Exercise 3.4

Find the H.C.F and L.C.M of the following.

Q.1 $x^3 + x^2 + x + 1, x^3 - x^2 + x - 1$

Sol:

First we find H.C.F

$$
\begin{array}{r|l}
 & \qquad 1 \\
\hline
x^3 + x^2 + x + 1 & x^3 - x^2 + x - 1 \\
 & \pm x^3 \pm x^2 \pm x \pm 1 \\
\hline
-2 & -2x^2 - 2
\end{array}
$$

$$
\begin{array}{r|l}
 & \qquad x + 1 \\
\hline
x^2 + 1 & x^3 + x^2 + x + 1 \\
 & \pm x^3 \qquad \pm x \\
\hline
 & \quad x^2 \qquad + 1 \\
 & \pm x^2 \qquad \pm 1 \\
\hline
 & \qquad 0
\end{array}
$$

H.C.F $= x^2 + 1$

Now we will find L.C.M

$$\text{L.C.M} = \frac{(x^3 + x^2 + x + 1)\overset{(x-1)}{\cancel{(x^3 - x^2 + x - 1)}}}{\cancel{(x^2+1)}}$$

$$= (x^3 + x^2 + x + 1)(x - 1)$$

$$= x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1$$

L.C.M $= x^4 - 1$

Working

$$
\begin{array}{r|l}
 & \qquad x - 1 \\
\hline
x^2 + 1 & x^3 - x^2 + x - 1 \\
 & \pm x^3 \qquad \pm x \\
\hline
 & -x^2 \qquad -1 \\
 & +x^2 \qquad +1 \\
\hline
 & \qquad 0
\end{array}
$$

Q.2 $x^3 - 3x^2 - 4x + 12,\ x^3 - x^2 - 4x + 4$

Sol: First we find H.C.F

$$\begin{array}{r|l} & \quad 1 \\ x^3 - 3x^2 - 4x + 12 & x^3 - x^2 - 4x + 4 \\ & \pm x^3 \mp 3x^2 \mp 4x \pm 12 \\ \hline 2 & 2x^2 - 8 \qquad x - 3 \end{array}$$

$$\begin{array}{r|l} x^2 - 4 & x^3 - 3x^2 - 4x + 12 \\ & \pm x^3 \qquad \mp 4x \\ \hline & -3x^2 \qquad + 12 \\ & \mp 3x^2 \qquad \pm 12 \\ \hline & \qquad 0 \end{array}$$

H.C.F $= x^2 - 4$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{x-3}{\cancel{(x^3 - 3x^2 - 4x + 12)}}(x^3 - x^2 - 4x + 4)}{\cancel{(x^2 - 4)}}$$

$= (x - 3)(x^3 - x^2 - 4x + 4)$

$= x^4 - x^3 - 4x^2 + 4x - 3x^3 + 3x^2 + 12x - 12$

$= x^4 - 4x^3 - x^2 + 16x - 12$

Q.3 $2x^3 + 2x^2 + x + 1, 2x^3 - 2x^2 + x - 1$

Sol: First we find H.C.F

$$\begin{array}{r|l} & \quad 1 \\ 2x^3 + 2x^2 + x + 1 & 2x^3 - 2x^2 + x - 1 \\ & \pm 2x^3 \pm 2x^2 \pm x \pm 1 \\ \hline -2 & -4x^2 - 2 \qquad x + 1 \end{array}$$

$$
\begin{array}{r|l}
2x^2+1 & 2x^3+2x^2+x+1 \\
 & \pm 2x^3 \qquad \pm x \\
\hline
 & \qquad 2x^2 \qquad +1 \\
 & \quad \pm 2x^2 \qquad \pm 1 \\
\hline
 & \qquad 0
\end{array}
$$

H.C.F $= 2x^2 + 1$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{x+1}{\cancel{(2x^3+2x^2+x+1)}}(2x^3-2x^2+x-1)}{\cancel{(2x^2+1)}}$$

$= (x+1)(2x^3 - 2x^2 + x - 1)$

$= 2x^4 - 2x^3 + x^2 - x + 2x^3 - 2x^2 + x - 1$

$= 2x^4 - x^2 - 1$

Q.4 $6x^3 + 7x^2 - 9x + 2,\ 8x^4 + 6x^3 - 15x^2 + 9x - 2$

Sol:

First we find H.C.F

$$
\begin{array}{r|l}
 & \qquad 4x+5 \\
6x^3+7x^2-9x+2 & 8x^4+6x^3-15x^2+9x-2 \\
 & \times 3 \\
\hline
 & 24x^4+18x^3-45x^2+27x-6 \\
 & \pm 24x^4 \pm 28x^3 \mp 36x^2 \pm 8x \\
\hline
 & -1 \,|\, -10x^3-9x^2+19x-6 \\
\hline
 & 10x^3+9x^2-19x+6 \\
 & \times 3 \\
\hline
 & 30x^3+27x^2-57x+18 \\
 & \pm 30x^3 \pm 35x^2 \mp 45x \pm 10
\end{array}
$$

$$-1\,\big|\,-8x^2-12x+8$$

$$4\,\big|\,8x^2+12x-8 \qquad 3x-1$$

$$2x^2+3x-2\,\big|\,6x^3+7x^2-9x+2$$
$$\pm 6x^3 \pm 9x^2 \mp 6x$$
$$-2x^2-3x+2$$
$$\mp 2x^2 \mp 3x \pm 2$$
$$0$$

H.C.F $= 2x^2+3x-2$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{3x-1}{\cancel{(6x^3+7x^2-9x+2)}}(8x^4+6x^3-15x^2+9x-2)}{\cancel{(2x^2+3x-2)}}$$

$$= (3x-1)(8x^4+6x^3-15x^2+9x-2)$$

Q.5 $3x^4+17x^3+27x^2+7x-6,\ 6x^4+7x^3-27x^2+17x-3$

Sol: First we find H.C.F

$$2$$
$$3x^4+17x^3+27x^2+7x-6\,\big|\,6x^4+7x^3-27x^2+17x-3$$
$$\pm 6x^4 \pm 34x^3 \pm 54x^2 \pm 14x \mp 12$$
$$-3\,\big|\,-27x^3-81x^2+3x+9$$
$$9x^3+27x^2-x-3$$

$$x+2+2$$
$$9x^3+27x^2-x-3\,\big|\,3x^4+17x^3+27x^2+7x-6$$
$$\times 3$$
$$9x^4+51x^3+81x^2+21x-18$$
$$\pm 9x^4 \pm 27x^3 \mp x^2 \mp 3x$$

$$24x^3+82x^2+24x-18$$
$$\pm 18x^3 \pm 54x^2 \mp 2x \mp 6$$
$$6x^3+28x^2+26x-12$$
$$\times 3$$
$$18x^3+84x^2+78x-36$$
$$\pm 18x^3 \pm 54x^2 \mp 2x \mp 6$$
$$2\,|\,30x^2+80x-30$$
$$5\,|\,15x^2+40x-15 \qquad 3x+1$$

$$3x^2+8x-3 \,|\, 9x^3+27x^2-x-3$$
$$\pm 9x^3 \pm 24x^2 \mp 9x$$
$$3x^2+8x-3$$
$$\pm 3x^2 \pm 8x \mp 3$$
$$0$$

H.C.F $= 3x^2+8x-3$

Now we will find L.C.M

$$\text{L.C.M} = \frac{(3x^4+17x^3+27x^2+7x-6)\overset{2x^2-3x+1}{\cancel{(6x^4+7x^3-27x^2+17x-3)}}}{\cancel{(3x^2+8x-3)}}$$

$$= (2x^2-3x+1)(3x^4+17x^3+27x^2+7x-6)$$

Working

$$2x^2-3x+1$$
$$3x^2+8x-3 \,\big|\, 6x^4+7x^3-27x^2+17x-3$$
$$\pm 6x^4 \pm 16x^3 \mp 6x^2$$
$$-9x^3-21x^2+17x-3$$
$$\mp 9x^3 \mp 24x^2 \pm 9x$$
$$3x^2+8x-3$$
$$\pm 3x^2 \pm 8x \mp 3$$
$$0$$

