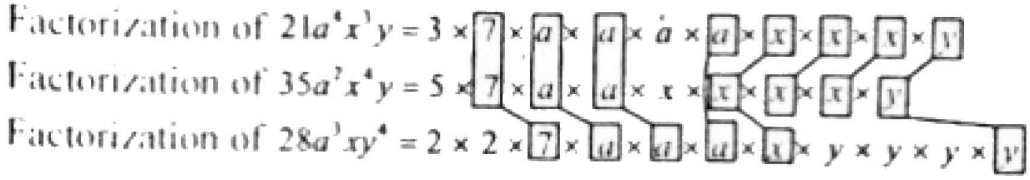
EXERCISE 4.1

Find L.C.M by Factorization.

Q.1 $21a^4x^3y,\ 35a^2x^4y,\ 28a^3xy^4$

Sol:



Product of common factors $= 7 \times a \times a \times a \times x \times y \times x \times x$

$= 7a^3x^3y$ .....(i)

Product of uncommon factors $= 3 \times 5 \times 2 \times 2 \times a \times x \times y \times y \times y$

$= 60axy^3$.....(ii)

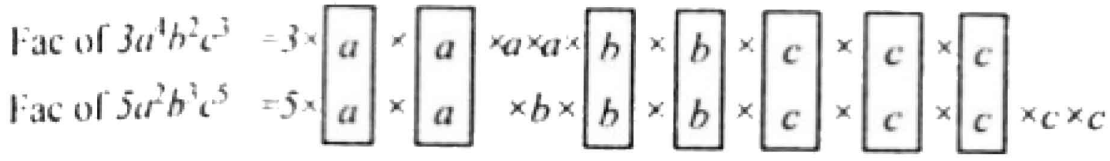
L.C.M = (i) × (ii)

$= 7a^3x^3y \times 60axy^3$

$= 420a^4x^4y^4$

Q.2 $3a^4b^2c^3,\ 5a^2b^3c^5$

Sol:



Product of common factors = a × a × b × b × c × c × c

$= a^2b^2c^3$ ..... (i)

Product of uncommon factors = 3 × 5 × a × a × b × c × c

$= 15a^2bc^2$ ..... (ii)

L.C.M = (i) × (ii)

$= (a^2b^2c^3)(15a^2bc^2)$

$= 15a^4b^3c^5$

Q.3 $2ab, 3ab, 4ca$

Sol:

$$\begin{aligned}
\text{Factorization of } 2ab &= \boxed{2} \times \boxed{a} \times \boxed{b} \\
\text{Factorization of } 3ab &= 3 \times \times \boxed{a} \times \boxed{b} \\
\text{Factorization of } 4ca &= 2 \times \boxed{2} \times \boxed{a} \times c
\end{aligned}$$

Product of common factors= $2 \times a \times b = 2ab$.....(i)

Product of uncommon factors= $3 \times 2 \times c = 6c$.....(ii)

$$\begin{aligned}
\text{L.C.M} &= \text{(i)} \times \text{(ii)} \\
&= (2ab)(6c) \\
&= 12abc
\end{aligned}$$

Q.4 x^2yz, xy^2z, xyz^2

Sol:

$$\begin{aligned}
\text{Factorization of } x^2yz &= \boxed{x} \times x \times \boxed{y} \times \boxed{z} \\
\text{Factorization of } xy^2z &= \boxed{x} \times \boxed{y} \times \boxed{y} \times \boxed{z} \\
\text{Factorization of } xyz^2 &= \boxed{x} \times \boxed{y} \times z \times \boxed{z}
\end{aligned}$$

Product of common factors= $x \times y \times y \times z = xy^2z$.....(i)

Product of uncommon factors= $x \times z = xz$.....(ii)

$$\begin{aligned}
\text{L.C.M} &= \text{(i)} \times \text{(ii)} \\
&= xy^2z \times xz \\
&= x^2y^2z^2
\end{aligned}$$

Q.5 $p^3q - pq^3, p^5q^2 - p^2q^5$

Sol:

$$\begin{aligned}
\text{Factorization of } p^3q - pq^3 &= pq(p^2 - q^2) \\
&= pq(p - q)(p + q).....\text{(i)}
\end{aligned}$$

