$$7x^2+8x-11=0$$
(a, b, c)

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{8^2-4(7)(-11)}}{2(7)}$$

UNIT 1
QUADRATIC EQUATIONS

Quadratic Equation (U.B)

(LHR 2014, 16, GRW 2014, FSD 2016, 17, MTN 2015, BWP 2015, D.G.K 2016)

"A polynomial equation, which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation, or an equation of degree two is called quadratic equation".

i.e., $ax^2+bx+c=0$ where $a,b,c\in R$ and $a\neq 0$.

For example: $5x^2+2x+1=0$, $x^2-1=0$ etc.

General or Standard Form of Quadratic Equation (U.B)

Standard form of quadratic equation in one variable x is: $ax^2+bx+c=0$ where $a,b,c\in R$ and $a\neq o$. Here a is the coefficient of x^2, b is coefficient of x and c is constant term.

Pure Quadratic Equation (U.B)

A quadratic equation in which coefficient of 'x' is zero is called pure quadratic equation.

OR

In standard form of quadratic equation $ax^2+bx+c=0$, if $b=0$, then it is called pure quadratic equation. i.e. $ax^2+c=0$

For example: $x^2-1=0$

Linear Equation (U.B)

An equation of degree one is called linear equation, or if $a=0$ in standard form of quadratic equation $ax^2+bx+c=0$ then it reduces to linear equation. i.e. $bx+c=0$ where $b,c\in R\wedge b\neq 0$. For example $3x+2=0$

Methods to Solve a Quadratic Equation (K.B)

(LHR 2017, GRW 2016, 17, SWL 2016, SGD 2013, 14, 15, MTN 2015, 17, RWP, 2016, D.G.K 2014, 17)

There are three methods to solve a quadratic equation.

(i) Factorization method

(ii) Completing square method

(iii) Using quadratic formula

Note (K.B)

- For factorization of $ax^2+bx+c=0$, we make two factors r and s of ac, such that $r+s=b\wedge rs=ac$.
- Cancelling of x on both sides of an equation (for example $5x^2=30x$) means the loss of one root. i.e. $x=0$
- For convenience, in the method of completing square, we make the coefficient of x^2 equal to 1.

Example 1: (Page # 2) **(FSD 2015)** **(A.B)**

Solve the quadratic equation $3x^2-6x=x+20$ by factorization.

Solution:

$3x^2-6x=x+20$

$3x^2-6x-x-20=0$

$3x^2-7x-20=0$

$\because -12+5=-7,\ -12\times 5=-60$

$3x^2-12x+5x-20=0$

$3x(x-4)+5(x-4)=0$

$\Rightarrow (x-4)(3x+5)=0$

Either $x-4=0$ *or* $3x+5=0,$

$\Rightarrow x=4$ or $3x=-5$

$\Rightarrow x=-\frac{5}{3}$

$x=-\frac{5}{3},4$ are the solutions of the given equation

Thus, the solution set is $\left\{-\frac{5}{3},4\right\}$

Example 2: (Page # 4) **(A.B)**

Solve the equation $2x^2-5x-3=0$ **by completing square.**

Solution:

$2x^2-5x-3=0$

Dividing each term by 2

$x^2-\frac{5}{2}x-\frac{3}{2}=0$

Now adding $\left(-\frac{5}{4}\right)^2$ on both sides

$x^2-\frac{5}{2}x+\left(-\frac{5}{4}\right)^2=\frac{3}{2}+\left(-\frac{5}{4}\right)^2$

$\left(x-\frac{5}{4}\right)^2=\frac{3}{2}+\frac{25}{16}=\frac{24+25}{16}$

$\left(x-\frac{5}{4}\right)^2=\frac{49}{16}$

Taking square root on both sides

$\sqrt{\left(x-\frac{5}{4}\right)^2}=\pm\sqrt{\frac{49}{16}}$

$x-\frac{5}{4}=\pm\frac{7}{4}$

Either $x-\frac{5}{4}=\frac{7}{4}$ or $x-\frac{5}{4}=-\frac{7}{4}$

$x=\frac{7}{4}+\frac{5}{4}$ or $x=-\frac{7}{4}+\frac{5}{4}$

$=\frac{7+5}{4}$ or $=\frac{-7+5}{4}$

$=\frac{12}{4}$ or $=\frac{-2}{4}$

$x=3$ or $x=-\frac{1}{2}$

$\therefore x=-\frac{1}{2},3$ are the roots of the given equation

Thus, the solution set is $\left\{-\frac{1}{2},3\right\}$

Exercise 1.1

Q.1 Write the following quadratic equations in the standard form and point out pure quadratic equations.

(SGD 2015, 17, RWP 2016) (A.B)

(i) $(x+7)(x-3)=-7$

Solution:

$(x+7)(x-3)=-7$

$x^2+7x-3x-21=-7$

$x^2+4x-21+7=0$

$x^2+4x-14=0$

Which is the required standard form of quadratic equation.

(ii) $\frac{x^2+4}{3}-\frac{x}{7}=1$ **(A.B)**

Solution:

$\frac{x^2+4}{3}-\frac{x}{7}=1$

$\frac{x^2+4}{3}-\frac{x}{7}-1=0$

$\frac{7(x^2+4)-3(x)-1(21)}{21}=0$

$\frac{7x^2+28-3x-21}{21}=0$

$7x^2-3x+7=0$

Which is the required standard form of quadratic equation.

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$ **(A.B)**

(BWP 2014, D.G.K 2014, 15)

Solution:

$$\frac{x}{x+1}+\frac{x+1}{x}=6$$
$$\frac{x}{x+1}+\frac{x+1}{x}-6=0$$
$$\frac{x(x)+(x+1)^2-6(x)(x+1)}{(x+1)(x)}=0$$
$$\frac{x^2+(x^2+2x+1)-6x(x+1)}{(x+1)(x)}=0$$
$$x^2+x^2+2x+1-6x(x+1)=0$$
$$2x^2+2x+1-6x^2-6x=0$$
$$-4x^2-4x+1=0$$
$$-(4x^2+4x-1)=0$$
$$4x^2+4x-1=0$$

Which is the required standard form of quadratic equation.

(iv) $\dfrac{x+4}{x-2}-\dfrac{x-2}{x}+4=0$ **(A.B)**

Solution:

$$\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$$

Multiplying both sides by $x(x-2)$, we get

$$x(x+4)-(x-2)(x-2)+4x(x-2)=0$$
$$x^2+4x-(x^2-4x+4)+4x^2-8x=0$$
$$x^2+4x-x^2+4x-4+4x^2-8x=0$$
$$4x^2+8x-8x-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$x^2-1=0 \qquad \because 4\neq 0$$

Is the required standard form and it is a pure quadratic equation.

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$ **(A.B)**

Solution:

$$\frac{x+3}{x+4}-\frac{x-5}{x}=1$$
$$\frac{x(x+3)-(x+4)(x-5)}{(x+4)(x)}=1$$
$$\frac{x^2+3x-(x^2-5x+4x-20)}{(x+4)(x)}=1$$
$$\frac{x^2+3x-x^2+5x-4x+20}{(x+4)(x)}=1$$
$$\frac{4x+20}{(x+4)(x)}=1$$
$$4x+20=x(x+4)$$
$$4x+20-x(x+4)=0$$
$$4x+20-x^2-4x=0$$
$$-x^2+20=0 \quad \text{Or} \quad x^2-20=0$$

Is the required standard form of quadratic equation and it is a pure quadratic.

(vi) $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$ **(A.B)**

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$
$$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{x^2+x+3x+3+x^2+4x+4}{(x+2)(x+3)}=\frac{25}{12}$$
$$\frac{2x^2+8x+7}{(x+2)(x+3)}-\frac{25}{12}=0$$
$$\frac{12(2x^2+8x+7)-[25(x+3)(x+2)]}{12(x+2)(x+3)}=0$$
$$24x^2+96x+84-[25(x^2+5x+6)]=0$$
$$24x^2+96x+84-[25x^2+125x+150]=0$$
$$24x^2+96x+84-25x^2-125x-150=0$$
$$-x^2-29x-66=0$$
$$\text{Or} \quad x^2+29x+66=0$$

Is the required standard form of quadratic equation.

Q.2 Solve by factorization:

(i) $x^2-x-20=0$ **(A.B)**

(LHR 2014, 15, FSD 2016, SGD 2016, SWL 2016, RWP 2016)

Solution:

$x^2-x-20=0$

$x^2-5x+4x-20=0$

$\because -5+4=-1,\ -5\times4=-20$

$x(x-5)+4(x-5)=0$

Either

$x-5=0$ or $x+4=0$

$x=5$ $x=-4$

$\therefore$ **Solution Set** $=\{5,-4\}$

(ii) $3y^2=y(y-5)$ **(A.B)**

(FSD 2015, BWP 2017)

Solution:

$3y^2=y(y-5)$

$3y^2=y^2-5y$

$3y^2-y^2+5y=0$

$2y^2+5y=0$

$y(2y+5)=0$

Either

$y=0$ *or* $2y+5=0$

$y=0$ *or* $2y=-5$

$y=\frac{-5}{2}$

$\therefore$ **Solution Set** $=\left\{0,-\frac{5}{2}\right\}$

Note

The cancelling of y on both sides of $3y^2=y(y-5)$ means the lose of 1 root i.e $y=0$

(iii) $4-32x=17x^2$ **(K.B, A.B)**

Solution:

$4-32x=17x^2$

$17x^2+32x-4=0$

$17x^2+34x-2x-4=0$

$17x(x+2)-2(x+2)=0$

$(x+2)(17x-2)=0$

$x+2=0$ or $17x-2=0$

$x=-2$ or $17x=2$

$x=\frac{2}{17}$

$\therefore$ **Solution Set** $=\left\{-2,\frac{2}{17}\right\}$

(iv) $x^2-11x=152$ **(A.B)**

Solution:

$x^2-11x=152$

$x^2-11x-152=0$

$x^2-19x+8x-152=0$

$x(x-19)+8(x-19)=0$

$(x-19)(x+8)=0$

Either

$x-19=0$ or $x+8=0$

$x=19$ $x=-8$

$\therefore$ **Solution Set** $=\{-8,19\}$

(v) $\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$ **(A.B)**

(GRW 2014, 17, BWP 2016)

Solution:

$\frac{x+1}{x}+\frac{x}{x+1}=\frac{25}{12}$

$\frac{(x+1)^2+(x)^2}{(x+1)(x)}=\frac{25}{12}$

$\frac{x^2+2x+1+x^2}{(x+1)(x)}=\frac{25}{12}$

$\frac{2x^2+2x+1}{(x+1)(x)}=\frac{25}{12}$

$12(2x^2+2x+1)=25(x)(x+1)$

$24x^2+24x+12=25x(x+1)$

$24x^2+24x+12=25x^2+25x$

$24x^2+24x+12-25x^2-25x=0$

$-x^2-x+12=0$

$x^2+x-12=0$

$x^2+4x-3x-12=0$

$x(x+4)-3(x+4)=0$

$(x+4)(x-3)=0$

Either

$x+4=0$ or $x-3=0$

$x=-4$ $x=3$

$\therefore$ **Solution Set** $=\{-4,3\}$

(vi) $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$ **(A.B)**

(LHR 2014, FSD 2017, RWP 2017, BWP 2016, D.G.K 2016)

Solution:

$$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$
$$\frac{2}{x-9}=\frac{1(x-4)-1(x-3)}{(x-3)(x-4)}$$
$$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-4x-3x+12}$$
$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$
$$2(x^2-7x+12)=-1(x-9)$$
$$2x^2-14x+24=-x+9$$
$$2x^2-14x+x+24-9=0$$
$$2x^2-13x+15=0$$
$$2x^2-10x-3x+15=0$$
$$2x(x-5)-3(x-5)=0$$
$$(x-5)(2x-3)=0$$

Either

$x-5=0$ or $2x-3=0$

$x=5$ or $2x=3$

$x=\dfrac{3}{2}$

$\therefore$ **Solution Set** $=\left\{\dfrac{3}{2},5\right\}$

Q.3 Solve the following equations by completing square

(i) $7x^2+2x-1=0$ **(A.B)**

Solution:

$$7x^2+2x-1=0$$
$$7x^2+2x=1$$

Divide by '7' on both sides

$$\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7}$$
$$x^2+\frac{2x}{7}=\frac{1}{7}$$

