$H + L = A + B$

$(H + L)^3 = (A + B)^3$ Taking cube

$H^3 + L^3 + 3HL(H + L) = A^3 + B^3 + 3AB(A + B)$

$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3HL(H + L)$ (i)

Now $H + L = A + B$

and $H \times L = A \times B$

Putting values in (i) H + L and HL

$H^3 + L^3 = A^3 + B^3 + 3AB(A + B) - 3AB(A + B)$

$H^3 + L^3 = A^3 + B^3$ Proved

## Exercise 3.5

*Simplify*

**Q.1** $\frac{1}{a} + \frac{2}{a+1} - \frac{3}{a+2}$

Sol: $= \frac{1(a+1)(a+2) + 2(a)(a+2) - 3(a)(a+1)}{(a)(a+1)(a+2)}$

$= \frac{a^2 + 3a + 2 + 2a^2 + 4a - 3a^2 - 3a}{(a)(a+1)(a+2)}$

$= \frac{4a + 2}{(a)(a+1)(a+2)}$

$= \frac{2(2a+1)}{(a)(a+1)(a+2)}$

**Q.2** $\frac{2a}{(x-2a)} - \frac{x-a}{x^2 - 5ax + 6a^2} + \frac{2}{x-3a}$

Sol: $= \frac{2a}{(x-2a)} - \frac{x-a}{x^2 - 2ax - 3ax + 6a^2} + \frac{2}{x-3a}$

$$= \frac{2a}{(x-2a)} - \frac{x-a}{x(x-2a)-3a(ax-2a)} + \frac{2}{x-3a}$$

$$= \frac{2a}{(x-2a)} - \frac{x-a}{(x-2a)(x-3a)} + \frac{2}{x-3a}$$

$$= \frac{2a(x-3a)-(x-a)+2(x-2a)}{(x-2a)(x-3a)}$$

$$= \frac{2ax-6a^2-x+a+2x-4a}{(x-2a)(x-3a)}$$

$$= \frac{2ax+x-3a-6a^2}{(x-2a)(x-3a)}$$

**Q.3** $\frac{1}{a^2+1} - \frac{a^4}{a^2+1} + \frac{a^6}{a^2-1} - \frac{1}{a^2-1}$

**Sol:** $= \frac{1(a^2-1)-a^4(a^2-1)+a^6(a^2+1)-1(a^2+1)}{(a^2+1)(a^2-1)}$

$$= \frac{a^2-1-a^6+a^4+a^8+a^6-a^2-1}{(a^2+1)(a^2-1)}$$

$$= \frac{a^8+a^4-2}{(a^2+1)(a^2-1)}$$

$$= \frac{a^8+2a^4-a^4-2}{(a^2+1)(a^2-1)}$$

$$= a^4(a^4+2)-1(a^4+2)$$

$$= \frac{(a^4+2)(a^4-1)}{(a^4-1)}$$

$$= a^4+2$$

**Q.4** $\dfrac{1}{x^2+x+1}-\dfrac{1}{x^2-x+1}+\dfrac{2x+1}{x^4+x^2+1}$

Sol: $=\dfrac{1}{x^2+x+1}-\dfrac{1}{x^2-x+1}+\dfrac{2x+1}{(x^2+x+1)(x^2-x+1)}$

$=\dfrac{1(x^2-x+1)-1(x^2+x+1)+2x+1}{(x^2+x+1)(x^2-x+1)}$

$=\dfrac{x^2-x+1-x^2-x-1+2x+1}{(x^2+x+1)(x^2-x+1)}$

$=\dfrac{1}{(x^2+x+1)(x^2-x+1)}$

$=\dfrac{1}{(x^4+x^2+1)}$

**Q.5** $\dfrac{a^2(b-c)}{(a+b)(a+c)}-\dfrac{b^2(c-a)}{(b+c)(b+a)}+\dfrac{c^2(a-b)}{(c+a)(c+b)}$

Sol: $=\dfrac{a^2(b-c)(b+c)-b^2(c-a)(c+a)+c^2(a-b)(a+b)}{(a+b)(a+c)(b+c)}$