Q.6 $2x^4 + 3x^3 - 13x^2 - 7x + 15,\ 2x^4 + x^3 - 20x^2 - 7x + 24$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & \quad 1 \\
\hline
2x^4+3x^3-13x^2-7x+15 & 2x^4 + x^3 - 20x^2 - 7x + 24 \\
 & \pm 2x^4 \pm 3x^3 \mp 13x^2 \mp 7x \pm 15 \\
\hline
-1 & -2x^3 - 7x^2 + 9 \qquad\qquad x - 2
\end{array}
$$

$$
\begin{array}{r|l}
2x^3 + 7x^2 - 9 & 2x^4+3x^3-13x^2-7x+15 \\
 & \pm 2x^4 \pm 7x^3 \qquad \mp 9x \\
\hline
 & -4x^3-13x^2+2x+15 \\
 & \mp 4x^3 \mp 14x^2 \quad \pm 18 \\
\hline
 & x^2 + 2x - 3
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad 2x^2 - x - 5 \\
\hline
x^2 + 2x - 3 & 2x^4 + 3x^3 - 13x^2 - 7x + 15 \\
 & \pm 2x^4 \pm 4x^3 \mp 6x^2 \\
\hline
 & -x^3 - 7x^2 - 7x + 15 \\
 & \mp x^3 \mp 2x^2 \pm 3x \\
\hline
 & -5x^2 - 10x + 15 \\
 & \mp 5x^2 \mp 10x \pm 15 \\
\hline
 & 0
\end{array}
$$

H.C.F $= x^2 + 2x - 3$

Now we will find L.C.M

$$\text{L.C.M} = \frac{\overset{2x^2 - x - 5}{\cancel{(2x^4 + 3x^3 - 13x^2 - 7x + 15)}}(2x^4 + x^3 - 20x^2 - 7x + 24)}{\cancel{x^2 + 2x - 3}}$$

$$= (2x^2 - x - 5)(2x^4 + x^3 - 20x^2 - 7x + 24)$$

Q.7 $x^4 - x^3 - x + 1,\ x^4 + x^3 - x - 1$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & \quad 1 \\
x^4 - x^3 - x + 1 & x^4 + x^3 - x - 1 \\
 & \pm x^4 \mp x^3 \mp x \pm 1 \\
\hline
2 & 2x^3 - 2
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad x - 1 \\
x^3 - 1 & x^4 - x^3 - x + 1 \\
 & \pm x^4 \qquad \mp x \\
\hline
 & \quad - x^3 + 1 \\
 & \quad - x^3 + 1 \\
\hline
 & \quad 0
\end{array}
$$

H.C.F $= x^3 - 1$

Now we will find L.C.M

$$
\text{L.C.M} = \frac{\overset{(x-1)}{\cancel{(x^4 - x^3 - x + 1)}}(x^4 + x^3 - x - 1)}{\cancel{(x^3 - 1)}}
$$

$$= (x - 1)(x^4 + x^3 - x - 1)$$

Q.8 $x^4 + x^3 + x + 1,\ x^4 + x^3 - x - 1$

Sol: First we find H.C.F

$$
\begin{array}{r|l}
 & \quad 1 \\
x^4 + x^3 + x + 1 & x^4 + x^3 - x - 1 \\
 & \pm x^4 \pm x^3 \pm x \pm 1 \\
\hline
-2 & -2x - 2
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad x^3 + 1 \\
x + 1 & x^4 + x^3 + x + 1 \\
 & \pm x^4 \pm x^3 \\
\hline
 & \quad x + 1 \\
 & \quad \pm x \pm 1 \\
\hline
 & \quad 0
\end{array}
$$

H.C.F $= x + 1$

Now we will find L.C.M.

$$\text{L.C.M} = \frac{\overset{(x^3+1)}{\cancel{(x^4+x^3+x+1)}}(x^4+x^3-x-1)}{\cancel{(x+1)}}$$

$$= (x^3+1)(x^4+x^3-x+1)$$

Find the Required Polynomial.

Q.9. **$A = x^2 - 5x - 14$, $H = x-7$, $L = x^3 - 10x^2 + 11x + 70$, $B = ?$**

Sol: Formula: $B = \dfrac{H \times L}{A}$

$$= \frac{(x-7)\overset{(x-5)}{\cancel{(x^3-10x^2+11x+70)}}}{\cancel{(x^2-5x-14)}}$$

$$= (x-7)(x-5)$$

$$= x^2 - 12x + 35$$

Working

$$\begin{array}{r|l} & \quad x - 5 \\ \hline x^2 - 5x - 14 & x^3 - 10x^2 + 11x + 70 \\ & \pm x^3 \mp 5x^2 \mp 14x \\ \hline & \quad -5x^2 + 25x + 70 \\ & \quad \mp 5x^2 \pm 25x \pm 70 \\ \hline & \qquad 0 \end{array}$$