Adding $\left(\dfrac{1}{7}\right)^2$ on both sides

$$x^2+\frac{2x}{7}+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$
$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$
$$\left(x+\frac{1}{7}\right)^2=\frac{1(7)+1}{49}$$
$$\left(x+\frac{1}{7}\right)^2=\frac{7+1}{49}$$
$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\sqrt{\frac{8}{49}}$$
$$x+\frac{1}{7}=\pm\frac{\sqrt{8}}{7}$$
$$x=-\frac{1}{7}\pm\frac{\sqrt{8}}{7}$$
$$x=\frac{-1\pm\sqrt{8}}{7}$$

Or $x=\dfrac{-1\pm2\sqrt{2}}{7}$

$\therefore$ **Solution Set =** $\left\{\dfrac{-1\pm2\sqrt{2}}{7}\right\}$

(ii) $ax^2+4x-a=0$, where $a\neq0$ **(A.B)**

Solution:

$$ax^2+4x-a=0$$
$$ax^2+4x=a$$

Divide by 'a'

$$\frac{ax^2}{a}+\frac{4x}{a}=\frac{a}{a}$$
$$x^2+\frac{4x}{a}=1$$

Adding $\left(\dfrac{2}{a}\right)^2$ on both sides

$$x^2+\frac{4x}{a}+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2}=\sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a}=\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=-\frac{2}{a}\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

$\therefore$ **Solution Set =** $\left\{\dfrac{-2\pm\sqrt{a^2+4}}{a}\right\}$

(iii) $11x^2-34x+3=0$ **(A.B)** **(FSD 2018)**

Solution:

$$11x^2-34x+3=0$$

$$11x^2-34x=-3$$

Divide by '11' on both sides

$$\frac{11x^2}{11}-\frac{34x}{11}=\frac{-3}{11}$$

$$x^2-\frac{34x}{11}=-\frac{3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides

$$x^2-\frac{34}{11}x+\left(\frac{17}{11}\right)^2=\frac{-3}{11}=\left(\frac{17}{11}\right)^2$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-3}{11}+\frac{289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-3(11)+289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{-33+289}{121}$$

$$\left(x-\frac{17}{11}\right)^2=\frac{256}{121}$$

Taking square root on both sides

$$\sqrt{\left(x-\frac{17}{11}\right)^2}=\sqrt{\frac{256}{121}}$$

$$x-\frac{17}{11}=\pm\frac{16}{11}$$

$$x=\frac{17}{11}\pm\frac{16}{11}$$

Either

$$x=\frac{16}{11}+\frac{17}{11} \quad \text{or} \quad x=\frac{-16}{11}+\frac{17}{11}$$

$$x=\frac{16+17}{11} \quad \text{or} \quad x=\frac{-16+17}{11}$$

$$x=\frac{33}{11} \quad \text{or} \quad x=\frac{1}{11}$$

$$x=3$$

$\therefore$ **Solution Set =** $\left\{\dfrac{1}{11},3\right\}$

(iv) $lx^2+mx+n=0$ **(FSD 2015)** **(A.B)**

Solution:

$$lx^2+mx+n=0$$

$$lx^2+mx=-n$$

Divide by 'l'on both sides

$$\frac{lx^2}{l}+\frac{mx}{l}=\frac{-n}{l}$$

$$x^2+\frac{mx}{l}=\frac{-n}{l}$$

Adding $\left(\frac{m}{2l}\right)^2$ on both sides

$$x^2+\frac{m}{l}x+\left(\frac{m}{2l}\right)^2=\left(\frac{m}{2l}\right)^2-\frac{n}{l}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{m^2}{4l^2}-\frac{n}{l}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{m^2-4ln}{4l^2}$$

Taking square roots on both sides

$$\sqrt{\left(x+\frac{m}{2l}\right)^2}=\pm\sqrt{\frac{m^2-4ln}{4l^2}}$$

$$x+\frac{m}{2l}=\pm\frac{\sqrt{m^2-4ln}}{2l}$$

$$x=\frac{-m}{2l}\pm\frac{\sqrt{m^2-4ln}}{2l}$$

Or $x=\dfrac{-m\pm\sqrt{m^2-4ln}}{2l}$

$\therefore$ **Solution Set =** $\left\{\dfrac{-m\pm\sqrt{m^2-4ln}}{2l}\right\}$

(v) $3x^2+7x=0$ **(A.B)**

Solution:

$3x^2+7x=0$

Dividing by 3

$$x^2+\frac{7}{3}x=0$$

$$\because \frac{1}{2}(\text{Coefficient of } x)=\frac{1}{2}\times\frac{7}{3}=\frac{7}{6}$$

Adding $\left(\frac{7}{6}\right)^2$ on both sides, we get

$$x^2+\frac{7}{3}x+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\frac{49}{36}$$

Taking square roots on both sides

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\frac{49}{36}}$$

$$x+\frac{7}{6}=\pm\frac{7}{6}$$

Either

$x=-\dfrac{7}{6}+\dfrac{7}{6}$ or $x=-\dfrac{7}{6}-\dfrac{7}{6}$

$x=0$ $\qquad x=\dfrac{-7-7}{6}$

$$x=\frac{-14}{6}$$

$$x=-\frac{7}{3}$$

$\therefore$ **Solution Set** $=\left\{0,-\dfrac{7}{3}\right\}$

(vi) Given:

$x^2-2x-195=0$ **(A.B)**

Solution:

$$x^2-2x-195=0$$

$$x^2-2x=195$$

Adding $(1)^2$ on both sides

$$x^2-2x+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root on both sides

$$\sqrt{(x-1)^2}=\sqrt{196}$$

$$x-1=\pm14$$

Either

$x-1=14$ or $x-1=-14$

$x=14+1$ $\qquad x=-14+1$

$x=15$ $\qquad x=-13$

$\therefore$ **Solution Set** $=\{-13,15\}$

(vii) $-x^2+\dfrac{15}{2}=\dfrac{7}{2}x$ **(A.B)**

Solution:

$$-x^2+\frac{15}{2}=\frac{7}{2}x$$

$$x^2+\frac{7}{2}x-\frac{15}{2}=0$$

$$x^2+\frac{7}{2}x=\frac{15}{2}$$

$$\because \frac{1}{2}(\text{Coefficient of } x)=\frac{1}{2}\left(\frac{7}{2}\right)=\frac{7}{4}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$x^2+\frac{7}{2}x+\left(\frac{7}{4}\right)^2=\left(\frac{15}{2}\right)+\left(\frac{7}{4}\right)^2$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$=\frac{169}{16}$$

Taking square root on both sides

$$x+\frac{7}{4}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=-\frac{7}{4}\pm\frac{13}{4}$$

Either

$$x=-\frac{7}{4}+\frac{13}{4} \quad \text{or} \quad x=-\frac{7}{4}-\frac{13}{4}$$

$$=\frac{-7+13}{4} \quad \text{or} \quad x=\frac{-7-13}{4}$$

$$=\frac{6}{4} \quad \text{or} \quad =-\frac{20}{4}$$

$$x=\frac{3}{2} \quad \text{or} \quad x=-5$$

$\therefore$ **Solution Set** $=\left\{-5,\frac{3}{2}\right\}$

(viii) $x^2+17x+\frac{33}{4}=0$ **(A.B)**

Solution:

$$x^2+17x+\frac{33}{4}=0$$

$$x^2+17x=-\frac{33}{4}$$

Adding $\left(\frac{17}{2}\right)^2$ on both sides

$$x^2+17x+\left(\frac{17}{2}\right)^2=\frac{-33}{4}+\left(\frac{17}{2}\right)^2$$

$$\left(x+\frac{17}{2}\right)^2=\frac{-33}{4}+\frac{289}{4}$$

$$\left(x+\frac{17}{2}\right)^2=\frac{-33+289}{4}$$

$$\left(x+\frac{17}{2}\right)^2=\frac{256}{4}$$

Taking square root on both sides

$$\sqrt{\left(x+\frac{17}{2}\right)^2}=\sqrt{\frac{256}{4}}$$

$$x+\frac{17}{2}=\pm\frac{16}{2}$$

$$x=\pm\frac{16}{2}-\frac{17}{2}$$

Either

$$x=\frac{16}{2}-\frac{17}{2} \quad \text{or} \quad x=\frac{-16}{2}\frac{-17}{2}$$

$$x=\frac{16-17}{2} \quad \text{or} \quad x=\frac{-16-17}{2}$$

$$x=\frac{-1}{2} \quad \text{or} \quad x=\frac{-33}{2}$$

$\therefore$ **Solution Set** = $\left\{-\frac{1}{2},-\frac{33}{2}\right\}$

(ix) $4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$ **(A.B + K.B + U.B)**

Solution:

$$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$$

$$4=\frac{3x^2+5}{3x+1}+\frac{8}{3x+1}$$

$$\text{Or } \frac{3x^2+5}{3x+1}+\frac{8}{3x+1}=4$$

$$\frac{3x^2+5+8}{3x+1}=4$$

$$\frac{3x^2+13}{3x+1}=4$$
$3x^2 + 13 = 4(3x + 1)$
$3x^2 + 13 = 12x + 4$
$3x^2 - 12x = 4 - 13$
$3x^2 - 12x = -9$
Divide by '3' on both sides
$x^2 - 4x = -3$
Adding $(2)^2$ on both side
$x^2 - 4x + (2)^2 = -3 + (2)^2$
$(x-2)^2 = -3+4$
$(x-2)^2 = 1$
Taking square root on both sides
$\sqrt{(x-2)^2} = \sqrt{1}$
$x - 2 = \pm 1$
$x = \pm 1 + 2$
Either
$x = 1+2$ or $x = -1+2$
$x = 3$ or $x = 1$
$\therefore$ **Solution Set** $= \{1,3\}$

(x) $7(x+2a)^2 + 3a^2 = 5a(7x+23a)$ **(A.B + K.B + U.B)**

Solution:
$7(x+2a)^2 + 3a^2 = 5a(7x+23a)$
$7(x^2+4ax+4a^2)+3a^2 = 35ax+115a^2$
$7x^2+28ax+28a^2+3a^2 = 35ax+115a^2$
$7x^2+28ax+31a^2-35ax-115a^2 = 0$
$7x^2-7ax-84a^2 = 0$
$7x^2-84a^2-7ax = 0$
$7(x^2-ax-12a^2) = 0$
$x^2-ax-12a^2 = 0 \qquad \because 7 \neq 0$
$x^2-ax = 12a^2$
Adding $\left(\frac{a}{2}\right)^2$ on both sides
$$x^2-ax+\left(\frac{a}{2}\right)^2 = 12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2 = 12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2 = \frac{4(12a^2)+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2 = \frac{48a^2+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2 = \frac{49a^2}{4}$$
Taking square root on both sides
$$\sqrt{\left(x-\frac{a}{2}\right)^2} = \sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2} = \pm\frac{7a}{2}$$
$$x = \pm\frac{7a}{2}+\frac{a}{2}$$
Either
$$x = \frac{7a}{2}+\frac{a}{2} \quad \text{or} \quad x = \frac{-7a}{2}+\frac{a}{2}$$
$$= \frac{7a+a}{2} \qquad = \frac{-7a+a}{2}$$
$$= \frac{8a}{2} \qquad = \frac{-6a}{2}$$
$x = 4a$ or $x = -3a$
$\therefore$ **Solution Set** $= \{-3a, 4a\}$

Quadratic Formula (U.B)

For a standard quadratic equation
$ax^2+bx+c=0$ where $a \neq 0$
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Use of Quadratic Formula (U.B)

The quadratic formula is useful tool for solving all those equations which can or cannot be factorized.

Derivation of Quadratic Formula by using Completing Square Method

(K.B + U.B + A.P)

The quadratic equation in standard form is
$ax^2+bx+c=0,\ a\neq 0$
Shifting constant term c to the right, we have
$ax^2+bx=-c$
Dividing each term by a
$$x^2+\frac{b}{a}x = -\frac{c}{a}$$
Adding $\left(\frac{b}{2a}\right)^2$ on both sides

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

Or $\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

Taking square root on both sides

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

Or $x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **,** $a\neq 0$ **is known as "Quadratic Formula".**

Example 1: (Page # 6) **(A.B)**

Solve the quadratic equation $2+9x=5x^2$ by using quadratic formula.

