Exercise 3.6

Find the Square Root of the Following.

Q.1 $16x^2 + 24xy + 9y^2$

Sol: $= 16x^2 + 12xy + 12xy + 9y^2$

$= (16x^2 + 12xy) + (12xy + 9y^2)$

$= 4x(4x + 3y) + 3y(4x + 3y)$

$= (4x + 3y)(4x + 3y)$

$= (4x + 3y)^2$

$\sqrt{16x^2 + 24xy + 9y^2} = \sqrt{(4x + 3y)^2}$

$= \pm(4x + 3y)$

Q.2 $(x^2 - 7x + 12)(x^2 - 9x + 20)(x^2 - 8x + 15)$

Sol: $= [x^2 - 3x - 4x + 12][x^2 - 4x - 5x + 20][x^2 - 3x - 5x + 15]$

$= [(x^2 - 3x) - (4x - 12)][(x^2 - 4x) - (5x - 20)][(x^2 - 3x) - (5x - 15)]$

$= [x(x - 3) - 4(x - 3)][x(x - 4) - 5(x - 4)][x(x - 3) - 5(x - 5)]$

$= (x - 3)(x - 4)(x - 4)(x - 5)(x - 3)(x - 5)$

$= (x - 3)^2(x - 4)^2(x - 5)^2$

Now we will take square root.

$= \sqrt{(x - 3)^2(x - 4)^2(x - 5)^2}$

$= \pm(x - 3)(x - 4)(x - 5)$

Q.3 $(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$

Sol: $= (x^2 + x + 7x + 7)(2x^2 - 3x + 2x - 3)(2x^2 + 14x - 3x - 21)$

$= [(x^2 + x) + (7x + 7)][(2x^2 - 3x) + (2x - 3)][(2x^2 + 14x) - (3x + 21)]$

$= [x(x + 1) + 7(x + 1)][x(2x - 3) + 1(2x - 3)][2x(x + 7) - 3(x + 7)]$

$= [(x + 1)(x + 7)][(2x - 3)(x + 1)][(x + 7)(2x - 3)]$

$= (x + 1)^2(x + 7)^2(2x - 3)^2$

$= \sqrt{(x + 1)^2(x + 7)^2(2x - 3)^2}$ (Taking square root)

$= \pm(x + 1)(x + 7)(2x - 3)$

Q.4 $x(x+2)(x+4)(x+6)+16$

Sol: $= x(x+6)(x+2)(x+4)+16$ (Rearranging)

$= [x(x+6)][(x+2)(x+4)]+16$

$= (x^2+6x)(x^2+6x-8)+16$......................(i)

Let $x^2+6x = y$

$= (y)(y+8)+16$ from (i)

$= y^2+8y+16$

$= y^2+4y+4y+16$

$= (y^2+4y)+(4y+16)$

$= y(y+4)+4(y+4)$

$= (y+4)(y+4)$

$= (y+4)^2$

Putting values of y in x^2+6x

$= (x^2+6x+4)^2$

$= \sqrt{(x^2+6x+4)^2}$ (Taking square root)

$= \pm(x^2+6x+4)$

Q.5 $(2x+1)(2x+3)(2x+5)(2x+7)+16$

Sol: (Rearranging)

$= (2x+1)(2x+7)(2x+3)(2x+5)+16$

$= [(2x+1)(2x+7)][(2x+3)(2x+5)]+16$

$= [4x^2+16x+7][(4x^2+16x+15]+16$

Let $4x^2+16x = y$

$= (y+7)(y+15)+16$

$= y^2+22y+105+16$

$= y^2+22y+121$

$= y^2+11y+11y+121$

$= (y^2+11y)+(11y+121)$

$= y(y + 11) + 11(y + 11)$

$= (y + 11)(y + 11)$

$= (y + 11)^2$

Putting value of y

$= (4x^2 + 16x + 11)^2$

$= \sqrt{(4x^2 + 16x + 11)^2}$ (Taking square root)

$= \pm(4x^2 + 16x + 11)$

Q.6 $\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27$ **; $x \neq 0$**

Sol: $\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27$ (i)

Let $x + \frac{1}{x} = y$

$x^2 + \frac{1}{x^2} + 2 = y^2$ (Squaring both sides.)

$x^2 + \frac{1}{x^2} = y^2 - 2$

Putting values $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in (i).

