UNIT 3 Algebraic Manipulation

- H.C.F and L.C.M
- Basic Operations on Algebraic Fractions
- Square Roots of Algebraic Fractions

After complation of this unit, the students will be able to:

- find highest common factor (HCF) and least common multiple (LCM) of algebraic expressions.
- use factor or division method to determine HCF and LCM.
- know the relationship between HCF and LCM.
- use HCF and LCM to reduce fractional expressions involving +, − , ×, ÷.
- find square root of an algebraic expression by factorization and division.

Exercise 3.1

Find H.C.F by factorization.

Q.1 $abxy$, a^2bc.

Sol.

Factorization of $abxy$ $= a \times b \times x \times y$

Factorization of a^2bc $= a \times b \times a \times c$

common factors $= a, b$

Thus, H.C.F $= a \times b$

$= ab$

Q.2 $6pqr,\ 15qrs$

Sol.

Factorization of $6pqr$ $= 2 \times \boxed{3} \times p \times \boxed{q} \times \boxed{r}$

Factorization of $15qrs$ $= 5 \times \boxed{3} \times \boxed{q} \times \boxed{r} \times s$

Common factors $= 3,\ q,\ r$

Thus, H.C.F $= 3 \times q \times r$

$= 3qr$

Q.3 $8xy^2z^3,\ 12x^2y^2z^2$

Sol.

Factorization of $8xy^2z^3$ $= \boxed{2} \times \boxed{2} \times 2 \times \boxed{x} \times \boxed{y} \times \boxed{y} \times z \times \boxed{z} \times \boxed{z}$

Factorization of $12x^2y^2z^2$ $= \boxed{2} \times \boxed{2} \times 3 \times \boxed{x} \times x \times \boxed{y} \times \boxed{y} \times \boxed{z} \times \boxed{z}$

Common factors $= 2,\ 2,\ x,\ y,\ y,\ z,\ z$

Thus, H.C.F $= 2 \times 2 \times x \times y \times y \times z \times z$

$= 4xy^2z^2$

Q.4 $14a^2bc,\ 21ab^2$

Sol.

Factorization of $14a^2bc$ $= 2 \times \boxed{7} \times \boxed{a} \times a \times \boxed{b} \times c$

Factorization of $21ab^2$ $= 3 \times \boxed{7} \times \boxed{a} \times \boxed{b} \times b$

Common factors $= 7,\ a,\ b$

Thus, H.C.F $= 7 \times a \times b$

$= 7ab$

Q.5 $3x^5y^2,\ 12x^2y^4,\ 15x^3y^2$

Sol.

Factorization of $3x^5y^2$ $= \boxed{3} \times x \times x \times \boxed{x} \times \boxed{x} \times x \times \boxed{y} \times \boxed{y}$

Factorization of $12x^2y^4$ $= \boxed{3} \times 2 \times 2 \times \boxed{x} \times \boxed{x} \times \boxed{y} \times \boxed{y} \times y \times y$

Factorization of $15x^3y^2$ $= \boxed{3} \times 5 \times \boxed{x} \times \boxed{x} \times x \times \boxed{y} \times \boxed{y}$

Common factors $= 3,\ x,\ x,\ y,\ y$

Thus, H.C.F $= 3 \times x \times x \times y \times y$

$= 3x^2y^2$

Q.6 $4abc^3, 8a^3bc, 6ab^3c$

Sol.

$$\text{Factorization of } 4abc^3 = \boxed{2} \times 2 \times \boxed{a} \times \boxed{b} \times c \times \boxed{c} \times c$$
$$\text{Factorization of } 8a^3bc = 2 \times \boxed{2} \times 2 \times \boxed{a} \times a \times a \times \boxed{b} \times \boxed{c}$$
$$\text{Factorization of } 6ab^3c = \boxed{2} \times 3 \times \boxed{a} \times b \times \boxed{b} \times b \times \boxed{c}$$

Common factors $= 2, a, b, c$

Thus, H.C.F $= 2 \times a \times b \times c$

$= 2abc$

Q.7 $x^3 + 64, x^2 - 16$

Sol.

Factorization of $x^3 + 64 = (x)^3 + (4)^3$

$= (x + 4)[(x)^2 - (x)(4) + (4)^2]$

$= (x + 4)(x^2 - 4x + 16)$ .... (i)

Factorization of $x^2 - 16 = (x)^2 - (4)^2$

$= (x + 4)(x - 4)$..................................(ii)

Common factors are: $(x + 4)$

Thus, H.C.F $= x + 4$

Q.8 $x^2 - y^2, x^4 - y^4, x^6 - y^6$

Factorization of $x^2 - y^2 = (x + y)(x - y)$...................(i)

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)(x + y)(x - y)$.............(ii)

Factorization of $x^6 - y^6 = (x^3)^2 - (y^3)^2$

$= (x^3 - y^3)(x^3 + y^3)$

$= (x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$.....(iii)

Common factors are : $(x + y), (x - y)$

Thus, H.C.F $= (x + y)(x - y)$

$= x^2 - y^2$

Q.9 $t^2 - 9, (t + 3)^2, t^2 + t - 6$

Sol.

Factorization of $t^2 - 9 = (t)^2 - (3)^2$

$= (t + 3)(t - 3)$..........................(i)

Factorization of $(t + 3)^2 = (t + 3)(t + 3)$..........................(ii)

Factorization of $t^2 + t - 6 = t^2 + 3t - 2t - 6$

$= t(t + 3) - 2(t + 3)$

$= (t + 3)(t - 2)$..........................(iii)

Common factor is : $(t + 3)$

Thus, H.C.F $= (t + 3)$

Q.10 $x^2 - x - 2, x^2 + x - 6, x^2 - 3x + 2$

Sol.

