Thus, H.C.F $= x(x + 3)$

$= x^2 + 3x$

Q.15 $35a^2c^3b, 45a^3cb^2, 30ac^2b^3$

Sol:

Factorization of $35a^2c^3b = \boxed{5} \times 7 \times \boxed{a} \times a \times c \times \boxed{c} \times c \times \boxed{b}$

Factorization of $45a^3cb^2 = \boxed{5} \times 3 \times 3 \times \boxed{a} \times a \times a \times \boxed{c} \times \boxed{b} \times b$

Factorization of $30ac^2b^3 = \boxed{5} \times 2 \times 3 \times \boxed{a} \times \boxed{c} \times c \times \boxed{b} \times b \times b$

Common factors = 5, *a*, *b*, *c*

Thus, H.C.F $= 5 \times a \times b \times c$

$= 5abc$

Exercise 3.2

Find the H.C.F by Division Method.

Q.1 $x^4 + x^2 + 1, x^4 + x^3 + x + 1$

Sol:

$$
\begin{array}{r|l}
 & 1 \\
\hline
x^4 + x^2 + 1 & x^4 + x^3 + x + 1 \\
 & \pm x^4 \quad \pm x^2 \quad \pm 1 \\
\hline
\end{array}
$$

Take common x

$$
\begin{array}{r|l}
 & x^3 - x^2 + x \\
\hline
x^2 - x + 1 & x^4 + x^2 + 1 \\
 & -x^4 + x^3 - x^2 \\
\hline
 & x^3 + 1 \\
 & -x^3 + x^2 - x \\
\hline
 & x^2 - x + 1 \\
 & \pm x^2 \mp x \pm 1 \\
\hline
 & 0
\end{array}
\qquad (x^2 + x + 1)
$$

H.C.F $= x^2 - x + 1$

Q.2. $6x^3 + 7x^2 - 9x + 2, 8x^4 + 6x^3 - 15x^2 + 9x - 2$

Sol:

$$
\begin{array}{r|l}
6x^3 + 7x^2 - 9x + 2 & 4x + 5 \\
\hline
8x^4 + 6x^3 - 15x^2 + 9x - 2 & \\
\times 3 & \text{multiply by 3} \\
\hline
24x^4 + 18x^3 - 45x^2 + 27x - 6 & \\
\underline{-}\,24x^4 \underline{+}\, 28x^3 \underline{-}\, 36x^2 \underline{+}\, 8x & \\
\hline
-10x^3 - 9x^2 + 19x - 6 & \\
\times(-3) & \\
\hline
30x^3 + 27x^2 - 57x + 18 & \\
\underline{+}\,30x^3 \underline{+}\, 35x^2 \underline{-}\, 45x \underline{+}\, 10 & \\
\hline
-4 \;\big|\; -8x^2 - 12x + 8 &
\end{array}
$$

$$
\begin{array}{r|l}
2x^2 + 3x - 2 & 3x - 1 \\
\hline
6x^3 + 7x^2 - 9x + 2 & \\
\underline{-}\,6x^3 \underline{-}\, 9x^2 \underline{+}\, 6x & \\
\hline
-2x^2 - 3x + 2 & \\
\underline{+}\,2x^2 \underline{+}\, 3x \underline{-}\, 2 & \\
\hline
0 &
\end{array}
$$

H.C.F $= 2x^2 + 3x - 2$

Q.3 $4x^3 + 2x^2 - 6x, 4x^3 - 8x + 4$

Sol: $2\,|\,4x^3 + 2x^2 - 6x$ $\qquad$ $4\,|\,4x^3 - 8x + 4$

$2x^3 + x^2 - 3x$ $\qquad$ $x^3 - 2x + 1$

Now we find H.C.F of $x^3 - 2x + 1$ and $2x^3 + x^2 - 3x$ a n d

H.C.F of 2, 4 is 2.

$$
\begin{array}{r|l}
 & 2 \\
\hline
x^2 - 2x + 1 & 2x^3 + x^2 - 3x \\
 & \pm 2x^3 \mp 4x \pm 2 \\
\hline
 & x^2 + x - 2 \;\big|\; x^2 - 2x + 1 \;\big|\; x \\
 & \pm x^2 \mp 2x \pm x^2 \\
\hline
 & -x^2 + 1
\end{array}
$$

$$
\begin{array}{r|l}
 & -1 \\
\hline
-x^2 + 1 & x^2 + x - 2 \\
 & \pm x^2 \mp 1 \\
\hline
 & x - 1 \;\big|\; -x^2 + 1 \;\big|\; -x - 1 \\
 & \mp x^2 \pm x \\
\hline
 & -x + 1 \\
 & \mp x \pm 1 \\
\hline
 & 0
\end{array}
$$