$=\dfrac{a^2(b^2-c^2)-b^2(c^2-a^2)+c^2(a^2-b^2)}{(a+b)(a+c)(b+c)}$

$=\dfrac{a^2b^2-a^2c^2-b^2c^2+a^2b^2+a^2c^2-b^2c^2}{(a+b)(a+c)(b+c)}$

$=\dfrac{2a^2b^2-2b^2c^2}{(a+b)(a+c)(b+c)}$

$=\dfrac{2b^2(a^2-c^2)}{(a+b)(a+c)(b+c)}$

$=\dfrac{2b^2(a+c)(a-c)}{(a+b)(a+c)(b+c)}$

$=\dfrac{2b^2(a-c)}{(a+b)(b+c)}$

**Q.6** $\dfrac{1}{x-1}+\dfrac{1}{x+1}-\dfrac{x+2}{x^2+x+1}-\dfrac{x-2}{x^2-x+1}$

Sol: Changing order

$$=\left(\frac{1}{x-1}-\frac{x+2}{x^2+x+1}\right)+\left(\frac{1}{x+1}-\frac{x-2}{x^2-x+1}\right)$$

$$=\frac{(x^2+x+1)-(x-1)(x+2)}{(x-1)(x^2+x+1)}+\frac{(x^2-x+1)-(x-2)(x+1)}{(x+1)(x^2-x+1)}$$

$$=\frac{(x^2+x+1)-(x^2+x-2)}{x^3-1}+\frac{(x^2-x+1)-(x^2-x-2)}{x^3+1}$$

$$=\frac{x^2+x+1-x^2-x+2}{x^3-1}+\frac{x^2-x+1-x^2+x+2}{x^3+1}$$

$$=\frac{3}{x^3-1}+\frac{3}{x^3+1}$$

$$=\frac{3(x^3+1)+3(x^3-1)}{(x^3-1)(x^3+1)}$$

$$=\frac{3x^3+3+3x^3-3}{x^6-1}$$

$$=\frac{6x^3}{x^6-1}$$

**Q.7** $\dfrac{a^2+ab+b^2}{a+b}+\dfrac{a^2-ab+b^2}{a-b}$

Sol: $$=\frac{(a-b)(a^2+ab+b^2)+(a+b)(a^2-ab+b^2)}{(a+b)(a-b)}$$

$$=\frac{(a^3-b^3)+(a^3+b^3)}{a^2-b^2}$$

$$=\frac{a^3-b^3+a^3+b^3}{a^2-b^2}$$

$$=\frac{2a^3}{a^2-b^2}$$

**Q.8** $\dfrac{x^4-y^4}{x^2-2xy+y^2}\times\dfrac{x-y}{x(x+y)}\div\dfrac{x^2+y^2}{x}$

Sol: $=\dfrac{(x^2+y^2)(x^2-y^2)}{(x-y)^2}\times\dfrac{x-y}{x(x+y)}\times\dfrac{x}{(x^2+y^2)}$

$=\dfrac{(x+y)(x-y)}{(x-y)(x-y)}\times\dfrac{(x-y)}{x(x+y)}\times x$

$=1$

**Q.9** $\dfrac{x^2-1}{x^2+x-2}\times\dfrac{x^3+8}{x^4+4x^2+16}\div\dfrac{x^2+x}{x^3+2x^2+4x}$

Sol: $=\dfrac{(x+1)(x-1)}{x^2+2x-x-2}\times\dfrac{(x)^3+(2)^3}{x^4+4x^2+16}\times\dfrac{x^3+2x^2+4x}{x^2+x}$

$=\dfrac{(x+1)(x-1)}{x(x+2)-1(x+2)}\times\dfrac{(x+2)(x^2-2x+4)}{x^4+8x^2+16-4x^2}\times\dfrac{x(x^2+2x+4)}{x(x+1)}$

(Completing square)

$=\dfrac{(x+1)(x-1)}{(x+2)(x-1)}\times\dfrac{(x+2)(x^2-2x+4)}{(x^2+4)^2-(2x)^2}\times\dfrac{x^2+2x+4}{x+1}$