Q.10. $B = 3x^2 + 14x + 8$, $H = 3x + 2$, $L = 6x^3 + 25x^2 + 2x - 8$, $A = ?$

Sol: Formula: $A = \dfrac{H \times L}{B}$

$$= \frac{(3x+2)\overset{(2x-1)}{\cancel{(6x^3+25x^2+2x-8)}}}{\cancel{(3x^2+14x+8)}}$$

$= (3x + 2)(2x - 1)$

$= 6x^2 + x - 2$

Working

$$\begin{array}{r|l} & 2x - 1 \\ \hline 3x^2 + 14x + 18 & 6x^3 + 25x^2 + 2x - 8 \\ & \pm 6x^3 \pm 28x^2 \pm 16x \\ \hline & -3x^2 - 14x - 8 \\ & +3x^2 + 14x + 8 \\ \hline & 0 \end{array}$$

Q.11. The product of two polynomials and their L.C.M. are $x^4 + 6x^3 - 3x^2 - 56x - 48$ and $x^3 + 2x^2 - 11x - 12$ respectively. Find their H.C.F.

Sol:

$A \times B = x^4 + 6x^3 - 3x^2 - 56x - 48$

$H = x^3 + 2x^2 - 11x - 12$

$L = ?$

Formula: $L = \dfrac{A \times B}{H}$

$$= \frac{x^4 + 6x^3 - 3x^2 - 56x - 48}{x^3 + 2x^2 - 11x - 12}$$

$= x + 4$

Working

$$\begin{array}{r|l} & x + 4 \\ \hline x^3 + 2x^2 - 11x - 12 & x^4 + 6x^3 - 3x^2 - 56x - 48 \\ & \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\ \hline & +4x^3 + 8x^2 - 44x - 48 \\ & \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\ \hline & 0 \end{array}$$

Q.12. The product of two polynomials and their L.C.M. are $x^4 + 5x^3 - x^2 - 17x + 12$ and $x^3 + 6x^2 + 5x - 12$ respectively.
Find their H.C.F.

Sol: $A \times B = x^4 + 5x^3 - x^2 - 17x + 12$

$L = x^3 + 6x^2 + 5x - 12$

$H = ?$

Formula $H = \dfrac{A \times B}{L}$

$$= \frac{x^4 + 5x^3 - x^2 - 17x + 12}{x^3 + 6x^2 + 5x - 12}$$

$H = x - 1$

Working

$$\begin{array}{r|l} & x - 1 \\ \hline x^3 + 6x^2 + 5x - 12 & x^4 + 5x^3 - x^2 - 17x + 12 \\ & \pm x^4 \pm 6x^3 \pm 5x^2 \mp 12x \\ \hline & -x^3 - 6x^2 - 5x + 12 \\ & \underline{+}\,x^3 \underline{+}\,6x^2 \underline{+}\,5x \pm 12 \\ \hline & 0 \end{array}$$

Q.13. The product of two polynomials and their H.C.F. are $x^4 - 12x^3 + 53x^2 - 102x + 72$ and $x - 3$ respectively. Find L.C.M.

Sol:

$A \times B = x^4 - 12x^3 + 53x^2 - 102x + 72$

$L = x - 3$

$H = ?$

Formula: $H = \dfrac{A \times B}{L}$

$$= \frac{x^4 - 12x^3 + 53x^2 - 102x + 72}{(x - 3)}$$

H $= x^3 - 9x^2 + 26x - 24$

Working

$$
\begin{array}{r|l}
 & x^3 - 9x^2 + 26x - 24 \\
\hline
x - 3 & x^4 - 12x^3 + 53x^2 - 102x + 72 \\
 & \pm x^4 \mp 3x^3 \\
\hline
 & - 9x^3 + 53x^2 - 102x + 72 \\
 & \mp 9x^3 \pm 27x^2 \\
\hline
 & 26x^2 - 102x + 72 \\
 & \pm 26x^3 \mp 78x \\
\hline
 & - 24x + 72 \\
 & \mp 24x \pm 72 \\
\hline
 & 0
\end{array}
$$

Q.14. The product of two polynomials and their H.C.F. is $x^4 - 5x^3 + 2x^2 + 20x - 24$ and $x + 2$ respectively. Find their L.C.M.