Solution:

$$2+9x=5x^2$$

$$\Rightarrow 5x^2-9x-2=0$$

By comparing given equation with standard quadratic equation $ax^2+bx+c=0$, we have

$a=5,\ b=-9,\ c=-2$

Quadratic formula is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Putting the values in quadratic formula

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(5)(-2)}}{2(5)}$$

$$x=\frac{9\pm\sqrt{81+40}}{10}$$

$$=\frac{9\pm\sqrt{121}}{10}$$

$$=\frac{9\pm 11}{10}$$

Either

$x=\frac{9+11}{10}$ or $x=\frac{9-11}{10}$

$x=\frac{20}{10}=2$ or $x=\frac{-2}{10}=-\frac{1}{5}$

$\therefore 2,-\frac{1}{5}$ are the roots of the given equation.

Thus, the solution set is $\left\{-\frac{1}{5},2\right\}$

Exercise 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2-x^2=7x$ **(A.B)**

Solution:

$$2-x^2=7x$$

$$0=x^2+7x-2$$

$$\Rightarrow x^2+7x-2=0$$

By comparing given equation with ax^2+bx+c, we get

$a=1, b=7, c=-2$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-7\pm\sqrt{(7)^2-4(1)(-2)}}{2\times 1}$$

$$=\frac{-7\pm\sqrt{49+8}}{2}=\frac{-7\pm\sqrt{57}}{2}$$

$\therefore$ **Solution Set** $=\left\{\frac{-7\pm\sqrt{57}}{2}\right\}$

(ii) $5x^2+8x+1=0$ **(A.B)**

Solution:

$5x^2+8x+1=0$

Here $a=5, b=8, c=1$

Putting the values in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-8\pm\sqrt{(8)^2-4\times5\times1}}{2\times5}$$

$$=\frac{-8\pm\sqrt{64-20}}{10}=\frac{-8\pm\sqrt{44}}{10}$$

$$=\frac{-8\pm\sqrt{4\times11}}{10}=\frac{-8\pm2\sqrt{11}}{10}$$

$$=\frac{\cancel{2}\left(-4\pm\sqrt{11}\right)}{\cancel{10}_5}$$

$$=\frac{-4\pm\sqrt{11}}{5}$$

$\therefore$ **Solution Set** $=\left\{\frac{-4\pm\sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2+x=4\sqrt{3}$ **(A.B + U.B + K.B)**

Solution:

$\sqrt{3}x^2+x=4\sqrt{3}$

$\sqrt{3}x^2+x-4\sqrt{3}=0$

Here $a=\sqrt{3},\ b=1,\ c=-4\sqrt{3}$

Putting in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{(1)^2-4\sqrt{3}\left(-4\sqrt{3}\right)}}{2\times\sqrt{3}}$$

$$x=\frac{-1\pm\sqrt{1+16\times\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x=\frac{-1\pm\sqrt{1+48}}{2\sqrt{3}}$$

$$x=\frac{-1\pm\sqrt{49}}{2\sqrt{3}}$$

$$x=\frac{-1\pm7}{2\sqrt{3}}$$

Either

$x=\frac{-1+7}{2\sqrt{3}}$ or $x=\frac{-1-7}{2\sqrt{3}}$

$x=\frac{\cancel{6}^3}{\cancel{2}\sqrt{3}}$ $\quad x=\frac{\cancel{-8}^4}{\cancel{2}\sqrt{3}}$

$x=\frac{3}{\sqrt{3}}$ $\quad x=-\frac{4}{\sqrt{3}}$

$x=\frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$ **or** $x=-\frac{4}{\sqrt{3}}$

$x=\sqrt{3}$

$\therefore$ **Solution of Set** $=\left\{\sqrt{3},-\frac{4}{\sqrt{3}}\right\}$

(iv) $4x^2-14=3x$ **(A.B)**

Solution:

$4x^2-14=3x$

$4x^2-3x-14=0$

Here $a=4,\ b=-3,\ c=-14$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4\times4\times(-14)}}{2\times4}$$

$$x=\frac{3\pm\sqrt{9+224}}{8}$$

$$x=\frac{3\pm\sqrt{233}}{8}$$

$\therefore$ **Solution Set** $=\left\{\frac{3\pm\sqrt{233}}{8}\right\}$

(v) $6x^2-3-7x=0$ **(A.B)**

Solution:

$6x^2-3-7x=0$

$6x^2-7x-3=0$

Here $a=6,\quad b=-7,\quad c=-3$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-3)}}{2\times6}$$

$$x=\frac{7\pm\sqrt{49+72}}{12}$$

$$x=\frac{7\pm\sqrt{121}}{12}$$

$$x=\frac{7\pm11}{12}$$

Either

$x=\frac{7+11}{12}$ or $x=\frac{7-11}{12}$

$x=\frac{18}{12}$, $x=\frac{-4}{12}$

$x=\frac{3}{2}$, $x=\frac{-1}{3}$

$\therefore$ **Solution Set** $=\left\{\frac{3}{2},\frac{-1}{3}\right\}$

(vi) $3x^2+8x+2=0$ **(A.B)**

Solution:

$3x^2+8x+2=0$

Here

$a=3,\quad b=8,\quad c=2$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4\times3\times2}}{2\times3}$$

$$x=\frac{-8\pm\sqrt{64-24}}{6}$$

$$x=\frac{-8\pm\sqrt{40}}{6}$$

$$x=\frac{-8\pm2\sqrt{10}}{6}$$

$$x=\frac{2\left(-4\pm\sqrt{10}\right)}{6_3}$$

$$x=\frac{-4\pm\sqrt{10}}{3}$$

$\therefore$ **Solution Set** $=\left\{\frac{-4\pm\sqrt{10}}{3}\right\}$

(vii) $\frac{3}{x-6}-\frac{4}{x-5}=1$ **(A.B + K.B)**

Solution:

$\frac{3}{x-6}-\frac{4}{x-5}=1$

Multiplying both sides by $(x-6)(x-5)$

$3(x-5)-4(x-6)=1(x-6)(x-5)$

$3x-15-4x+24=x^2-6x-5x+30$

$-x+9=x^2-11x+30$

$0=x^2-11x+30+x-9$

$0=x^2-10x+21$

$x^2-10x+21=0$

Here

$a=1,\quad b=-10,\quad c=21$

Putting values of a,b,c in Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4\times1\times21}}{2\times1}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm4}{2}$$

Either

$x=\frac{10+4}{2}$ or $x=\frac{10-4}{2}$

$x=\frac{14}{2}$, $x=\frac{6}{2}$

$x=7$, $x=3$

$\therefore$ **Solution Set** $=\{3,7\}$

(viii) $\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$ **(A.B + K.B)**

Solution:

$\frac{x+2}{x-1}-\frac{4-x}{2x}=2\frac{1}{3}$

$\frac{x+2}{x-1}-\frac{4-x}{2x}=\frac{7}{3}$

Multiplying throughout by $3(2x)(x-1)$

$3(2x)(x+2)-3(x-1)(4-x)=7(2x)(x-1)$

$6x(x+2)+3(x-1)(x-4)=14x(x-1)$

$6x^2+12x+3(x^2-5x+4)=14x^2-14x$

$6x^2+12x+3x^2-15x+12=14x^2-14x$

$9x^2-3x+12=14x^2-14x$

$0=14x^2-14x-9x^2+3x-12$

$0=5x^2-11x-12$

$5x^2-11x-12=0$

Here

$a=5,\quad b=-11,\quad c=-12$

Putting values of a,b,c in Quadratic Formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-11)\pm\sqrt{(-11)^2-4\times5\times(-12)}}{2\times5}$

$x=\frac{11\pm\sqrt{121+240}}{10}$

$x=\frac{11\pm\sqrt{361}}{10}$

$x=\frac{11\pm19}{10}$

Either

$x=\frac{11+19}{10}$ or $x=\frac{11-19}{10}$

$x=\frac{30}{10}$, $x=\frac{-8}{10}$

$x=3$, $x=-\frac{4}{5}$

$\therefore$ **Solution Set** $=\left\{-\frac{4}{5},3\right\}$

(ix) $\frac{a}{x-b}+\frac{b}{x-a}=2$ **(A.B + K.B)**

(FSD 2016)

Solution:

$\frac{a}{x-b}+\frac{b}{x-a}=2$

Multiplying throughout by $(x-a)(x-b)$

$a(x-a)+b(x-b)=2(x-a)(x-b)$

$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$

$ax+bx-a^2-b^2=2x^2-2ax-2bx+2ab$

$0=2x^2-2ax-2bx+2ab-ax-bx+a^2+b^2$

$0=2x^2-3ax-3bx+a^2+b^2+2ab$

$2x^2-3(a+b)x+(a+b)^2=0$

Here

$A=2,\quad B=-3(a+b), C=(a+b)^2$

Putting values of A, B, C in Quadratic Formula

$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$x=\frac{-[-3(a+b)]\pm\sqrt{[-3(a+b)]^2-4\times2(a+b)^2}}{2\times2}$

$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$

$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$

$x=\frac{3(a+b)\pm(a+b)}{4}$

Either

$x=\frac{3(a+b)+(a+b)}{4}$ or $x=\frac{3(a+b)-(a+b)}{4}$

$x = \frac{4(a+b)}{4}$, $x = \frac{2(a+b)}{4}$

$x = a+b$, $x = \frac{a+b}{2}$

$\therefore$ **Solution Set** $= \left\{a+b, \frac{a+b}{2}\right\}$

(x) $-(l+m) - lx^2 + (2l+m)x = 0$ **(A.B + K.B + U.B)**

Solution:

$-(l+m) - lx^2 + (2l+m)x = 0$

$-lx^2 + (2l+m)x - (l+m) = 0$

$lx^2 - (2l+m)x + (l+m) = 0$

Here $a = l$, $b = -(2l+m)$, $c = l+m$

Putting the values of a, b, c in Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-(2l+m)] \pm \sqrt{[-(2l+m)]^2 - 4\times l \times (l+m)}}{2\times l}$$

$$x = \frac{(2l+m) \pm \sqrt{(2l+m)^2 - 4l(l+m)}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{4l^2 + m^2 + 4lm - 4l^2 - 4lm}}{2l}$$

$$x = \frac{(2l+m) \pm \sqrt{m^2}}{2l}$$

$$x = \frac{2l+m \pm m}{2l}$$

$x = \frac{2l+m+m}{2l}$, $x = \frac{2l+m-m}{2l}$

$x = \frac{2l+2m}{2l}$, $x = \frac{2l}{2l}$

$x = \frac{2(l+m)}{2l}$, $x = 1$

$x = \frac{l+m}{l}$

$\therefore$ **Solution Set** $= \left\{1, \frac{l+m}{l}\right\}$

Reciprocal Equation (U.B)

(LHR 2015, 16, 17, FSD 2017, MTN 2015, 17, RWP 2017, BWP 2017, D.G.K 2015, 17)

An equation, which remains unchanged when 'x' is replaced by $\frac{1}{x}$ is called reciprocal equation.

e.g. $a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$

$ax^4 + bx^3 + cx^2 + bx + a = 0$

Exponential Equation (U.B)

(LHR 2015, GRW 2017 SWL 2017, SGD 2017, D.G.K 2015, MTN 2016)

An equation, in which a variable or an algebraic expression occurs in exponent is called exponential equation.

e.g. $2^x + 64.2^{-x} - 20 = 0$

Or

$3^{2x+2} = 12.3^x - 3$ etc.

