Exercise 9.1

1. ***Find the third side of each right triangle with legs a and b and hypotenuse c.***

(i) $a = 3, \quad b = 4, \quad c = ?$

(ii) $a = 5, \quad c = 13, \; b = ?$

(iii) $b = 5, \quad c = 61, \; a = ?$

Solution:

(i) $a = 3$

$b = 4$

$c = ?$

$c^2 = a^2 + b^2$ By Pythagoras theorem

$c^2 = (3)^2 + (4)^2$ Putting values of a, b

$c^2 = 9 + 16$

$c^2 = 25$

$\sqrt{c^2} = \sqrt{25}$ Taking square root

$\boxed{c = 5}$

(ii) $a = 5, \quad c = 13, \quad b = ?$

By Pythagoras theorem

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$ or

Putting values of a, c

$b^2 = (13)^2 - (5)^2$

$b^2 = 169 - 25$

$b^2 = 144$

$\sqrt{b^2} = \sqrt{144}$ Taking square root

$\boxed{b = 12}$

(iii) b = 5

c = 61

a = ?

By Pythagoras theorem

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$ therefore

Putting values of b, c

$$a^2 = (61)^2 - (5)^2$$

$$a^2 = 3721 - 25$$

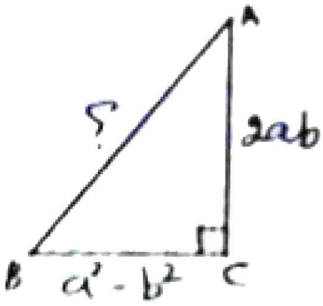
$$a^2 = 3696$$

$$a^2 = 3696 \quad \text{Taking square root}$$

$$\sqrt{a^2} = \sqrt{16 \times 231}$$

$$\boxed{a = 4\sqrt{231}}$$

2 If the legs of a right triangle are 2ab and $a^2 - b^2$, prove that hypotenuse is $a^2 + b^2$.



Sol

one side = $2ab$

2nd side = $a^2 - b^2$

Hypotenuse = ?

By Pythagoras theorem

$$(\text{Hypotenuse})^2 = (2ab)^2 + (a^2 - b^2)^2$$

$$\text{Hypotenuse} = \sqrt{(a^2 - b^2)^2 + (2ab)^2}$$

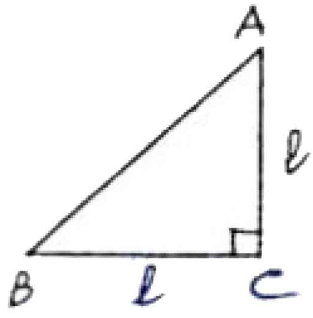
$$= \sqrt{a^4 + b^4 - 2a^2b^2 + 4a^2b^2}$$

$$= \sqrt{a^4 + b^4 + 2a^2b^2}$$

$$= \sqrt{(a^2 + b^2)^2}$$

$$\text{Hypotenuse} = (a^2 + b^2)$$

3. Find the hypotenuse of the right isosceles triangle each of whose legs is l.



Sol:

$$m\overline{AC} = l$$

$$m\overline{BC} = l$$

$$m\overline{AB} = ?$$

By Pythagoras theorem

$$\left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2$$

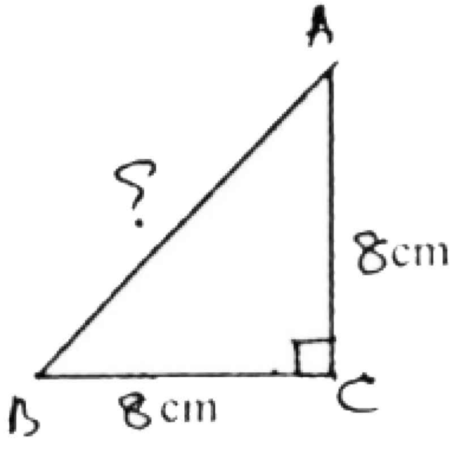
$$= l^2 + l^2$$

$$= 2l^2$$

$$\sqrt{m\overline{AB}^2} = \sqrt{2l^2} \quad \text{Taking square root}$$

$$m\overline{AB} = \sqrt{2}\, l \text{ units}$$

4. Find the hypotenuse of a right isosceles triangle whose legs are 8cm



Sol:

$a = 8$ cm here

$b = 8$ cm

$c = ?$

By Pythagoras theorem

$c^2 = a^2 + b^2$

Putting values of a, b

$c^2 = (8)^2 + (8)^2$

$c^2 = 64 + 64$

$c^2 = 128$

$\sqrt{c^2} = \sqrt{128}$ Taking square root

$c = \sqrt{8 \times 8 \times 2}$

$c = 8\sqrt{2}$ cm

5. If the numbers represent the lengths of the sides of a triangle, which triangles are right triangles?

(i) 3, 4, 5

(ii) 9, 17, 25

(iii) 11, 61, 60

Sol: Length of sides 3, 4, 5

We observer that, is the square of big side equal or not equal the square of the other to sides? Then these sides will be right angle Δ otherwise not.

$(5)^2 = 25$	$(3)^2 + (4)^2$
	$= 9 + 16$
	$= 25$

These sides are sides of right angle Δ.

(ii)

$(25)^2$	$(9)^2 + (17)^2$
$=625$	$= 81 + 289$
	$= 370$

$$625 \neq 370$$

Sides 9, 17, 25 are not the sides of right angle triangle.