$=\dfrac{(x^2-2x+4)(x^2+2x+4)}{(x^2+4-2x)(x^2+4+2x)}$

$=\dfrac{(x^2-2x+4)(x^2+2x+4)}{(x^2-2x+4)(x^2+2x+4)}$

$=1$

**Q.10** $\dfrac{a^3+64b^3}{a^2+20ab+64b^2}\div\dfrac{a^2-4ab+16b^2}{a^2+4ab+16b^2}\times\dfrac{a^2+12ab-64b^2}{a^3-64b^3}$

Sol:

$=\dfrac{(a)^3+(4b)^3}{a^2+4ab+16ab+64b^2}\times\dfrac{a^2+4ab+16b^2}{a^2-4ab+16b^2}\times\dfrac{a^2+16ab-4ab-64b^2}{(a)^3-(4b)^3}$

$=\dfrac{(a+4b)(a^2-4ab+16b^2)}{a(a+4b)+16b(a+4b)}\times\dfrac{a^2+4ab+16b^2}{a^2-4ab+16b^2}\times\dfrac{a(a+16b)-4b(a+16b)}{(a-4b)(a^2+4ab+16b^2)}$

$=\dfrac{(a+4b)(a^2-4ab+16b^2)}{(a+4b)(a+16b)}\times\dfrac{a^2+4ab+16b^2}{a^2-4ab+16b^2}\times\dfrac{(a+16b)(a-4b)}{(a-4b)(a^2+4ab+16b^2)}$

$=1$

**Q.11** $\dfrac{a}{(a+b)^2-2ab}\times\dfrac{a^4-b^4}{(a+b)^3-3ab(a+b)}\div\dfrac{(a+b)^2-4ab}{(a+b)^2-3ab}$

Sol: $=\dfrac{a}{a^2+b^2+2ab-2ab}\times\dfrac{(a^2)^2-(b^2)^2}{a^3+b^3+3ab(a+b)-3ab(a+b)}\times\dfrac{(a+b)^2-3ab}{(a+b)^2-4ab}$

$=\dfrac{a}{(a^2+b^2)}\times\dfrac{(a^2+b^2)(a^2-b^2)}{a^3+b^3}\times\dfrac{a^2+b^2+2ab-3ab}{a^2+b^2+2ab-4ab}$

$=a\times\dfrac{(a+b)(a-b)}{(a+b)(a^2-ab+b^2)}\times\dfrac{(a^2+b^2-ab)}{a^2+b^2-2ab}$

$=\dfrac{a\times(a-b)}{a^2-2ab+b^2}=\dfrac{a(a-b)}{(a-b)^2}$

$=\dfrac{a(a-b)}{(a-b)(a-b)}$

$=\dfrac{a}{a-b}$

**Q.12** $\dfrac{a^2-1}{a^2-a-2}\div\dfrac{a^2+5a+6}{a^2-5a+6}\div\dfrac{a^2-4a+3}{a^2+4a+3}$

Sol: $=\dfrac{(a)^2-(1)^2}{a^2-2a+a-2}\times\dfrac{a^2-5a+6}{a^2+5a+6}\times\dfrac{a^2+4a+3}{a^2-4a+3}$

$=\dfrac{(a-1)(a+1)}{a(a-2)+1(a-2)}\times\dfrac{a^2-2a-3a+6}{a^2+2a+3a+6}\times\dfrac{a^2+a+3a+3}{a^2-a-3a+3}$

$=\dfrac{(a-1)(a+1)}{(a-2)(a+1)}\times\dfrac{a(a-2)-3(a-2)}{a(a+2)+3(a+2)}\times\dfrac{a(a+1)+3(a+1)}{a(a-1)-3(a-1)}$

$=\dfrac{(a-1)(a+1)}{(a-2)(a+1)}\times\dfrac{(a-2)(a-3)}{(a+2)(a+3)}\times\dfrac{(a+1)(a+3)}{(a-1)(a-3)}$

$=\dfrac{a+1}{a+2}$