Factorization of $p^5q^2 - p^2q^5 = p^2q^2(p^3 - q^3)$

$= ppqq(p - q)(p^2 + pq + q^2)$.....(ii)

In (i) and (ii)

Product of common factors= $pq(p - q)$.....(iii)

Product of uncommon factors = $pq(p + q)(p^2 + pq + q^2)$.....(iv)

L.C.M = (iii) × (iv)

$= \left[pq(p - q)\right]\left[pq(p + q)(p^2 + pq + q^2)\right]$

$= p^2q^2(p - q)(p + q)(p^2 + pq + q^2)$

Q. $x^3 + 64,\ x^2 - 16$

Sol:

Factorization of $x^3 + 64 = (x)^3 + (4)^3$

$= (x + 4)\left[(x)^2 - (x)(4) + (4)^2\right]$

$= (x + 4)(x^2 - 4x + 16)$.....(i)

Factorization of $x^2 - 16 = (x)^2 - (4)^2$

$= (x + 4)(x - 4)$.....(ii)

In (i) and (ii)

Product of common factors = $(x + 4)$..... (iii)

Product of uncommon factors = $(x - 4)(x^2 - 4x + 16)$.....(iv)

L.C.M = (iii) × (iv)

$= (x + 4)(x - 4)(x^2 - 4x + 16)$

Q. $x^2 - x - 2,\ x^2 + x - 6, x^2 - 3x + 2$

Sol:

Factorization of $x^2 - x - 2 = x^2 - 2x + x - 2$

$= (x^2 - 2x) + (x - 2)$

$= x(x - 2) + 1(x - 2)$

$= (x - 2)(x + 1)$.....(i)

Factorization of $x^2 + x - 6 = x^2 + 3x - 2x - 6$

$= (x^2 + 3x) - (2x + 6)$

$= x(x + 3) - 2(x + 3)$

$= (x + 3)(x - 2)$.....(ii)

Factorization of $x^2 - 3x + 2 = x^2 - x - 2x + 2$

$= (x^2 - x) - (2x + 2)$

$= x(x - 1) - 2(x - 1)$

$= (x - 1)(x - 2)$.....(iii)

In (i), (ii) and (iii)

Product of common factors = $(x - 2)$.....(iv)

Product of uncommon factors = $(x + 1)(x + 3)(x - 1)$.....(v)

L.C.M = (iv) × (v)

$= (x - 2)(x + 1)(x + 3)(x - 1)$

Q.8 $y^2 - 9,\ (y + 3)^2, y^2 + y - 6$

Sol:

Factorization of $y^2 - 9 = (y)^2 - (3)^2$

$= (y + 3)(y - 3)$.....(i)

Factorization of $(y + 3)^2 = (y + 3)(y + 3)$.....(ii)

Factorization of $y^2 + y - 6 = y^2 + 3y - 2y - 6$

$= (y^2 + 3y) - (2y + 6)$

$= y(y + 3) - 2(y + 3)$

$= (y + 3)(y - 2)$.....(iii)

In (i), (ii) and (iii)

Product of common factors = $(y + 3)$.....(iv)

Product of uncommon factors = $(y - 3)(y + 3)(y - 2)$.....(v)

L.C.M = (iv) × (v)

$= (y + 3)(y - 3)(y + 3)(y - 2)$

Q.9 $1 - x^2, x^3 + 1, 1 - x - 2x^2$

Sol:

Factorization of $1 - x^2 = (1)^2 - (x)^2$

$= (1 + y)(1 - y)$.....(i)

Factorization of $y^3 + 1 = (y)^3 + (1)^3$

$= (y + 1)(y^2 - y + 1)$.....(ii)

Factorization of $1 - y - 2y^2 = 1 - 2y + y - 2y^2$

$= (1 - 2y) + (y - 2y^2)$

$= 1(1 - 2y) + y(1 - 2y)$

$= (1 - 2y)(1 + y)$.....(iii)

In (i), (ii) and (iii)

Product of common factors = $(1 + y)$.....(iv)