Factorization of $x^2 - x - 2 = x^2 - 2x + x - 2$

$= (x^2 - 2x) + (x - 2)$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$..............................(i)

Factorization of $x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= (x^2 + 3x) - (2x + 6)$

$= x(x + 3) - 2(x + 3)$

$= (x + 3)(x - 2)$..............................(ii)

Factorization of $x^2 - 3x + 2 = x^2 - x - 2x + 2$

$= (x^2 - x) - (2x - 2)$

$= x(x - 1) - 2(x - 1)$

$= (x - 1)(x - 2)$..............................(iii)

Common factor is : $(x - 2)$

Thus, H.C.F $= (x - 2)$

Q.11 $1 - x^2, x^3 + 1, 1 - x - 2x^2$

Sol.

Factorization of $1 - x^2 = (1)^2 - (x)^2$

$= (1 + x)(1 - x)$..............................(i)

Factorization of $x^3 + 1 = (x)^3 + (1)^3$

$= (x + 1)(x^2 - x + 1)$......................(ii)

Factorization of $1 - x - 2x^2 = 1 - 2x + x - 2x^2$

$= (1 - 2x) + (x - 2x^2)$

$= 1(1 - 2x) + x(1 - 2x)$

$= (1 - 2x)(1 + x)$

Common factor is : $1 + x$

Thus, H.C.F $= 1 + x$

Q.12 $x^3 - 8, x^2 - 7x + 10$

Sol:

Factorization of $x^3 - 8 = (x)^3 - (2)^3$

$= (x - 2)[(x)^2 + (x)(2) + (2)^2]$

$= (x - 2)(x^2 + 2x + 4)$.........................(i)

Factorization of $x^2 - 7x + 10 = x^2 - 2x - 5x + 10$

$= (x^2 - 2x) - (5x - 10)$

$= x(x - 2) - 5(x - 2)$

$= (x - 2)(x - 5)$................................(ii)

Common factors are: $x - 2$

Thus, H.C.F $= x - 2$

Q.13 $x^2 + 3x + 2, x^2 + 4x + 3, x^2 + 5x + 4$

Sol:

Factorization of $x^2 + 3x + 2 = x^2 + x + 2x + 2$

$= (x^2 + x) + (2x + 2)$

$= x(x + 1) + 2(x + 1)$

$= (x + 1)(x + 2)$..........................(i)

Factorization of $x^2 + 4x + 3 = x^2 + x + 3x + 3$

$= (x^2 + x) + (3x + 3)$

$= x(x + 1) + 3(x + 1)$

$= (x + 1)(x + 3)$............................(ii)

Factorization of $x^2 + 5x + 4 = x^2 + x + 4x + 4$

$= (x^2 + x) + (4x + 4)$

$= x(x + 1) + 4(x + 1)$

$= (x + 1)(x + 4)$ ....(iii)

Common factors are: $(x + 1)$

Thus, H.C.F $= x + 1$

Q.14: $x^4 + x^3 - 6x^2,\ x^4 - 9x^2,\ x^3 + x^2 - 6x$

Sol:

Factorization of $x^4 + x^3 - 6x^2 = x^2(x^2 + x - 6)$

$= x^2(x^2 + 3x - 2x - 6)$

$= x^2\left[(x^2 + 3x) - (2x + 6)\right]$

$= x^2\left[x(x + 3) - 2(x + 3)\right]$

$= x^2(x + 3)(x - 2)$

$= x \times x(x + 3)(x - 2)$ ....(i)

Factorization of $x^4 - 9x^2 = x^2(x^2 - 9)$

$= x^2\left[(x)^2 - (3)^2\right]$

$= x^2(x - 3)(x + 3)$

$= x \times x(x - 3)(x + 3)$ ....(ii)

Factorization of $x^3 + x^2 - 6x = x(x^2 + x - 6)$

$= x(x^2 + 3x - 2x - 6)$

$= x\left[(x^2 + 3x) - (2x + 6)\right]$

$= x\left[x(x + 3) - 2(x + 3)\right]$

$= x(x + 3)(x - 2)$ ....(iii)

Common factors are: $x,\ x + 3$

Thus, H.C.F $= x(x + 3)$

$= x^2 + 3x$

Q.15 $35a^2c^3b, 45a^3cb^2, 30ac^2b^3$

Sol:

Factorization of $35a^2c^3b$ =	5	×7×	a	×a×c×	c	×c×	b			
Factorization of $45a^3cb^2$ =	5	×3×3×	a	×a×a×	c	×	b	×b		
Factorization of $30ac^2b^3$ =	5	×2×3×	a	×	c	×c×	b	×b×b		

Common factors = $5, a, b, c$

Thus, H.C.F $= 5 \times a \times b \times c$

$= 5abc$

Exercise 3.2

Find the H.C.F by Division Method.

Q.1 $x^4 + x^2 + 1, x^4 + x^3 + x + 1$

Sol:

$x^4 + x^2 + 1 \,\big)\, x^4 + x^3 + x + 1 \,\big(\, 1$

$\underline{\pm x^4 \pm x^2 \pm 1}$

Take common x $\big)\, x^3 - x^2 + x$

$x^2 - x + 1 \,\big)\, x^4 + x^2 + 1 \,\big(\, x^2 + x + 1$

$\underline{\pm x^4 \mp x^3 \pm x^2}$

$x^3 + 1$

$\underline{\pm x^3 \mp x^2 \pm x}$

$x^2 - x + 1$

$\underline{\pm x^2 \mp x \pm 1}$

0

H.C.F $= x^2 - x + 1$