$= y^2 - 2 - 10y + 27$

$= y^2 - 10y + 25$

$= y^2 - 5y - 5y + 25$

$= y(y - 5) - 5(y - 5)$

$= (y - 5)(y - 5)$

$= (y - 5)^2$

$= \left(x + \frac{1}{x} - 5\right)^2$

$= \sqrt{\left(x + \frac{1}{x} - 5\right)^2}$ (Taking square root)

$$= \pm\left(x+\frac{1}{x}-5\right)$$

Q.7 $\left(t-\frac{1}{t}\right)^2-4\left(t+\frac{1}{t}\right)+8, (t \neq 0)$

Sol: Let $t+\frac{1}{t}=y$

We know that

$$\left(t-\frac{1}{t}\right)^2=\left(t+\frac{1}{t}\right)^2-4t\times\frac{1}{t}$$

$$=y^2-4$$

Now $t+\frac{1}{t}=y$ and $\left(t+\frac{1}{t}\right)^2=y^2-4$ putting in given values

$= y^2 - 4 - 4y + 8$
$= y^2 - 4y + 4$
$= y^2 - 2y - 2y + 4$
$= y(y - 2) - 2(y - 2)$
$= (y - 2)(y - 2)$
$= (y - 2)^2$

Putting value of y.

$$=\left(t+\frac{1}{t}-2\right)^2$$

$$=\sqrt{\left(t+\frac{1}{t}-2\right)^2} \quad \text{(Taking square root)}$$

$$=\pm\left(t+\frac{1}{t}-2\right)$$

Q.8 $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$; $x \neq 0$

Sol: Let $x+\frac{1}{x}=y$

$$x + \frac{1}{x} = y$$

Squaring both sides

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \quad \text{.............................(i)}$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in the given statement.

$= (y^2 - 2)^2 - 4y^2 + 12$
$= y^4 - 4y^2 + 4 - 4y^2 + 12$
$= y^4 - 8y^2 + 16$
$= y^4 - 4y^2 - 4y^2 + 16$
$= y^2(y^2 - 4) - 4(y^2 - 4)$
$= (y^2 - 4)(y^2 - 4)$
$= (y^2 - 4)^2$

From (i), putting values $y^2 = x^2 + \frac{1}{x^2} + 2$.

$$= \left(x^2 + \frac{1}{x^2} + 2 - 4\right)^2$$

$$= \left(x^2 + \frac{1}{x^2} - 2\right)^2$$

$$= \sqrt{\left(x^2 + \frac{1}{x^2} - 2\right)^2} \quad \text{(Taking square root)}$$

$$= \pm\left(x^2 + \frac{1}{x^2} - 2\right)$$

Q.9 $4x^4 + 12x^3 + 25x^2 + 24x + 16$

Sol:

$$
\begin{array}{r|l}
 & 2x^2 + 3x + 4 \\
\hline
2x^2 & 4x^4 + 12x^3 + 25x^2 + 24x + 16 \\
 & \pm 4x^4 \\
\hline
4x^2 + 3x & 12x^3 + 25x^2 + 24x + 16 \\
 & \pm 12x^3 \pm 9x^2 \\
\hline
4x^2 + 6x + 4 & +16x^2 + 24x + 16 \\
 & \pm 16x^2 \pm 24x \pm 16 \\
\hline
 & 0
\end{array}
$$

Square root $= \pm(2x^2 + 3x + 4)$

Q.10 $\dfrac{9x^2}{4y^2} - \dfrac{3x}{2y} - \dfrac{7}{4} + \dfrac{2y}{3x} + \dfrac{4y^2}{9x^2} \; (x \neq 0,\ y \neq 0)$

Sol:

$$\text{Square root} = \pm\left(\frac{3x}{2y} - \frac{1}{2} - \frac{2y}{3x}\right)$$

Q.11. For what value of x, $x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$ is a complete square, where $x \neq 0$

Sol: First we will find square root

$$x^2 + 2 + \frac{2}{x^2}$$

$$x^2 \;\Big|\; x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$$

$$\pm x^4$$

$$2x^2 + 2 \;\Big|\; 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$$

$$\pm 4x^2 \pm 4$$

$$2x^2 + 4 + \frac{2}{x^2} \;\Big|\; -4 + x + \frac{8}{x^2} + \frac{4}{x^4}$$

$$\pm 4 \pm \frac{8}{x^2} \pm \frac{4}{x^4}$$

$$-8 + x$$

For completing square remainder must be zero.

Therefore $-8 + x = 0$

$\Rightarrow \quad x = 8$

Q.12. If $x^4 + lx^3 + mx^2 + 12x + 9$ is a complete square then find the values of l and m.

Sol:

$$x^2 + 2x + 3$$

$$x^2 \;\Big|\; x^4 + lx^3 + mx^2 + 12x + 9$$

$$\pm x^4$$

$$2x^2 + 2x \;\Big|\; lx^3 + mx^2 + 12x + 9$$

$$\pm 4x^3 \pm 4x^2$$

$$2x^2 + 4x + 3 \;\Big|\; (l-4)x^3 + mx^2 - 4x^2 + 12x + 9$$

$$\pm 6x^2 \pm 12x \pm 9$$

$$(l-4)x^3 + mx^2 - 10x^2$$

For completing square remainder must be zero

$(l - 4)x^3 + (m - 10)x^2 = 0$

Therefore, $l - 4 = 0$ and $m - 10 = 0$

$l = 4 , m = 10$