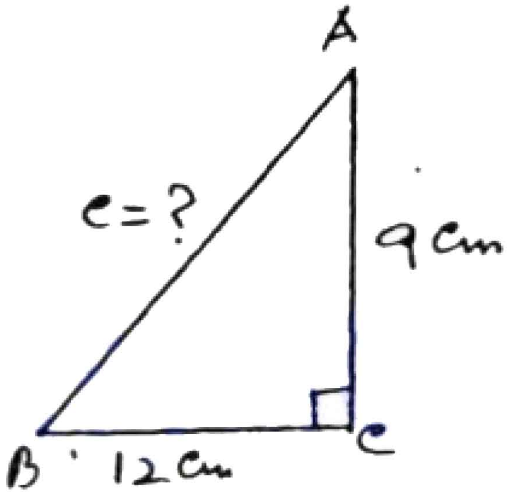
(iii)

$(61)^2$	$(11)^2 + (60)^2$
$=3721$	$= 121 + 3600$
	$= 3721$

Sides 11, 61, 60 are the sides of right angle triangle.

6. ΔABC is right angled at C. If $m\overline{AC} = 9cm$ and $m\overline{BC} = 12cm$, find the length $\overline{AB}$, using Pythagoras theorem.

Sol: In ΔABC



b = 9 cm

a = 12 cm

c = ?

By Pythagoras theorem

$c^2 = a^2 + b^2$

Putting values of *a*, *b*

$c^2 = (12)^2 + (9)^2$

$= 144 + 81$

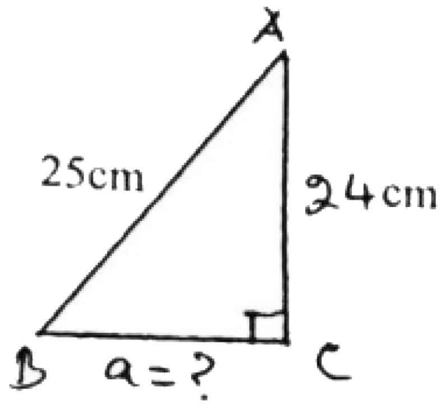
$c^2 = 225$

$\sqrt{c^2} = \sqrt{225}$ Taking square root

$\boxed{c = 15}$ cm

7. The hypotenuse of a right triangle is 25cm. If one of the sides are of length 24cm, find the length of the other side.

Sol:



$c = 25$ cm here

$b = 24$ cm

$a = ?$

By Pythagors theorem

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$ or

Putting values of b, c

$a^2 = (25)^2 - (24)^2$

$= 625 - 576$

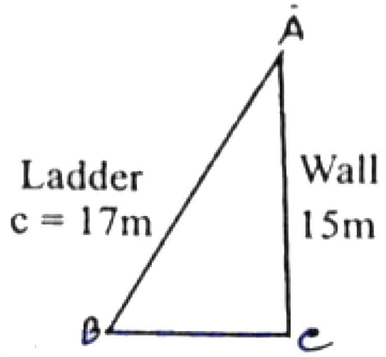
$a^2 = 49$

$\sqrt{a^2} = \sqrt{49}$ Taking square root

$\boxed{a = 7cm}$

8. A ladder 17m long when set against the wall of a house just reaches a window at a height of 15m from the ground. How far is the lower end of the ladder from the base of the wall?

Sol:



Lower end of the ladder from the wall = a

$b = 15\text{m}$ here

$c = 17\text{m}$

$a = ?$

By Pythagoras theorem

$$a^2 = c^2 - b^2$$

Putting values of c, b

$$a^2 = (17)^2 - (15)^2$$

$$= 289 - 225$$

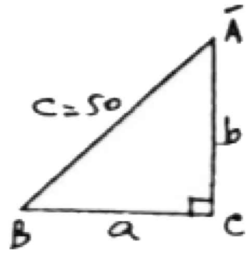
$$a^2 = 64$$

$$\sqrt{a^2} = \sqrt{64} \quad \text{Taking square root}$$

$$\boxed{a = 8}\text{m}$$

9. The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.

Sol:



$c = 50$ here

$a = b$

By Pythagoras theorem

$$c^2 = a^2 + b^2$$

$$c^2 = a^2 + a^2 \quad (a = b)$$

$$c^2 = 2a^2$$

$$2a^2 = c^2$$ or

Putting values of c^2

$$2a^2 = (50)^2$$

$$a^2 = \frac{2500}{2}$$

$$a^2 = 1250$$

$$\sqrt{a^2} = \sqrt{1250}$$ taking square root

$$a = \sqrt{625 \times 2}$$

$$a = 25\sqrt{2}$$

Length of each side $= 25\sqrt{2}$ units

10. The sides of a triangle are 15cm, 36cm and 39cm. Show that it is a right angled triangle.

Sol:

The length of big side = 39 cm

The length of small side = 15cm, 36cm

$$(\text{Lengths of big side})^2 = (39)^2$$

$$= 1521 \quad \text{(i)}$$

$$\text{The sum of the square of small sides} = (15)^2 + (36)^2$$

$$= 225 + 1296$$

$$\text{Now its prove that} = 1521 \quad \text{(ii)}$$

These lengths are the lengths of right angle triangle.

Area

The surface inside the boundary of a shape is called area. We see that, Is the big side of square equal or not equal the square of the other two sides? Then these sides will be sides of right angle triangle otherwise not.

Area of a Triangle when all the three sides are given

A triangle ABC with sides a, b, c and

$$2S = a + b + c \Rightarrow \quad S = \frac{a+b+c}{2},$$

where 'S' is half the perimeter of a triangle.

Then area of any triangle is $A = \sqrt{S(S-a)(S-b)(S-c)}$.

This is called ***Hero's Formula*** for finding the area of a triangle.