Equations Reducible to Quadratic Form (K.B + A.B + U.B)

Type (i)

The Equations of the type

$ax^4 + bx^2 + c = 0$

Example 1: (Page # 8)

Solve the equation $x^4 - 13x^2 + 36 = 0$

Solution:

$x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13x^2 + 36 = 0 \rightarrow$ (i)

Let $x^2 = y \rightarrow$ (ii)

Equation (i) $\Rightarrow$

$y^2 - 13y + 36 = 0$

$y^2 - 9y - 4y + 36 = 0$

$y(y-9) - 4(y-9) = 0$

$(y-9)(y-4) = 0$

Either

$y - 9 = 0$ or $y - 4 = 0$

$\Rightarrow y = 9$ or $y = 4$

Putting the value of y in equation (ii)

$\Rightarrow x^2 = 9$ or $x^2 = 4$

Taking square root on both sides

$\Rightarrow x = \pm 3$ or $x = \pm 2$

$\therefore$ **Solution Set** $= \{\pm 3, \pm 2\}$

Type (ii)

The Equations of the type

$$ap(x)+\frac{b}{p(x)}=c$$

Example 2: (Page # 8) **(K.B + A.B)**

Solve the equation $2(2x-1)+\frac{3}{2x-1}=5$

Solution:

$$2(2x-1)+\frac{3}{2x-1}=5\rightarrow\text{(i)}$$

Let $2x-1=y\rightarrow$ (ii)

Equation (i) $\Rightarrow$

$$2y+\frac{3}{y}=5$$

$$2y^2+3=5y$$

(by Mul both sides by y)

$$2y^2-5y+3=0$$

$$2y^2-2y-3y+3=0$$

$$2y(y-1)-3(y-1)=0$$

$$(y-1)(2y-3)=0$$

Either

$y-1=0$ or $2y-3=0$

$\Rightarrow y=1$ or $2y=3$

$\Rightarrow y=\frac{3}{2}$

Putting the values of y in equation (ii)

$\Rightarrow 2x-1=1$ or $2x-1=\frac{3}{2}$

$2x=1+1$ or $2x=\frac{3}{2}+1$

$2x=2$ or $2x=\frac{5}{2}$

$x=1$ or $x=\frac{5}{4}$

$\therefore$ **Solution Set** $=\left\{1,\frac{5}{4}\right\}$

Type (iii) **(A.B + K.B)**

Reciprocal Equation of the type:

$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$ or

$$ax^4+bx^3+cx^2+bx+a=0$$

Example 3: (Page # 9) **(A.B)**

Solve the equation

$$2x^4-5x^3-14x^2-5x+2=0$$

Solution:

$$2x^4-5x^3-14x^2-5x+2=0$$

$$2x^4+2-5x^3-5x-14x^2=0$$

Dividing each term by x^2

$$2x^2+\frac{2}{x^2}-5x-\frac{5}{x}-14=0$$

$$2\left(x^2+\frac{1}{x^2}\right)-5\left(x+\frac{1}{x}\right)-14=0\rightarrow\text{(i)}$$

Let $x+\frac{1}{x}=y\rightarrow$ (ii)

Squaring both sides

$y=-2$

$$x^2+\frac{1}{x^2}+2=y^2$$

$$x^2+\frac{1}{x^2}=y^2-2$$

Equation (i) $\Rightarrow$

$$2(y^2-2)-5y-14=0$$

$$2y^2-4-5y-14=0$$

$$2y^2-5y-18=0$$

$$2y^2-9y+4y-18=0$$

$$y(2y-9)+2(2y-9)=0$$

$$(2y-9)(y+2)=0$$

Either

$2y-9=0$ or $y+2=0$

$\Rightarrow 2y=9$ or $y=-2$

$\Rightarrow y=\frac{9}{2}$

Putting the values of y in equation (ii)

When $y=\frac{9}{2}$

$$x+\frac{1}{x}=\frac{9}{2}$$

$$2x^2+2=9x$$

$$2x^2-9x+2=0$$

By using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(2)}}{2(2)}$$

$$x=\frac{9\pm\sqrt{81-16}}{4}$$

$$x=\frac{9\pm\sqrt{65}}{4}$$

When $y=-2$

$$x+\frac{1}{x}=-2$$

$$x^2+1=-2x$$

$$x^2+2x+1=0$$

$$(x+1)^2=0$$

$$\Rightarrow x+1=0$$

Or $x=-1$

Thus, Solution Set $=\left\{-1,\frac{9\pm\sqrt{65}}{4}\right\}$

Type (iv) **(A.B + K.B + U.B)**

Exponential Equation:

$$5y+\frac{5}{y}=26 \quad \text{or} \quad 5^{1+x}+5^{1-x}=26$$

Example 4: (Page # 10)

Solve the equation $5^{1+x}+5^{1-x}=26$

Solution:

$$5^{1+x}+5^{1-x}=26$$

$$5^1.5^x+5^1.5^{-x}=26$$

$$5.5^x+\frac{5}{5^x}=26\rightarrow(\text{i})$$

Let $5^x=y\rightarrow(\text{ii})$

Equation (i) $\Rightarrow$

$$5y+\frac{5}{y}=26$$

$$5y^2-26y+5=0$$

$$5y^2-26y+5=0$$

$$5y^2-25y-y+5=0$$

$$(y-5)(5y-1)=0$$

$$(y-5)(5y-1)=0$$

Either

$$y-5=0 \quad \text{or} \quad 5y-1=0$$

$$y=5 \quad \text{or} \quad y=\frac{1}{5}$$

Putting the values of y in equation (ii)

$$\Rightarrow 5^x=5 \quad \text{or} \quad 5^x=\frac{1}{5}$$

$$\Rightarrow 5^x=5^1 \quad \text{or} \quad 5^x=5^{-1}$$

$\because$ Bases are same

$$x=1 \quad \text{or} \quad x=-1$$

Thus, Solution Set $=\{\pm1\}$

Type (v) **(A.B + U.B + K.B)**

The Equation of the type of

$(x+a)(x+b)(x+c)(x+d)=k$ where $a+b=c+d$

Example 5: (Page # 11)

Solve the equation

$$(x-1)(x+2)(x+8)(x+5)=19$$

Solution:

$$(x-1)(x+2)(x+8)(x+5)=19$$

$$(x-1)(x+8)(x+2)(x+5)=19 \quad (\because -1+8=2+5)$$

$$(x^2+7x-8)(x^2+7x+10)-19=0\rightarrow(\text{i})$$

Let $x^2+7x=y\rightarrow(\text{ii})$

Equation (i) $\Rightarrow$

$$(y-8)(y+10)-19=0$$

$$y^2+10y-8y-80-19=0$$

$$y^2+2y-99=0$$

$$y^2+11y-9y-99=0$$

$$y(y+11)-9(y+11)=0$$

$$(y+11)(y-9)=0$$

Putting the values of y

$$(x^2+7x+11)(x^2+7x-9)=0$$

Either

$$x^2+7x+11=0 \quad \text{or} \quad x^2+7x-9=0$$

When $x^2+7x+11=0$

Solving by quadratic formula, we have

$$x=\frac{\sqrt{-7\pm-(7)^2-(1)(11)}}{2(1)}$$

$$=\frac{-7\pm\sqrt{49-44}}{2}$$

$= \dfrac{-7 \pm \sqrt{5}}{2}$

When $x^2 + 7x - 9 = 0$

$x = \dfrac{-7 \pm \sqrt{(7)^2 - 4(1)(-9)}}{2(1)}$

$= \dfrac{-7 \pm \sqrt{49 + 36}}{2}$

$= \dfrac{-7 \pm \sqrt{85}}{2}$

$\therefore$ **Solution Set** $= \left\{ \dfrac{-7 \pm \sqrt{5}}{2}, \dfrac{-7 \pm \sqrt{85}}{2} \right\}$

Exercise 1.3

Solve the following equations.

Q.1 $2x^4 - 11x^2 + 5 = 0$ **(A.B)**

Solution:

$2x^4 - 11x^2 + 5 = 0$

$2(x^2)^2 - 11x^2 + 5 = 0 \rightarrow (\text{i})$

Let $x^2 = y$

Putting $x^2 = y$ in equation (i)

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y - 5) - 1(y - 5) = 0$

$(y - 5)(2y - 1) = 0$

Either

$y - 5 = 0$ or $2y - 1 = 0$

$y = 5$ $\quad 2y = 1$

$y = \dfrac{1}{2}$

Putting $y = x^2$

$x^2 = 5$ or $x^2 = \dfrac{1}{2}$

Taking square root on both sides

$x = \pm\sqrt{5}$ $\quad x = \pm \dfrac{1}{\sqrt{2}}$

$\therefore$ **Solution Set** = $\left\{ \pm\sqrt{5}, \pm \dfrac{1}{\sqrt{2}} \right\}$

Q.2 $2x^4 = 9x^2 - 4$ **(A.B)**

Solution:

$2x^4 = 9x^2 - 4$

$2(x^2)^2 - 9x^2 + 4 = 0 \rightarrow (i)$

Let $x^2 = y$

Putting $x^2 = y$ in equation (i)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y(y - 4) - 1(y - 4) = 0$

$(y - 4)(2y - 1) = 0$

Either

$y - 4 = 0$ or $2y - 1 = 0$

$y = 4$ $\quad 2y = 1$

$y = \dfrac{1}{2}$

Putting $y = x^2$

$x^2 = 4$ or $x^2 = \dfrac{1}{2}$

Taking square root on both sides

$x = \pm 2$ $\quad x = \pm \dfrac{1}{\sqrt{2}}$

$\therefore$ **Solution set** = $\left\{ \pm 2, \pm \dfrac{1}{\sqrt{2}} \right\}$

Q.3 $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$ **(FSD 2016)** **(A.B)**

Solution:

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5(x^{\frac{1}{4}})^2 - 7x^{\frac{1}{4}} + 2 = 0 \rightarrow (\text{i})$

Let $x^{\frac{1}{4}} = y$

Putting $x^{\frac{1}{4}} = y$ in equation (i)

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y - 1) - 2(y - 1) = 0$

$(y - 1)(5y - 2) = 0$

Either

$y - 1 = 0$ or $5y - 2 = 0$

$y = 1$ or $5y = 2$

$$y=\frac{2}{5}$$

Putting $y=x^{1/4}$

$$x^{\frac{1}{4}}=1 \quad \text{or} \quad x^{\frac{1}{4}}=\frac{2}{5}$$

$$x=(1)^4 \qquad \left(x^{\frac{1}{4}}\right)^4=\left(\frac{2}{5}\right)^4$$

$$x=1 \qquad x=\frac{16}{625}$$

$\therefore$ **Solution Set** $=\left\{1,\frac{16}{625}\right\}$

Q.4 $x^{2/3}+54=15x^{1/3}$ **(A.B)**

Solution:

$$x^{2/3}+54=15x^{1/3}$$

$$\left(x^{\frac{1}{3}}\right)^2-15x^{\frac{1}{3}}+54=0\rightarrow(\text{i})$$

Let $x^{\frac{1}{3}}=y$

Putting $x^{\frac{1}{3}}=y$ in equation (i)

$$y^2-15y+54=0$$

$$y^2-9y-6y+54=0$$

$$y(y-9)-6(y-9)=0$$

$$(y-9)(y-6)=0$$

Either

$$y-9=0 \quad \text{or} \quad y-6=0$$

$$y=9 \quad \text{or} \quad y=6$$

Putting $y=x^{\frac{1}{3}}$

$$x^{\frac{1}{3}}=9 \quad \text{or} \quad x^{\frac{1}{3}}=6$$

Taking cube on both sides

$$\left(x^{\frac{1}{3}}\right)^3=(9)^3 \quad \text{or} \quad \left(x^{\frac{1}{3}}\right)^3=(6)^3$$

$$x=729 \quad \text{or} \quad x=216$$

$\therefore$ **Solution Set** $=\{729,216\}$

Q.5 $3x^{-2}+5=8x^{-1}$ **(SWL 2014)** **(A.B)**

Solution:

$$3x^{-2}+5=8x^{-1}$$

$$3(x^{-1})^2-8x^{-1}+5=0\rightarrow(\text{i})$$

Let $x^{-1}=y$

Putting $x^{-1}=y$ in equation (i)

$$3y^2-8y+5=0$$

$$3y^2-3y-5y+5=0$$

$$3y(y-1)-5(y-1)=0$$

$$(y-1)(3y-5)=0$$

Either

$$y-1=0 \quad \text{or} \quad 3y-5=0$$

$$y=1 \quad \text{or} \quad 3y=5$$

$$y=\frac{5}{3}$$

Putting $y=x^{-1}$

$$x^{-1}=1\ , \qquad x^{-1}=\frac{5}{3}$$

$$\frac{1}{x}=1\ , \qquad \frac{1}{x}=\frac{5}{3}$$

Taking reciprocal on both sides

$$\Rightarrow x=1 \qquad x=\frac{3}{5}$$

$\therefore$ **Solution Set** $=\left\{1,\frac{3}{5}\right\}$

Q.6 $(2x^2+1)+\frac{3}{2x^2+1}=4$ **(A.B)**

Solution:

$$(2x^2+1)+\frac{3}{2x^2+1}=4\rightarrow(\text{i})$$

Let $a=2x^2+1$

Putting the value of 'a' in equation (i)

$$a+\frac{3}{a}-4=0 \quad \text{(Mul both sides by } a\text{)}$$

$$a^2+3-4a=0$$

$$a^2-4a+3=0$$

$$a^2-3a-a+3=0$$

$$a(a-3)-1(a-3)=0$$

$$(a-3)(a-1)=0$$

Either

$a-3=0$ or $a-1=0$

$a=3$ $\quad a=1$

Putting $a = 2x^2 + 1$

$2x^2+1=3$ or $2x^2+1=1$

$2x^2=3-1$ $\quad 2x^2=1-1$

$2x^2=2$ $\quad 2x^2=0$

$x^2=1$ $\quad x^2=0$

Taking square root on both sides

$x=\pm 1$ $\quad x=0$

$\therefore$ **Solution Set** $=\{0,\pm 1\}$

Q.7 $\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4$ **(A.B)**

Solution:

$$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4 \rightarrow \text{(i)}$$