Product of uncommon factors = $(1 - y)(1 - 2y)(y^2 - y + 1)$ ...(v)

L.C.M = (iv) × (v)

$= (1 + y)(1 - y)(1 - 2y)(y^2 - y + 1)$

Q.10 $x^2 - y^2$, $x^4 - y^4$, $x^6 - y^6$

Sol:

Factorization of $x^2 - y^2 = (x + y)(x - y)$.....(i)

$= (x)^2 - (y)^2$

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)[(x)^2 - (y)^2]$

$= (x^2 + y^2)(x + y)(x - y)$.....(ii)

Factorization of $x^6 - y^6 = (x^3)^2 - (y^3)^2$

$= (x^3 + y^3)(x^3 - y^3)$

$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$(iii)

In (i), (ii) and (iii)

Product of common factors = $(x + y)(x - y)$.....(iv)

Product of uncommon factors = $(x^2+y^2)(x^2-xy+y^2)(x^2+xy+y^2)$ (v)

L.C.M = (iv) × (v)

$= (x + y)(x - y)(x^2 + y^2)$

$$(x^2 - xy + y^2)(x^2 + xy + y^2)$$
$$= (x + y)(x - y)(x^2 + y^2)$$
$$(x^4 + x^2y^2 + y^4)$$

Q.11 $x^3 + 1$, $x^4 + x^2 + 1$, $(x^2 + x + 1)^2$

Sol:

Factorization of $x^3 + 1 = (x)^3 + (1)^3$

$= (x + 1)(x^2 - x + 1)$.....(i)

Factorization of $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$ (completing square)

$= (x^2 + 1)^2 - (x)^2$

$= (x^2 + 1 + x)(x^2 + 1 - x)$

$= (x^2 + x + 1)(x^2 - x + 1)$.....(ii)

Factorization of $(x^2 + x + 1)^2 = (x^2 + x + 1)(x^2 + x + 1)$.....(iii)

In (i), (ii) and (iii)

Product of common factors $= (x^2 + x + 1)(x^2 - x + 1)$....(iv)

Product of uncommon factors $= (x + 1)(x^2 + x + 1)$.....(v)

L.C.M $= (iv) \times (v)$

$= (x^2 + x + 1)(x^2 - x + 1)(x + 1)(x^2 + x + 1)$

$= (x + 1)(x^2 - x + 1)(x^2 + x + 1)^2$

Q.12 $x^3 + y^3$, $x^4 - y^4$, $x^6 + y^6$

Sol:

Factorization of $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.....(i)

Factorization of $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$

$= (x^2 + y^2)\left[(x^2) - (y)^2\right]$

$= (x^2 + y^2)(x + y)(x - y)$.....(ii)

Factorization of $x^6 + y^6 = (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)[(x^2)^2 - (x^2)(y^2) + (y^2)^2]$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.....(iii)

In (i), (ii) and (iii)

Product of common factors = $(x + y)(x^2 + y^2)$.....(iv)

Product of uncommon factors = $(x^2-xy+y^2)(x-y)(x^4-x^2y^2+y^4)$..(v)

$$\text{L.C.M} = \text{(iv)} \times \text{(v)}$$

$$= (x + y)(x^2 + y^2)(x^2 - x + y^2)(x - y)(x^4 - x^2y^2 + y^4)$$

$$= (x + y)(x - y)(x^2 + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$$

Q.13 $2x^2 + 5x + 3,\ x^2 + 2x + 1,\ 2x^2 + 9x + 9$

Sol:

Factorization of $2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$

$$= (2x^2 + 2x) + (3x + 3)$$

$$= 2x(x + 1) + 3(x + 1)$$

$$= (x + 1)(2x + 3)\text{.....(i)}$$

Factorization of $x^2 + 2x + 1 = x^2 + x + x + 1$

$$= (x^2 + x) + (x + 1)$$

$$= x(x + 1) + 1(x + 1)$$

$$= (x + 1)(x + 1)\text{.....(ii)}$$

Factorization of $2x^2 + 9x + 9 = 2x^2 + 3x + 6x + 9$

$$= (2x^2 + 3x) + (6x + 9)$$

$$= x(2x + 3) + 3(2x + 3)$$

$$= (2x + 3)(x + 3)\text{.....(iii)}$$

In (i), (ii) and (iii)