Required H.C.F = $2(x - 1)$

Q.4 $x^3 + 7x^2 + 12x$, $x^3 - 2x^2 - 15x$

Sol:

$x \;|\; x^3 + 7x^2 + 12x$ $\qquad$ $x \;|\; x^3 - 2x^2 - 15x$

$|\; x^2 + 7x + 12$ $\qquad$ $|\; x^2 - 2x - 15$

H.C.F of x and x is x.

Now we find H.C.F of $x^2 - 2x - 15$ and $x^2 + 7x + 12$.

$$
\begin{array}{r|l}
 & 1 \\
\hline
x^2+7x+12 & x^2-2x-15 \\
 & +x^2\pm 7x\pm 12 \\
\hline
 & -9 \mid -9x-27 \\
\hline
 & x+3 \mid +x^2+7x+12 \mid x+4 \\
 & \quad \pm x^2 \pm 3x \\
\hline
 & \quad +4x+12 \\
 & \quad \pm 4x \pm 12 \\
\hline
 & \quad 0
\end{array}
$$

Required H.C.F = $x(x+3)$

Q.5 $x^3 - x^2 - x + 1$, $x^4 - 2x^3 + 2x - 1$

Sol:

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x^3-x^2-x+1 & x^4-2x^3+2x-1 \\
 & \pm x^4 \mp x^3 \mp x^2 \pm x \\
\hline
 & -x^3+x^2+x-1 \\
 & \mp x^3 \pm x^2 \pm x \mp 1 \\
\hline
 & 0
\end{array}
$$

Required H.C.F = $x^3 - x^2 - x + 1$

$= x^2(x-1) - 1(x-1)$

$= (x^2-1)(x-1)$

$= \left[(x)^2 - (1)^2\right](x-1)$

$= (x-1)(x+1)(x-1)$

$= (x+1)(x-1)^2$

Q.6 $x^3 - x^2 - x - 2$, $x^3 + 3x^2 - 6x - 8$

Sol:

$$x^3-x^2-x-2 \;\big|\; x^3+3x^2-6x-8$$
$$\pm x^3 \mp x^2 \mp x \mp 2$$
$$4x^2-5x-6 \;\big|\; x^3-x^2-x-2 \;\big|\; \times 4$$

$$4x^2-5x-6 \;\big|\; 4x^3-4x^2-4x-8 \;\big|\; x$$
$$\pm 4x^3 \mp 5x^2 \mp 6x$$
$$x^2+2x-8 \;\big|\; 4x^2-5x-6 \;\big|\; 4$$
$$\pm 4x^2 \pm 8x \mp 32$$
$$-13 \;\big|\; -13x+26$$
$$x-2$$

$$x-2 \;\big|\; x^2+2x-8 \;\big|\; x+4$$
$$\pm x^2 \mp 2x$$
$$4x-8$$
$$\pm 4x \mp 8$$
$$0$$

Required H.C.F = $x - 2$

Q.7 $x^2 + 3x - 4$, $x^3 - 2x^2 - 2x + 3$

Sol:

$$x^2+3x-4 \;\big|\; x^3-2x^2-2x+3 \;\big|\; x-5$$
$$\pm x^3 \pm 3x^2 \mp 4x$$
$$-5x^2+2x+3$$
$$\mp 5x^2 \mp 15x \pm 20$$
$$17 \;\big|\; 17x-17$$
$$x-1 \;\big|\; x^2+3x-4 \;\big|\; x+4$$
$$\pm x^2 \mp x$$
$$4x-4$$
$$\pm 4x \mp 4$$
$$0$$

Required H.C.F = $x - 1$

Q.8 $3x^3 - 14x^2 + 9x + 10,\ 15x^3 - 34x^2 + 21x - 10$

Sol:

$$
\begin{array}{r|l}
 & 5 \\
\hline
3x^3-14x^2+9x+10 & 15x^3 - 34x^2 + 21x - 10 \\
 & \pm 15x^3 \mp 70x^2 \pm 45x \pm 50 \\
\hline
 & 12 \mid 36x^2 - 24x - 60
\end{array}
$$

$$
\begin{array}{r|l}
 & x-1 \\
\hline
3x^2 - 2x - 5 & 3x^3-14x^2+9x+10 \\
 & \pm 3x^3 \mp 2x^2 \mp 5x \\
\hline
 & -12x^2+14x+10 \\
 & \mp 12x^2 \pm 8x \pm 20 \\
\hline
 & 2 \mid 6x - 10 \\
\hline
 & 3x - 5
\end{array}
$$

$$
\begin{array}{r|l}
 & x + 1 \\
\hline
3x - 5 & 3x^2 - 2x - 5 \\
 & \pm 3x^2 \mp 5x \\
\hline
 & 3x - 5 \\
 & \pm 3x \mp 5 \\
\hline
 & 0
\end{array}
$$