If $y=\frac{x}{x-3} \rightarrow \text{(ii)}$

Equation (i) $\Rightarrow y+\frac{4}{y}=4$

$y^2+4=4y$

$y^2-4y+4=0$

$(y^2)-2(y)(2)+(2)^2=0$

$(y-2)^2=0$

Taking square root on both sides

$y-2=0$

$y=2$

Putting value of y in equation (ii)

$\frac{x}{x-3}=2$

$x=2(x-3)$

$x=2x-6$

$6=2x-x$

$6=x$

Or $x=6$

$\therefore$ **Solution Set** $=\{6\}$

Q.8 $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$ **(A.B)**

Solution:

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6} \rightarrow \text{(i)}$$

Let $a=\frac{4x+1}{4x-1}$

Then,

$\frac{1}{a}=\frac{4x-1}{4x+1}$

Putting the values in equation (i)

$a+\frac{1}{a}=\frac{13}{6}$

$\frac{a^2+1}{a}=\frac{13}{6}$

$6a^2+6=13a$

$6a^2-13a+6=0$

$6a^2-9a-4a+6=0$

$3a(2a-3)-2(2a-3)=0$

$(2a-3)(3a-2)=0$

Either

$2a-3=0$ or $3a-2=0$

$2a=3$ $\quad 3a=2$

$a=\frac{3}{2}$ $\quad a=\frac{2}{3}$

Putting $a=\frac{4x+1}{4x-1}$

$\frac{4x+1}{4x-1}=\frac{3}{2}$ or $\frac{4x+1}{4x-1}=\frac{2}{3}$

$2(4x+1)=3(4x-1)$ or $3(4x+1)=2(4x-1)$

$8x+2=12x-3$ $\quad 12x+3=8x-2$

$12x-8x=2+3$ $\quad 12x-8x=-2-3$

$4x=5$ $\quad 4x=-5$

$x=\frac{5}{4}$ $\quad x=\frac{-5}{4}$

$\therefore$ **Solution Set** $=\left\{\pm\frac{5}{4}\right\}$

Q.9 $\dfrac{x-a}{x+a}-\dfrac{x+a}{x-a}=\dfrac{7}{12}$ **(A.B)**

Solution:

$$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}\rightarrow(\text{i})$$

Let $\dfrac{x-a}{x+a}=b$

$$\frac{1}{b}=\frac{x+a}{x-a}$$

Putting the values in equation (i)

$$b-\frac{1}{b}=\frac{7}{12}$$

$$b-\frac{1}{b}-\frac{7}{12}=0$$

Multiplying by $12b$

$$12b^2-12-7b=0$$

$$12b^2-7b-12=0$$

$$12b^2-16b+9b-12=0$$

$$4b(3b-4)+3(3b-4)=0$$

$$(3b-4)(4b+3)=0$$

Either

$3b-4=0$ or $4b+3=0$

$3b=4$ $\quad 4b=-3$

$b=\dfrac{4}{3}$ $\quad b=-\dfrac{3}{4}$

Put $b=\dfrac{x-a}{x+a}$

$\dfrac{x-a}{x+a}=-\dfrac{3}{4}$ $\quad \dfrac{x-a}{x+a}=+\dfrac{4}{3}$

$4(x-a)=-3(x+a)$ $\quad 3(x-a)=4(x+a)$

$4x-4a=-3x-3a$ $\quad 3x-3a=4x+4a$

$4x+3x=-3a+4a$ $\quad 3x-4x=4a+3a$

$7x=a$ $\quad -x=7a$

$x=\dfrac{a}{7}$ $\quad x=-7a$

$\therefore$ **Solution Set** = $\left\{-7a\ ,\ \dfrac{a}{7}\right\}$

Q.10 $x^4-2x^3-2x^2+2x+1=0$ **(A.B + K.B)**

Solution:

$$x^4-2x^3-2x^2+2x+1=0$$

Divide by x^2

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0\rightarrow(\text{i})$$

Let $x-\dfrac{1}{x}=\text{a}\rightarrow(\text{ii})$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=(a)^2$$

$$x^2+\frac{1}{x^2}-2=a^2$$

$$x^2+\frac{1}{x^2}=a^2+2$$

By putting values in equation (i)

$$(a^2+2)-2a-2=0$$

$$a^2+2-2a-2=0$$

$$a^2-2a=0$$

$$a(a-2)=0$$

Either

$a=0$ or $a-2=0$

$a=2$

putting the value of a in equation (ii)

when $a=0$ $\quad$ when $a=2$

$x-\dfrac{1}{x}=0\rightarrow(\text{iii})$ $\quad x-\dfrac{1}{x}=2\rightarrow(\text{iv})$

Equation (iii) $\Rightarrow x=\dfrac{1}{x}$

$$x^2=1$$

Taking square root on both sides

$$x=\pm1$$

Equation (iv) $\Rightarrow$

$x^2 - 1 = 2x$

$x^2 - 2x - 1 = 0$

Here

$a = 1, b = -2, c = -1$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{2\left(1 \pm \sqrt{2}\right)}{2}$$

$$= 1 \pm \sqrt{2}$$

$\therefore$ **Solution Set** = $\left\{\pm 1, 1 \pm \sqrt{2}\right\}$

Q.11 $2x^4 + x^3 - 6x^2 + x + 2 = 0$

(A.B + K.B)

Solution:

$2x^4 + x^3 - 6x^2 + x + 2 = 0 \rightarrow (\text{i})$

Dividing by x^2, we get

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0$$

Let $x + \frac{1}{x} = y \rightarrow (\text{ii})$

Squaring both sides

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting the values in equation (i)

$2\left(y^2 - 2\right) + y - 6 = 0$

$2y^2 - 4 + y - 6 = 0$

$2y^2 + y - 10 = 0$

$2y^2 + 5y - 4y - 10 = 0$

$y(2y+5) - 2(2y+5) = 0$

$(2y+5)(y-2) = 0$

Either

$2y + 5 = 0$ or $y - 2 = 0$

$2y = -5,$ $y = 2$

$$y = \frac{-5}{2}$$

Putting the values of y in equation (ii)

When $y = \frac{-5}{2}$

$$x + \frac{1}{x} = y$$

$$x + \frac{1}{x} = \frac{-5}{2}$$

$$\frac{x^2+1}{x} = \frac{-5}{2}$$

$2x^2 + 2 = -5x$

$2x^2 + 5x + 2 = 0$

$2x^2 + 4x + x + 2 = 0$

$2x(x+2) + 1(x+2) = 0$

$(x+2)(2x+1) = 0$

Either

$x + 2 = 0$ or $2x + 1 = 0$

$x = -2$ $2x = -1$

$$x = \frac{-1}{2}$$

When $y = 2$

$$x + \frac{1}{x} = y$$

$$x + \frac{1}{x} = 2$$

$$\frac{x^2+1}{x} = 2$$

$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

Taking square root on both sides

$x-1=0$

$x=1$

$\therefore$ **Solution Set** $=\left\{1,-2,-\frac{1}{2}\right\}$

Q.12 $4.2^{2x+1}-9.2^{x}+1=0$ **(A.B + K.B)**

Solution:

$4.2^{2x+1}-9.2^{x}+1=0$

$4.2^{2x}.2-9.2^{x}+1=0$

$8.(2^{x})^{2}-9.2^{x}+1=0\rightarrow(\text{i})$

Let $2^{x}=y\rightarrow(ii)$

Equation $(\text{i})\Rightarrow$

$8y^{2}-9y+1=0$

$8y^{2}-8y-y+1=0$

$8y(y-1)-1(y-1)=0$

$(y-1)(8y-1)=0$

Either

$y-1=0$ or $8y-1=0$

$y=1$ $\quad$ $8y=1$

$y=\frac{1}{8}$

Putting the value of y in equation (ii)

when $y=1$, when $y=\frac{1}{8}$

$2^{x}=y$, $\quad$ $2^{x}=y$

$2^{x}=1$, $\quad$ $2^{x}=\frac{1}{8}$

$2^{x}=2^{o}$, $\quad$ $2^{x}=\frac{1}{2^{3}}$

$2^{x}=2^{-3}$

$\therefore$ Bases are same

$\Rightarrow x=0$ $\quad$ $x=-3$

$\therefore$ **Solution Set** $=\{0,-3\}$

Q.13 $3^{2x+2}=12.3^{x}-3$ **(A.B + K.B)**

Solution:

$3^{2x+2}=12.3^{x}-3$

$3^{2x+2}-12.3^{x}+3=0$

$3^{2x}.\,3^{2}-12.3^{x}+3=0$

$9(3^{x})^{2}-12.3^{x}+3=0\rightarrow(\text{i})$

Let $3^{x}=y\rightarrow(\text{ii})$

Putting $3^{x}=y$ in equation (i)

$9y^{2}-12y+3=0$

$9y^{2}-9y-3y+3=0$

$9y(y-1)-3(y-1)=0$

$(y-1)(9y-3)=0$

Either

$y-1=0$ or $9y-3=0$

$y=1$ $\quad$ $9y=3$

$y=\frac{3}{9}$

$y=\frac{1}{3}$

Putting the value of y in equation (ii)

when $y=1$ $\quad$ when $y=\frac{1}{3}$

$3^{x}=y$ $\quad$ $3^{x}=y$

$3^{x}=1$ $\quad$ $3^{x}=\frac{1}{3}$

$3^{x}=3^{0}$ $\quad$ $3^{x}=\frac{1}{3^{1}}$

$3^{x}=3^{-1}$

$\therefore$ Bases are same

$x=0$ $\quad$ $x=-1$

$\therefore$ **Solution Set** $=\{0,-1\}$

Q.14 $2^{x}+64\times2^{-x}-20=0$ **(A.B + K.B)**

Solution:

$2^{x}+64\times2^{-x}-20=0$

$2^{x}+64\times2^{-x}-20=0$

$2^{x}+\frac{64}{2^{x}}-20=0\rightarrow(\text{i})$

Let $2^{x}=y\rightarrow(\text{ii})$

Put $2^{x}=y$ in equation (i)

$y+\frac{64}{y}-20=0$

Multiplying throughout by y

$y^{2}+64-20y=0$

$y^{2}-20y+64=0$

$y^2-4y-16y+64=0$

$y(y-4)-16(y-4)=0$

$(y-4)(y-16)=0$

Either

$y-4=0$ or $y-16=0$

$y=4$ $y=16$

Putting the value of y in equation (ii)

when $y=4$ when $y=16$

$2^x=y$ $2^x=y$

$2^x=4$ $2^x=16$

$2^x=2^2$ $2^x=2^4$

$x=2$ $x=4$

$\therefore$ **Solution Set** $=\{2,4\}$

Q.15 $(x+1)(x+3)(x-5)(x-7)=192$

(A.B + K.B + U.B)

Solution:

$(x+1)(x+3)(x-5)(x-7)=192$

$\because (1-5=-4 \quad 3-7=-4)$

$(x+1)(x-5)(x+3)(x-7)=192$

$(x^2-4x-5)(x^2-4x-21)=192 \rightarrow \text{(i)}$

Let $x^2-4x=y \rightarrow \text{(ii)}$

Equation (i) $\Rightarrow$

$(y-5)(y-21)=192$

$y^2-26y+105=192$

$y^2-26y+105-192=0$

$y^2-26y-87=0$

$y^2-29y+3y-87=0$

$y(y-29)+3(y-29)=0$

$(y-29)(y+3)=0$

Either

$y-29=0$ or $y+3=0$

$y=29$ or $y=-3$

Putting the values of y in equation (ii)

when $y=29$

$x^2-4x=y$

$x^2-4x=29$

$x^2-4x-29=0$

$a=1, b=-4, c=-29$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4\times1\times(-29)}}{2\times1}$$

$$x=\frac{4\pm\sqrt{16+116}}{2}$$

$$x=\frac{4\pm\sqrt{132}}{2}$$

$$x=\frac{4\pm\sqrt{4\times33}}{2}$$

$$x=\frac{4\pm2\sqrt{33}}{2}=2\left(\frac{2\pm\sqrt{33}}{2}\right)$$

$x=2\pm\sqrt{33}$

when $y=-3$

$x^2-4x=y$

$x^2-4x=-3$

$x^2-4x+3=0$

$x^2-3x-x+3=0$

$x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

Either

$x-3=0$ or $x-1=0$

$x=3$, $x=1$

$\therefore$ **Solution Set** = $\{1,3,2\pm\sqrt{33}\}$

Q.16 $(x-1)(x-2)(x-8)(x+5)+360=0$

(U.B + K.B)

Solution:

$(x-1)(x-2)(x-8)(x+5)+360=0$

$(x^2-2x-x+2)(x^2-8x+5x-40)+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0 \rightarrow \text{(i)}$

Let $x^2-3x=a \rightarrow \text{(ii)}$

Equation (i) $\Rightarrow$

$(a+2)(a-40)+360=0$

$a^2-40a+2a-80+360=0$

$a^2-38a+280=0$

$a^2-28a-10a+280=0$

$a(a-28)-10(a-28)=0$

$(a-28)(a-10)=0$

Either
$a - 28 = 0$ or $a - 10 = 0$
$a = 28$ $a = 10$
Putting the values of a in equation (ii)
When $a = 28$
$x^2 - 3x = 28$
$x^2 - 3x - 28 = 0$
$x^2 - 7x + 4x - 28 = 0$
$x(x-7) + 4(x-7) = 0$
$(x-7)(x+4) = 0$
Either
$x - 7 = 0$ or $x + 4 = 0$
$x = 7$ $x = -4$
when $a = 10$
$x^2 - 3x = 10$
$x^2 - 3x - 10 = 0$
$x^2 - 5x + 2x - 10 = 0$
$x(x-5) + 2(x-5) = 0$
$(x-5)(x+2) = 0$
Either
$x - 5 = 0$ or $x + 2 = 0$
$x = 5$ $x = -2$
$\therefore$ **Solution Set** $= \{7, -4, 5, -2\}$

Radical Equation (K.B)

(LHR 2016, GRW 2014, 17, BWP 2015, 17, MTN 2015, SWL 2016, SGD 2017, D.G.K 2016, 17)

An equation in which a variable or an algebraic expression occurs under radical sign is called radical equation.

e.g. $\sqrt{ax+b} = cx+d,\ 2\sqrt{x} - 3 = 0$ etc.

Extraneous Root (K.B)

Value of variable obtained by solving the equation not satisfying it is called extraneous root.

Note (U.B + K.B)

- Roots of radical equation must be verified.
- Extraneous is introduced by either squaring the given equation or clearing it of fractions.

Type 1 (K.B)

Equation of the type:
$\sqrt{ax+b} = cx+d$

Example 1: (Page # 12)

Solve the equation $\sqrt{3x+7} = 2x+3$

Solution:

$\left(\sqrt{3x+7}\right)^2 = (2x+3)^2$
$3x + 7 = 4x^2 + 12x + 9$
$4x^2 + 9x + 2 = 0$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4(4)(2)}}{2(4)}$$
$$= \frac{-9 \pm \sqrt{81-32}}{8}$$
$$= \frac{-9 \pm \sqrt{49}}{8}$$
$$= \frac{-9 \pm 7}{8}$$

Either
$x = \frac{-9+7}{8}$ or $x = \frac{-9-7}{8}$
$= \frac{-2}{8}$ $= \frac{-16}{8}$
$= \frac{-1}{4}$ $= -2$

Checking:

Putting $x = -\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right)+7} = 2\left(-\frac{1}{4}\right)+3$$
$$\sqrt{\frac{-3+28}{4}} = -\frac{1}{2}+3$$
$$\sqrt{\frac{25}{4}} = \frac{5}{2}$$
$$\frac{5}{2} = \frac{5}{2} \quad \text{(True)}$$

Now putting $x = -2$ in equation (i)
$$\sqrt{3(-2)+7} = 2(-2)+3$$
$$\sqrt{1} = -1 \quad \text{(False)}$$

Thus, the solution set is $\left\{-\frac{1}{4}\right\}$**.** -2 **is an extraneous root.**

Type 2

Equation of the type:

$$\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$$

Example 2: (Page # 13)

Solve the equation

$$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$$

(A.B + K.B + U.B)

Solution:

$$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$$

Squaring both sides

$$x+3+x+6+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$2\sqrt{x^2+9x+18}=-x+2$$

Squaring both sides of the equation (ii), we get

$$4\left(x^2+9x+18\right)=x^2-4x+4$$

$$3x^2+40x+68=0$$

Applying quadratic formula,

$$x=\frac{-40\pm\sqrt{(40)^2-4(3)(68)}}{2(3)}$$

$$=\frac{-40\pm\sqrt{1600-816}}{6}$$

$$=\frac{-40\pm\sqrt{784}}{6}$$

$$=\frac{-40\pm 28}{6}$$

Either

$$x=\frac{-40+28}{6} \quad \text{or} \quad x=\frac{-40-28}{6}$$

$$=\frac{-12}{6} \quad \text{or} \quad =\frac{-68}{6}$$

$$=-2 \quad\quad =\frac{-34}{3}$$

Checking

Putting $x=-2$ in the equation (i)

$$\sqrt{-2+3}+\sqrt{-2+6}=\sqrt{-2+11}$$

$$\sqrt{1}+\sqrt{4}=\sqrt{9}$$

$$1+2=3$$

$$3=3 \quad \text{(True)}$$

Now, putting $x=\frac{-34}{3}$ in the equation (i)

$$\sqrt{\frac{-34}{3}+3}+\sqrt{\frac{-34}{3}+6}=\sqrt{\frac{-34}{3}+11}$$

$$\sqrt{\frac{-34+9}{3}}+\sqrt{\frac{-34+18}{3}}=\sqrt{\frac{-34+33}{3}}$$

$$\sqrt{\frac{-25}{3}}+\sqrt{\frac{-16}{3}}=\sqrt{\frac{-1}{3}}$$

$$\frac{5i}{\sqrt{3}}+\frac{4i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$$\frac{9i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \quad \text{(False)}$$

Thus, solution set is $\{-2\}$, $\frac{-34}{3}$ is an extraneous root.

Type 3

Equation of the type:

$$\sqrt{x^2+px+m}+\sqrt{x^2+px+n}=q$$

Example 3: (Page # 14)

Solve the equation

$$\sqrt{x^2-3x+36}-\sqrt{x^2-3x+9}=3$$

(A.B + K.B + U.B)

Solution:

$$\sqrt{x^2-3x+36}-\sqrt{x^2-3x+9}=3 \rightarrow \text{(i)}$$

Let $x^2-3x=y \rightarrow \text{(ii)}$

Equation (i) $\Rightarrow$

$$\sqrt{y+36}-\sqrt{y+9}=3$$

$$\Rightarrow x=3$$

Squaring both sides

$$\left(\sqrt{y+36}-3\right)^2=\left(\sqrt{y+9}\right)^2$$

$$\left(\sqrt{y+36}\right)^2+(3)^2-2(3)\sqrt{y+36}=y+9$$

$$y+36+9-6\sqrt{y+36}=y+9$$

$$36=6\sqrt{y+36}$$

$$6=\sqrt{y+36}$$

Again squaring both sides

$36 = y + 36$

$\Rightarrow \; y = 0$

Putting the value of y in equation (ii)

$x^2 - 3x = 0$

$x(x-3) = 0$

Either

$x = 0$ or $x - 3 = 0$

$\Rightarrow \; x = 3$

On checking it is found that 0 and 3 are roots of the given equation.

Thus, solution set $= \{0, 3\}$

Exercise 1.4

Solve the following equations.

Q.1 $2x + 5 = \sqrt{7x+16}$ **(A.B)**

Solution:

$2x + 5 = \sqrt{7x+16} \longrightarrow$ (i)

Taking square on both sides

$(2x+5)^2 = \left(\sqrt{7x+16}\right)^2$

$4x^2 + 20x + 25 = 7x + 16$

$4x^2 + 20x - 7x + 25 - 16 = 0$

$4x^2 + 13x + 9 = 0$

$4x^2 + 9x + 4x + 9 = 0$

$x(4x+9) + 1(4x+9) = 0$

$(4x+9)(x+1) = 0$

Either

$4x + 9 = 0$ or $x + 1 = 0$

$4x = -9 \quad x = -1$

$x = \frac{-9}{4}$

Check

Put $x = \frac{-9}{4}$ in equation (i)

$2\left(\frac{-9}{4}\right) + 5 = \sqrt{7\left(\frac{-9}{4}\right) + 16}$

$\frac{-18}{4} + 5 = \sqrt{\frac{-63}{4} + 16}$

$\frac{-18+20}{4} = \sqrt{\frac{-63+64}{4}}$

$\frac{2}{4} = \sqrt{\frac{1}{4}}$

$\frac{1}{2} = \frac{1}{2}$ Satisfied

Put $x = -1$ in equation (i)

$2(-1) + 5 = \sqrt{7(-1)+16}$

$-2 + 5 = \sqrt{-7+16}$

$3 = \sqrt{9}$

$3 = 3$ Satisfied

$\therefore$ **Solution Set =** $\left\{\frac{-9}{4}, -1\right\}$

Q.2 $\sqrt{x+3} = 3x - 1$ **(FSD 2014)** **(A.B)**

Solution:

$\sqrt{x+3} = 3x - 1 \longrightarrow$ (i)

Taking square on both sides

$\left(\sqrt{x+3}\right)^2 = (3x-1)^2$

$x + 3 = 9x^2 - 6x + 1$

$-9x^2 + x + 6x + 3 - 1 = 0$

$-9x + 7x + 2 = 0$

$-(9x - 7x - 2) = 0$

$9x - 7x - 2 = 0$

$9x^2 - 9x + 2x - 2 = 0$

$9x(x-1) + 2(x-1) = 0$

$(x-1)(9x+2) = 0$

Either

$x - 1 = 0$ or $9x + 2 = 0$

$x = 1 \quad 9x = -2$

$x = \frac{-2}{9}$

Check

Put $x = 1$ in equation (i)

$\sqrt{(1)+3} = 3(1) - 1$

$\sqrt{4} = 3 - 1$

$2 = 2$ Satisfied

Put $x = \frac{-2}{9}$ in equation (i)

$\sqrt{\left(\frac{-2}{9}\right) + 3} = 3\left(\frac{-2}{9}\right) - 1$

$\sqrt{\frac{-2+27}{9}}=\frac{-6}{9}-1$

$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$

$\frac{5}{3}=\frac{-15}{9}$

$\frac{5}{3}=\frac{-5}{3}$ Not Satisfied

$\therefore$ **Solution Set = {1},**

$\frac{-2}{9}$ **is extraneous root.**

Q.3 $4x+3=\sqrt{13x+14}$ **(A.B)**

Solution:

$4x+3=\sqrt{13x+14}\rightarrow(\text{i})$

Squaring both sides, we get

$(4x+3)^2=\left(\sqrt{13x+14}\right)^2$

$(4x)^2+2(4x)(3)+(3)^2=13x+14$

$16x^2+24x+9-13x-14=0$

$16x^2+11x-5=0$

$16x^2+16x-5x-5=0$

$16x(x+1)-5(x+1)=0$

$(x+1)(16x-5)=0$

Either

$x+1=0$ or $16x-5=0$

$x=-1$ $\quad 16x=5$

$x=\frac{5}{16}$

Check

Put $x=-1$ in equation (i)