Sol:

A × B $= x^4 - 5x^3 + 2x^2 + 20x - 24$

H $= x + 2$

L $= ?$

Formula $L = \dfrac{A \times B}{H}$

$$= \frac{x^4 - 5x^3 + 2x^2 + 20x - 24}{(x+2)}$$

H $= x^3 - 7x^2 + 16x - 12$

Working

$$
\begin{array}{r|l}
 & x^3 - 7x^2 + 16x - 12 \\
\hline
x + 2 & x^4 - 5x^3 + 2x^2 + 20x - 24 \\
 & \pm x^4 \pm 2x^3 \\
\hline
 & - 7x^3 + 2x^2 + 20x - 24
\end{array}
$$

$$\mp 7x^3 \mp 14x^2$$

$$+16x^2 + 20x - 24$$
$$\pm 16x^3 \pm 32x$$

$$-12x - 24$$
$$\mp 12x \mp 24$$

$$0$$

Q.15. One algebraic expression is $x^3 + 3x^2 - 4x - 12$ and $x^3 + 5x^2 - 4x - 20$ other one is . Their H.C.F is $x^2 - 4$. Find their L.C.M.

Sol:

A $= x^3 + 3x^2 - 4x - 12$

B $= x^3 + 5x^2 - 4x - 20$

H $= x^2 - 4$

L $= ?$

Formula: $L = \dfrac{A \times B}{H}$

$$= \frac{\cancel{(x^3 + 3x^2 - 4x - 12)}^{x+3}(x^3 + 5x^2 - 4x - 20)}{\cancel{(x^2 - 4)}}$$

$$= (x+3)(x^3 + 5x^2 - 4x - 20)$$

$$= x^4 + 8x^3 + 11x^2 - 32x - 60$$

Working

$$x + 3$$

$$x^2 - 4 \,\big|\, x^3 + 3x^2 - 4x - 12$$
$$\pm x^3 \quad \mp 4x$$

$$3x^2 \quad - 12$$
$$\pm 3x^3 \quad \mp 12$$

$$0$$

Q.16. One algebraic expression is $x^3 - x^2 + 2x - 2$ and other one is $x^3 - x^2 - 2x + 2$. Their H.C.F is $x - 1$. Find their L.C.M.

Sol:

A $= x^3 - x^2 + 2x - 2$

B $= x^3 - x^2 - 2x + 2$

H $= x - 1$

L $= ?$

Formula: $L = \dfrac{A \times B}{H}$

$$= \frac{\overset{(x^2+2)}{\cancel{(x^3 - x^2 + 2x - 2)}}(x^3 - x^2 - 2x + 2)}{\cancel{(x-1)}}$$

$$= (x^2 + 2)(x^3 - x^2 - 2x + 2)$$

$$= x^5 - x^4 - 2x + 4$$

Working

$$\begin{array}{r|l} & x^2 + 2 \\ \hline x - 1 & x^3 - x^2 + 2x - 2 \\ & \pm x^3 \mp x^2 \\ \hline & 2x^2 - 2 \\ & \pm 2x^2 \mp 2 \\ \hline & 0 \end{array}$$

Q.17. Prove that $H^3 + L^3 = A^3 + B^3$ where $H + L = A + B$ 'H' and 'L' stand for H.C.F and L.C.M respectively and 'A, B' represent two polynomials.

Sol:

Proof: We know th(

$$H + L = A + B$$

$$(H + L)^3 = (A + B)^3 \quad \text{Taking cube}$$

$$H^3 + L^3 + 3HL(H + L) = A^3 + B^3 + 3AB(A + B)$$

$$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3HL(H + L) \quad \text{(i)}$$

Now $H + L = A + B$

and $H \times L = A \times B$

Putting values in (i) H + L and HL

$$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3AB(A + B)$$

$$H^3 + L^3 = A^3 + B^3 \quad \text{Proved}$$

Exercise 3.5

Simplify

Q.1 $\dfrac{1}{a} + \dfrac{2}{a+1} - \dfrac{3}{a+2}$

Sol: $= \dfrac{1(a+1)(a+2) + 2(a)(a+2) - 3(a)(a+1)}{(a)(a+1)(a+2)}$

$= \dfrac{a^2 + 3a + 2 + 2a^2 + 4a - 3a^2 - 3a}{(a)(a+1)(a+2)}$

$= \dfrac{4a+2}{(a)(a+1)(a+2)}$

$= \dfrac{2(2a+1)}{(a)(a+1)(a+2)}$

Q.2 $\dfrac{2a}{(x-2a)} - \dfrac{x-a}{x^2 - 5ax + 6a^2} + \dfrac{2}{x-3a}$

Sol: $= \dfrac{2a}{(x-2a)} - \dfrac{x-a}{x^2 - 2ax - 3ax + 6a^2} + \dfrac{2}{x-3a}$