Product of common factors = $(x + 1)(2x + 3)$.....(iv)

Product of uncommon factors = $(x + 1)(x + 3)$.....(v)

$$\text{L.C.M} = \text{(iv)} \times \text{(v)}$$

$$= (x + 1)(2x + 3)(x + 1)(x + 3)$$

$$= (x + 1)^2(2x + 3)(x + 3)$$

Q.14 $x^4 + x^3 - 6x^2$, $x^4 - 9x^2$, $x^3 + x^2 - 6x$

Sol:

Factorization of $x^4 + x^3 - 6x^2 = x^2(x^2 + x - 6)$

$= x^2(x^2 + 3x - 2x - 6)$

$= x^2[(x^2 + 3x) - (2x + 6)]$

$= x^2[x(x + 3) - 2(x + 3)]$

$= x^2(x + 3)(x - 2)$.....(i)

Factorization of $x^4 - 9x^2 = x^2(x^2 - 9)$

$= x^2[(x^2 - 3^2)]$

$= x^2(x + 3)(x - 3)$.....(ii)

Factorization of $x^3 + x^2 - 6x = x(x^2 + x - 6)$

$= x(x^2 + 3x - 2x - 6)$

$= x[x(x + 3) - 2(x + 3)]$

$= x(x + 3)(x - 2)$.....(iii)

In (i), (ii) and (iii)

Product of common factors $= x^2(x + 3)(x - 2)$.....(iv)

Product of uncommon factors $= (x - 3)$..... (v)

L.C.M = (iv) × (v)

$= x^2(x + 3)(x - 2)(x - 3)$

Q.15 $x^2 + 4xy + 4y^2$, $x^2 + 3xy + 2y^2$, $x^2 + 2xy + y^2$

Sol:

Factorization of $x^2 + 4xy + 4y^2 = x^2 + 2xy + 2xy + 4y^2$

$= (x^2 + 2xy) + (2xy + 4y^2)$

$= x(x + 2y) + 2y(x + 2y)$

$= (x + 2y)(x + 2y)$.....(i)

Factorization of $x^2 + 3xy + 2y^2 = x^2 + xy + 2xy + 2y^2$

$= (x^2 + xy) + (2xy + 2y^2)$

$= x(x + y) + 2y(x + y)$

$= (x + y)(x + 2y)$.....(ii)

Factorization of $x^2 + 2xy + y^2 = x^2 + xy + xy + y^2$

$= (x^2 + xy) + (xy + y^2)$

$= x(x + y) + y(x + y)$

$= (x + y)(x + y)$.....(iii)

In (i), (ii) and (iii)

Product of common factors = $(x + 2y)(x + y)$.....(iv)

Product of uncommon factors = $(x + 2y)(x + y)$.....(v)

L.C.M = (iv) × (v)

$= (x + 2y)(x + y)(x + 2y)(x + y)$

$= (x + y)(x + y)(x + 2y)(x + 2y)$

$= (x + y)^2(x + 2y)^2$

Relationship between HCF and LCM

If A and B are two algebraic expressions and ***H.C.F*** and ***L.C.M*** of these is represented by H and L respectively, then the relation among them can be expressed as:

$$\boxed{A \times B = H \times L}$$

It is called a formula between ***L.C.M*** and ***H.C.F.***

PROOF: Suppose that

$$\frac{A}{H} = x \qquad \text{and} \qquad \frac{B}{H} = y$$

$A = Hx$ .....(i)

$B = Hy$ .....(ii)

Since there is no common factor between x and y.

Therefore $L = H.\,x.\,y$

$HL = H\,(H.x.y)$ (multiplying both the sides by H)

$= (Hx).\,(Hy)$

$HL = A.B.$

$$(i) \quad L = \frac{A \times B}{H}$$

$$(ii) \quad H = \frac{A \times B}{L}$$

$$(iii) \quad A = \frac{H \times L}{B}$$