$4x=\sqrt{13x+14}-3$

$4(-1)=\sqrt{13(-1)+14}-3$

$-4=\sqrt{-13+14}-3$

$-4=\sqrt{1}-3$

$-4=1-3$

$-4\neq-2$ (Not true)

Put $x=\frac{5}{16}$ in equation (i)

$4x=\sqrt{13x+14}-3$

$4\left(\frac{5}{16}\right)=\sqrt{13\left(\frac{5}{16}\right)+14}-3$

$\frac{5}{4}=\sqrt{\frac{65}{16}+14}-3$

$\frac{5}{4}=\sqrt{\frac{65+224}{16}}-3$

$\frac{5}{4}=\sqrt{\frac{289}{16}}-3$

$\frac{5}{4}=\frac{17}{4}-3$

$\frac{5}{4}=\frac{17-12}{4}$

$\frac{5}{4}=\frac{5}{4}$ True

$\therefore$ **Solution Set** $=\left\{\frac{5}{16}\right\}$,

Extraneous root **=** –1

Q.4 $\sqrt{3x+100}-x=4$ **(A.B)**

Solution:

$\sqrt{3x+100}-x=4\rightarrow(\text{i})$

$\sqrt{3x+100}=4+x$

Taking square on both sides

$\left(\sqrt{3x+100}\right)^2=(4+x)^2$

$3x+100=16+8x+x^2$

$-x^2+3x-8x+100-16=0$

$-x^2-5x+84=0$

$-(x^2+5x-84)=0$

Or $x^2+5x-84=0$

$x^2+12x-7x-84=0$

$x(x+12)-7(x+12)=0$

$(x-7)(x+12)=0$

Either

$x-7=0$ or $x+12=0$

$x=7$ $\quad x=-12$

Check

Put x = 7 in equation (i)

$\sqrt{3(7)+100}-7=4$

$\sqrt{21+100}-7=4$

$\sqrt{121}-7=4$

$11-7=4$

4 = 4 Satisfied

Put x = -12 in equation (i)

$\sqrt{3(-12)+100}-(-12)=4$

$\sqrt{-36+100}+12=4$

$\sqrt{64}+12=4$

8 + 12 = 4

20 = 4 Not satisfied

$\therefore$ **Solution Set** $=\{7\}$, -12 **is extraneous root.**

Q.5 $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

(A.B + U.B)

Solution:

$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \rightarrow$ (i)

Taking square on both sides

$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$

$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$

$x+5+x+21+2\sqrt{(x+5)(x+21)}=x+60$

$2x+26+2\sqrt{x^2+26x+105}=x+60$

$2x-x+26-60=-2\sqrt{x^2+26x+105}$

$x-34=-2\sqrt{x^2+26x+105}$

Again squaring both sides

$(x-34)^2=\left(-2\sqrt{x^2+26x+105}\right)^2$

$x^2-68x+1156=4(x^2+26x+105)$

$x^2-68x+1156=4x^2+104x+420$

$x^2-4x^2-68x-104x+1156-420=0$

$-3x^2-172x+736=0$

$-(3x^2+172x-736)=0$

Or $3x^2+172x-736=0$

As $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here $a=3$, $b=172$, $c=-736$

Putting the values

$x=\dfrac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$

$=\dfrac{-172\pm\sqrt{29584+8832}}{6}$

$=\dfrac{-172\pm\sqrt{38416}}{6}$

$=\dfrac{-172\pm196}{6}$

Either

$x=\dfrac{-176-196}{6}$ or $x=\dfrac{-176+196}{6}$

$=\dfrac{-368}{6}$ $\quad x=\dfrac{24}{6}$

$x=\dfrac{-184}{3}$ $\quad x=4$

Check

Put $x=\dfrac{-184}{3}$ in equation (i)

$\sqrt{\dfrac{-184}{3}+5}+\sqrt{\dfrac{-184}{3}+21}=\sqrt{\dfrac{-184}{3}+60}$

$\sqrt{\dfrac{-184+15}{3}}+\sqrt{\dfrac{-184+63}{3}}=\sqrt{\dfrac{-184+180}{3}}$

$\sqrt{\dfrac{-169}{3}}+\sqrt{\dfrac{-121}{3}}=\sqrt{\dfrac{-4}{3}}$

$\dfrac{13}{\sqrt{3}}\text{i}+\dfrac{11}{\sqrt{3}}\text{i}=\dfrac{2}{3}\text{i}$

$\dfrac{24}{\sqrt{3}}i=\dfrac{2}{3}i$

Not satisfied

Put $x=4$ in equation (i)

$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$

$\sqrt{9}+\sqrt{25}=\sqrt{64}$

3 + 5 = 8

8 = 8 Satisfied

$\therefore$ **Solution Set** $=\{4\}$, $\dfrac{-184}{3}$

is extraneous root

Q.6 $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

(A.B + U.B)

Solution:

$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6} \rightarrow (i)$

Taking square on both sides

$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$

$\left(\sqrt{x+1}\right)^2+\left(\sqrt{x-2}\right)^2+2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right)=x+6$

$x+1+x-2+2\sqrt{x^2-x-2}=x+6$

$2x-1+2\sqrt{x^2-x-2}=x+6$

$2x-x-1-6=-2\sqrt{x^2-x-2}$

$x-7=-2\sqrt{x^2-x-2}$

Again taking square on both sides

$(x-7)^2=\left(-2\sqrt{x^2-x-2}\right)^2$

$x^2-14x+49=4(x^2-x-2)$

$x^2-14x+49=4x^2-4x-8$

$x^2-4x^2-14x+4x+49+8=0$

$-3x^2-10x+57=0$

$-(3x^2+10x-57)=0$

Or $3x^2+10x-57=0$

$3x^2+19x-9x-57=0$

$x(3x+19)-3(3x+19)=0$

$(x-3)(3x+19)=0$

Either

$x-3=0$ or $3x+19=0$

$x=3$ $3x=-19$

$x=\frac{-19}{3}$

Check

Put $x = 3$ in equation (i)

$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$

$\sqrt{4}+\sqrt{1}=\sqrt{9}$

2 + 1 = 3

3 = 3 Satisfied

Put $x=\frac{-19}{3}$ in equation (i)

$\sqrt{\frac{-19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{\frac{-19}{3}+6}$

$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$

$\sqrt{\frac{-16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{\frac{-1}{3}}$

$\frac{4}{\sqrt{3}}i+\frac{5}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i$

$\frac{4i+5i}{\sqrt{3}}=\frac{1}{\sqrt{3}}i$

$\frac{9}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i$, Not satisfied

$\therefore$ **Solution Set =** $\{3\}$**,** $\frac{-19}{3}$ **is extraneous root**

Q.7 $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

(A.B + U.B)

Solution:

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x} \rightarrow (i)$

Squaring both sides, we get

$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$

$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2-2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$

$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$

$17-2x-2\sqrt{66-11x-6x+x^2}=27-x$

$-2\sqrt{66-17x+x^2}=27-x-17+2x$

$-2\sqrt{x^2-17x+66}=x+10$

Again squaring both sides

$\left(-2\sqrt{x^2-17x+66}\right)^2=(x+10)^2$

$4(x^2-17x+66)=x^2+20x+100$

$4x^2-68x+264-x^2-20x-100=0$

$3x^2-88x+164=0$

$a=3,\quad b=-88,\quad c=164$

Using Quadratic Formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2\times 3}$

$x=\frac{88\pm\sqrt{7744-1968}}{6}$

$x=\frac{88\pm\sqrt{5776}}{6}$

$x=\frac{88\pm 76}{6}$

$x=\frac{88+76}{6}$, $x=\frac{88-76}{6}$

$x=\frac{164}{6}$, $x=\frac{12}{6}$

$x=\frac{82}{3}$, $x=2$

Check

When $x=2$

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$

$\sqrt{9}-\sqrt{4}=\sqrt{25}$

$3-2=5$

$1=5$ (Not true)

Put $x=\frac{82}{3}$ in equation (i)

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$

$\sqrt{\frac{33-82}{3}}-\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$

$\sqrt{\frac{-49}{3}}-\sqrt{\frac{-64}{3}}=\sqrt{\frac{-1}{3}}$

$\frac{7}{\sqrt{3}}i-\frac{8}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i$

$\frac{7i-8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$

$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$ (Not true)

$\therefore$ **Solution Set** $=\{\ \}$

Extraneous roots $=\frac{82}{3},2$

Q.8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

(A.B)

Solution:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}\rightarrow(i)$

Taking sq. on both sides

$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$

$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$

$4a+x+a-x-2\sqrt{(4a+x)(a-x)}=a$

$5a-2\sqrt{4a^2-3ax-x^2}=a$

$5a-a=2\sqrt{4a^2-3ax-x^2}$

$4a=2\sqrt{4a^2-3ax-x^2}$

Again taking square on both sides

$(4a)^2=(2\sqrt{4a^2-3ax-x^2})^2$

$16a^2=4(4a^2-3ax-x^2)$

$16a^2=16a^2-12ax-4x^2$

$16a^2-16a^2+12ax+4x^2=0$

$12ax+4x^2=0$

$4x(3a+x)=0$

Either

$4x=0$ or $3a+x=0$

$x=\frac{0}{4}$ $x=-3a$

$x=0$

Check

Put $x=0$ in equation (i)

$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$

$\sqrt{4a}-\sqrt{a}=\sqrt{a}$

$2\sqrt{a}-\sqrt{a}=\sqrt{a}$

$\sqrt{a}=\sqrt{a}$ $\therefore$ Satisfied

Put $x=-3a$ in equation (i)

$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$

$\sqrt{a}-\sqrt{4a}=\sqrt{a}$

$\sqrt{a}-2\sqrt{a}=\sqrt{a}$

$-\sqrt{a}=\sqrt{a}$ Not satisfied

$\therefore$ **Solution Set = {0},** $-3a$ **is extraneous root**

Q.9 $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ **(A.B)**

Solution:

$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1\rightarrow(\text{i})$

Let $x^2+x=y\rightarrow(\text{ii})$

Equation (i) $\Rightarrow\sqrt{y+1}-\sqrt{y-1}=1$

Taking square on both sides

$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=(1)^2$

$\left(\sqrt{y+1}\right)^2+\left(\sqrt{y-1}\right)^2-2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right)=1$

$y+1+y-1-2\sqrt{(y+1)(y-1)}=1$

$2y-2\sqrt{y^2-1}=1$

$2y-1=2\sqrt{y^2-1}$

Again taking square on both sides

$(2y-1)^2=(2\sqrt{y^2-1})^2$

$4y^2-4y+1=4(y^2-1)$

$4y^2-4y+1=4y^2-4$

$4y^2-4y^2-4y+1+4=0$

$-4y+5=0$

$-4y=-5$

$y=\frac{5}{4}$

Put $y=\frac{5}{4}$ in equation (ii)

$x^2+x=\frac{5}{4}$

$4\left(x^2+x\right)=5$

$4x^2+4x=5$

$4x^2+4x-5=0$

Here a = 4, b = 4, c = -5

As $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Putting the values

$x=\frac{-(4)\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$

$x=\frac{-4\pm\sqrt{16+80}}{8}$

$x=\frac{-4\pm\sqrt{96}}{8}$

$x=\frac{-4\pm4\sqrt{6}}{8}$

$x=4\frac{(-1\pm\sqrt{6})}{8}$

$x=\frac{-1\pm\sqrt{6}}{2}$

On checking, we know that these values satisfy the equation.

$\therefore$ **Solution Set** $=\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

Q.10 $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ **(A.B)**

Solution:

$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3\rightarrow(\text{i})$

Let $x^2+3x=y\rightarrow(\text{ii})$

Equation (i) $\Rightarrow\sqrt{y+8}+\sqrt{y+2}=3$

Taking square on both sides

$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$

$(\sqrt{y+8})^2+(\sqrt{y+2})^2+2(\sqrt{y+8})(\sqrt{y+2})=9$

$y+8+y+2+2\sqrt{(y+8)(y+2)}=9$

$2y+10+2\sqrt{y^2+10y+16}=9$

$2y+10-9=-2\sqrt{y^2+10y+16}$

$2y+1=-2\sqrt{y^2+10y+16}$

Again taking square on both sides

$(2y+1)^2=(-2\sqrt{y^2+10y+16})^2$

$4y^2+4y+1=4(y^2+10y+16)$

$4y^2+4y+1=4y^2+40y+64$

$4y^2-4y^2+4y-40y+1-64=0$

$-36y-63=0$

$-36y=63$

$y=\frac{63}{-36}$

$y=\frac{-7}{4}$

Put y = $\frac{-7}{4}$ in equation (i)

$$x^2+3x=\frac{-7}{4}$$
$$4(x^2+3x)=-7$$
$$4x^2+12x=-7$$
$$4x^2+12x+7=0$$

Here a = 4, b = 12, c = 7

As $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Putting the values

$$x=\frac{-12\pm\sqrt{(12)^2-4(4)(7)}}{2(4)}$$
$$x=\frac{-12\pm\sqrt{144-112}}{8}$$
$$x=\frac{-12\pm\sqrt{32}}{8}$$
$$x=\frac{-12\pm4\sqrt{2}}{8}$$
$$x=4\frac{(-3\pm\sqrt{2})}{8}$$
$$x=\frac{-3\pm\sqrt{2}}{2}$$

On checking, we know that these values satisfy the equation.