Required H.C.F = $3x - 5$

Q.9 $2x^4 + x^3 + 4x + 2,\ 6x^3 + 5x^2 + x,\ 2x^4 + 3x^3 + x^2 + 2x + 1$

Sol: First we will find the H.C.F of $2x^4 + x^3 + 4x + 2$ and $2x^4 + 3x^3 + x^2 + 2x + 1$

$$
\begin{array}{r|l}
 & 1 \\
\hline
2x^4 + x^3 + 4x + 2 & +2x^4+3x^3 + x^2 + 2x +1 \\
 & \pm 2x^4 \pm x^3 \quad \pm 4x \pm 2 \quad x \\
\hline
 & 2x^3 + x^2 - 2x - 1 \;\big|\; +2x^4+x^3 \quad +4x+2x \\
 & \qquad\qquad \big|\; \pm 2x^4 \pm x^3 \mp 2x^2 \mp x \\
\hline
 & \qquad\qquad\qquad\qquad 2x^2+5x+2
\end{array}
$$

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
2x^2 + 5x + 2 & 2x^3 + x^2 - 2x - 1 \\
 & \pm 2x^3 \pm 5x^2 \pm 2x \\
\hline
 & -4x^2 - 4x - 1 \\
 & \mp 4x^2 \mp 10x \mp 4 \\
\hline
 & 3 \;\big|\; 6x + 3
\end{array}
$$

$$
\begin{array}{r|l}
\text{H.C.F} & 2x + 1 \;\big|\; 2x^2 + 5x + 2 \\
 & \pm 2x^2 \pm x \\
\hline
 & 4x + 2 \\
 & \pm 4x \pm 2 \\
\hline
 & 0
\end{array}
$$

Now we will find H.C.F of $2x + 1$ and $6x^3 + 5x^2 + x$

$$
\begin{array}{r|l}
 & 3x^2 + x \\
\hline
2x + 1 & 6x^3 + 5x^2 + x \\
 & \pm 6x^3 \pm 3x^2 \\
\hline
 & 2x^2 + x \\
 & \pm 2x^2 \pm x \\
\hline
 & 0
\end{array}
$$

Required H.C.F = $2x + 1$

Q.10 $x^3 + x^2 - 5x + 3,\ x^3 - 7x + 6,\ x^3 + 2x^2 - 2x + 3$

Sol: First we will find the H.C.F of $x^3 - 7x + 6$ and $x^3 + x^2 - 5x + 3$.

$$
\begin{array}{l|l}
x^3 - 7x + 6 & x^3 + x^2 - 5x + 3 \\
 & \pm x^3 \mp 7x \pm 6 \qquad x - 2 \\
\hline
 & x^2 + 2x - 3 \;\Big|\; x^3 - 7x + 6 \\
 & \qquad \pm x^3 \pm 2x^2 \mp 3x \\
\hline
 & \qquad -2x^2 - 4x + 6 \\
 & \qquad \mp 2x^2 \mp 4x \pm 6 \\
\hline
 & \qquad 0
\end{array}
$$

Now we will find H.C.F of $x^2 + 2x - 3$ and $x^3 + 2x^2 + 3$.

$$
\begin{array}{l|l}
x^2 + 2x - 3 & x^3 + 2x^2 - 2x + 3 \\
 & \pm x^3 \pm 2x^2 \mp 3x \qquad x - 1 \\
\hline
 & x + 3 \;\Big|\; x^2 + 2x - 3 \\
 & \qquad \pm x^2 \pm 3x \\
\hline
 & \qquad -x - 3 \\
 & \qquad \mp x \mp 3 \\
\hline
 & \qquad 0
\end{array}
$$

H.C.F $x + 3$

Least Common Multiple (LCM)

The least common multiple of two or more algebraic expressions is the expression of lowest degree which is divisible by each of them without remainder.

The abbreviation of the words ***least common multiple*** is ***L.C.M.***