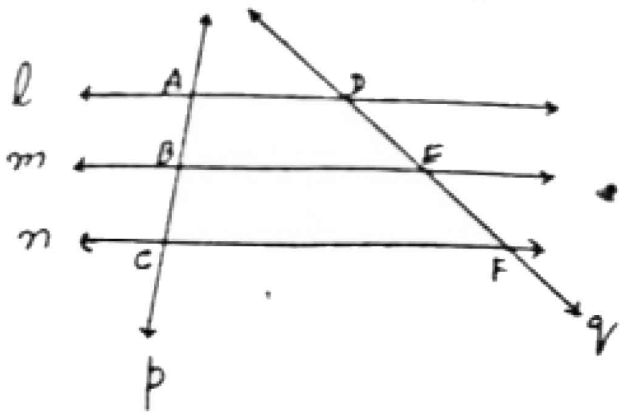
(ii) ***Properties of Parallel Lines***

(a) Two lines parallel to a third are parallel to each other.

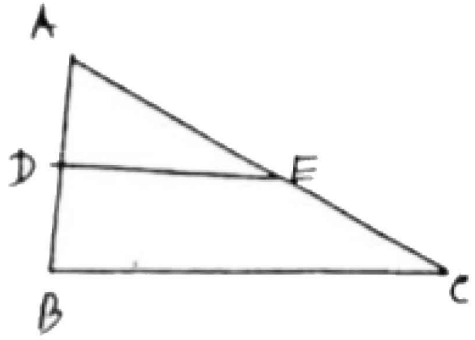


(b) If three parallel lines are intercepted by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.

i.e. if $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$



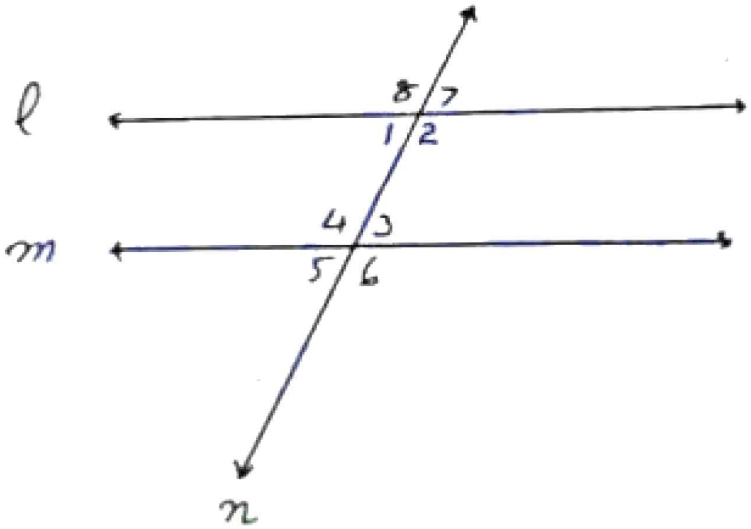
(c) If a line bisects one side of a triangle and is parallel to a second side, then it bisects the third side.



i.e. if ΔABC with $\overline{BD} \cong \overline{DA}$, $\overline{DE} \parallel \overline{BC}$ then $\overline{AE} \cong \overline{CE}$

Transversal

A transversal is a line that intersects two lines in different points.



If a transversal *"t"* intersects two parallel lines a and b, the angles formed are identified as follows:

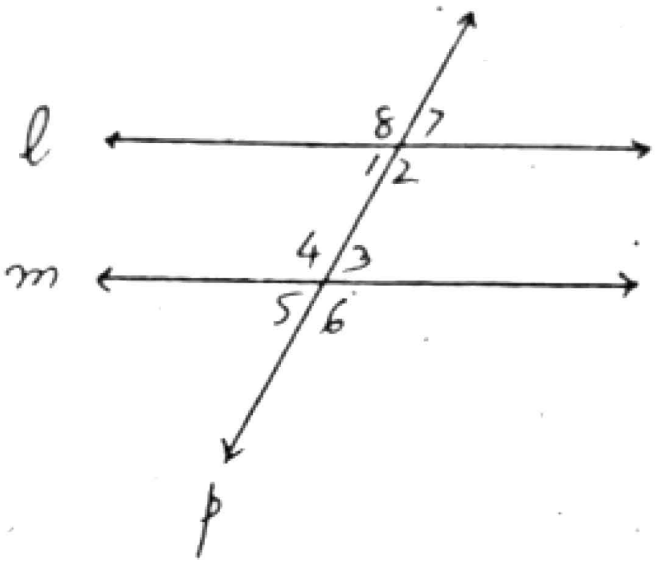
1. Four interior angles : $\angle 1, \angle 2, \angle 3, \angle 4$
2. Four exterior angles : $\angle 5, \angle 6, \angle 7, \angle 8$
3. Two pairs of alternate interior angles $\angle 1$ and $\angle 3$; and $\angle 4$
4. Two pairs of alternate exterior angles $\angle 5$ and $\angle 7$; $\angle 6$ and $\angle 8$
5. Two pairs of interior angles on the same side of the transversal: $\angle 2$ and $\angle 3$; $\angle 1$ and $\angle 4$.
6. Four pairs of corresponding angles: $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$; $\angle 2$ and $\angle 6$; $\angle 1$ and $\angle 5$.

Relation Between the Pairs of Angles

If two parallel lines are cut by a transversal, the corresponding angles are equal.

$[\angle 1 = \angle 2, \angle 2 = \angle 3, \angle 1 = \angle 3]$

d) If two parallel lines are cut by a transversal, the alternate interior angles are equal.



a || b, lines a and b are cut by the transversal c at point M and N to form the pairs of alternate interior angles.

$(\angle 1, \angle 2) and\ (\angle 3, \angle 4)$

$\angle 1 = \angle 2, \angle 3 = \angle 4$

e) If two parallel lines are intercepted by a transversal, then pairs of interior angles on the same side of transversal are supplementary.

$AB \parallel CD$, lines are cut by the transversal t, angles a, b, c and d are formed.

(*i*) $m\angle 2 = m\angle 4$

$m\angle 1 = m\angle 3$

(*ii*) $m\angle 3 = m\angle 7$

$m\angle 4 = m\angle 8$

$m\angle 6 = m\angle 2$

$m\angle 5 = m\angle 1$

(*iii*) $m\angle 7 = m\angle 5$

$m\angle 6 = m\angle 8$

(*iv*) $m\angle 2 + m\angle 3 = 180^{o}$

$m\angle 1 + m\angle 4 = 180^{o}$

(*v*) $m\angle 5 + m\angle 8 = 180^{o}$

$m\angle 6 + m\angle 7 = 180^{o}$