$\therefore$ **Solution Set** = $\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

Q.11 $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

(A.B)

Solution:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5\rightarrow(i)$$

Let $x^2+3x=y\rightarrow(ii)$

Put in equation (i) ⇒

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring on both sides, we get

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=(5)^2$$
$$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\sqrt{y+9}\sqrt{y+4}=25$$
$$y+9+y+4+2\sqrt{(y+9)(y+4)}=25$$
$$2y+13+2\sqrt{y^2+13y+36}=25$$
$$2\sqrt{y^2+13y+36}=25-13-2y$$
$$2\sqrt{y^2+13y+36}=12-2y$$
$$2\sqrt{y^2+13y+36}=2(6-y)$$
$$\sqrt{y^2+13y+36}=(6-y)$$

Again squaring both sides.

$$\left(\sqrt{y^2+13y+36}\right)^2=(6-y)^2$$
$$y^2+13y+36=36+y^2-12y$$
$$13y+12y=36-36$$
$$25y=0$$
$$y=0$$

Put in equation (ii)

$$x^2+3x=y$$
$$x^2+3x=0$$
$$x(x+3)=0$$

Either

$x=0$ *or* $x+3=0$

$x=0$ *or* $x=-3$

Check

When $x=0$

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$$
$$\sqrt{0^2+3(0)+9}+\sqrt{0^2+3(0)+4}=5$$
$$\sqrt{0+9}+\sqrt{0+4}=5$$
$$\sqrt{9}+\sqrt{4}=5$$
$$3+2=5$$

$5=5$ (True)

When $x=-3$

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$$
$$\sqrt{(-3)^2+3(-3)+9}+\sqrt{(-3)^2+3(-3)+4}=5$$
$$\sqrt{9-9+9}+\sqrt{9-9+4}=5$$
$$\sqrt{9}+\sqrt{4}=5$$
$$3+2=5$$

$5=5$ (True)

$\therefore$ **Solution Set** $=\{0,-3\}$

Miscellaneous Exercise 1

Q.1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is; **(K.B)**
(GRW 2014, 16, MTN 2017, SGD 2015, 17, FSD 2018, RWP 2015)

(a) $bx + c = 0, b \neq 0$ **(b)** $ax^2 + bx + c = 0, a \neq 0$
(c) $ax^2 = bx, a \neq 0$ **(d)** $ax^2 = 0, a \neq 0$

(ii) The number of terms in a standard quadratic equation $ax^2 + bx + c = 0$ **is;** **(K.B)**
(LHR 2015, BWP 2015, MTN 2015)

(a) 1 **(b)** 2
(c) 3 **(d)** 4

(iii) The number of methods to solve a quadratic equation is; **(K.B)**
(GRW 2017, FSD 2014, 17, SWL 2015, 16, RWP 2016, 17, D.G.K 2014, 15, 17)

(a) 1 **(b)** 2
(c) 3 **(d)** 4

(iv) The quadratic formula is; **(FSD 2015, D.G.K 2014, SWL 2014, 17)** **(A.B)**

(a) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **(b)** $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$
(c) $x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ **(d)** $x = \frac{b \pm \sqrt{b^2 + 4ac}}{2a}$

(v) Two linear factors of $x^2 - 15x + 56$ **are;** **(A.B + U.B)**
(LHR 2014, FSD 2016, RWP 2017, D.G.K 2016)

(a) $(x-7)$ and $(x+8)$ **(b)** $(x+7)$ and $(x-8)$
(c) $(x-7)$ and $(x-8)$ **(d)** $(x+7)$ and $(x+8)$

(vi) An equation, which remains unchanged when x **is replaced by** $\frac{1}{x}$ **is called a/an;** **(K.B)**

(a) Exponential equation **(b)** Reciprocal equation
(c) Radical equation **(d)** None of these

(vii) An equation of the type $3^x + 3^{2-x} + 6 = 0$ **is a/an;** **(K.B)**
(GRW 2016, SGD 2014, 16, D.G.K 2015, 17)

(a) Exponential equation **(b)** Radical equation
(c) Reciprocal equation **(d)** None of these

(viii) The solution set of equation $4x^2 - 16 = 0$ **is;** **(K.B + U.B + A.B)**

(a) $\{\pm 4\}$ **(b)** $\{4\}$
(c) $\{\pm 2\}$ **(d)** ± 2

(ix) The equation of the form $2x^4 - 3x^3 + 7x^2 - 3x + 2 = 0$ **is called a/an;** **(A.B+K.B+U.B)**

(a) Reciprocal equation **(b)** Radical equation
(c) Exponential equation **(d)** None of these

ANSWER KEY

i.	ii.	iii.	iv.	v.	vi.	vii.	viii.	ix.
b	c	c	a	c	b	a	c	a

Q.2 Write short answers of the following questions. **(A.B)**

(i) Solve $x^2+2x-2=0$

(LHR 2017, SWL 2017, SGD 2014, 17, RWP 2015)

Solution:

$x^2+2x-2=0$

$x^2+2x=2$

Adding $(1)^2$ on both sides

$x^2+2x+(1)^2=2+(1)^2$

$(x+1)^2=2+1$

$(x+1)^2=3$

Taking sq. root on both sides

$x+1=\pm\sqrt{3}$

$x=-1\pm\sqrt{3}$

$\therefore$ **Solution Set** = $\{-1\pm\sqrt{3}\}$

(ii) Solve by factorization $5x^2=15x$

(LHR 2015, 16, GRW 2014, 16, 17, SWL 2016, 17, BWP 2014, 16, D.G.K 2017)

(A.B)

Solution:

$5x^2=15x$

$5x^2-15x=0$

$5x(x-3)=0$

Either

$5x=0$ or $x-3=0$

$x=0$ $\quad$ $x=3$

$\therefore$ **Solution Set** $=\{0,3\}$

(iii) Write in standard form $\frac{1}{x+4}+\frac{1}{x-4}=3$

(FSD 2017, SWL 2016, SGD 2015, BWP 2015, MTN 2014, D.G.K 2014) **(A.B)**

Solution:

$\frac{1}{x+4}+\frac{1}{x-4}=3$

Multiply both sides by

$(x+4)(x-4)$,

we get

$x-4+x+4=3(x+4)(x-4)$

$2x=3(x^2-16)$

$2x=3x^2-48$

$3x^2-2x-48=0$

Which is in standard form of quadratic equation.

(iv) Write the name of the methods for solving a quadratic equation. **(A.B)**

Solution:

Following are the three methods:

(a) Factorization

(b) Completing square

(c) Quadratic Formula

(v) Solve $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Solution:

$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Taking square root on both sides

$\sqrt{\left(2x-\frac{1}{2}\right)^2}=\sqrt{\frac{9}{4}}$

$2x-\frac{1}{2}=\pm\frac{3}{2}$

Either

$2x-\frac{1}{2}=\frac{3}{2}$ or $2x-\frac{1}{2}=-\frac{3}{2}$

$2x=\frac{3}{2}+\frac{1}{2}$, $2x=-\frac{3}{2}+\frac{1}{2}$

$2x=\frac{3+1}{2}$, $2x=\frac{-3+1}{2}$

$2x=\frac{4}{2}$, $2x=\frac{-2}{2}$

$2x=2$, $2x=-1$

$x=\frac{2}{2}$, $x=-\frac{1}{2}$

$x=1$

$\therefore$ **Solution Set** $=\left\{-\frac{1}{2},1\right\}$

(vi) Solve $\sqrt{3x+18}=x$ **(A.B)**

Solution:

$\sqrt{3x+18}=x$

Squaring both sides

$\left(\sqrt{3x+18}\right)^2=(x)^2$

$3x+18=x^2$

$x^2-3x-18=0$

$x^2-6x+3x-18=0$

$x(x-6)+3(x-6)=0$

$(x-6)(x+3)=0$

Either

$x-6=0$ or $x+3=0$

$x=6$ or $x=-3$

Verification

When $x=6$ When $x=-3$

$\sqrt{3x+18}=x$ $\sqrt{3x+18}=x$

$\sqrt{3(6)+18}=6$ $\sqrt{3(-3)+18}=-3$

$\sqrt{18+18}=6$ $\sqrt{-9+18}=-3$

$\sqrt{36}=6$ $\sqrt{9}=-3$

$6=6$ (True) $3=-3$ (false)

$\therefore$ **Solution Set** $=\{6\}$

Extraneous root = -3

(vii) Define quadratic equation. **(U.B)**

Answer: See definition Page # 1

(viii) Define reciprocal equation. **(U.B)**

Answer: See definition Page # 14

(ix) Define exponential equation. **(U.B)**

Answer: See definition Page # 14

(x) Define radical equation. **(U.B)**

Answer: See definition Page # 24

Q.3 Fill in the blanks

(i) The standard form of the quadratic equation is________. **(K.B)**

(ii) The number of methods to solve a quadratic equation are________. **(K.B)**

(iii) The name of the method to derive a quadratic formula is________. **(K.B)**

(iv) The solution of the equation $ax^2+bx+c=0$, $a\neq 0$ is ________. **(K.B + A.B)**

(v) The solution set of $25x^2-1=0$ is ________.

(vi) An equation of the form $2^{2x}-3.2^x+5=0$ is called a/an ________equation. **(U.B)**

(GRW 2014, 17, SGD 2016, 14, BWP 2016)

(vii) The solution set of the equation $x^2-9=0$ is ________. **(U.B)**

(viii) An equation of the type $x^4+x^3+x^2+x+1=0$ called a/ an ________equation. **(U.B)**

(ix) A root of an equation, which do not satisfy the equation is called ________root. **(U.B)**

(x) An equation involving impression of the variable under ________ is called radical equation. **(U.B)**

ANSWER KEY

(i) $ax^2+bx+c=0$

(ii) 3

(iii) Completing square

(iv) $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

(v) $\left\{\pm\dfrac{1}{5}\right\}$

(vi) Exponential

(vii) $\{\pm 3\}$

(viii) Reciprocal

(ix) Extraneous

(x) Radical sign

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 A quadratic equation can be solved by:
(A) Factorization **(B)** Completing square
(C) Quadratic formula **(D)** All of these

2 In quadratic equation $ax^2+bx+c=0$, $a,b,c,\in$ _______
(A) N **(B)** Q'
(C) Z **(D)** R

3 The equation $2x^4-3x^3+7x^2-3x+2=0$ is called a/an
(A) Reciprocal equation **(B)** Radical equation
(C) Exponential equation **(D)** None of these

4 The solution set of the equation $4x^2-16=0$ is
(A) {±4} **(B)** {4}
(C) {±2} **(D)** {–2}

5 Two linear factors of $x^2-15x+56$ are:
(A) $(x-7)$, $(x-8)$ **(B)** $(x+7)$, $(x+8)$
(C) $(x+7)$, $(x-8)$ **(D)** $(x-7)$, $(x+8)$

6 The quadratic formula is:
(A) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ **(B)** $\frac{b\pm\sqrt{b^2-4ac}}{2a}$
(C) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ **(D)** $\frac{b\pm\sqrt{b^2+4ac}}{2a}$

7 Solution set of $5x^2=30x$
(A) {6} **(B)** {–6}
(C) {0,6} **(D)** {25}

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Define quadratic equation? Write the standard form of quadratic equation.
(ii) Find the solution set of the standard quadratic equation $ax^2+bx+c=0$ for $a=1, b=-3, c=-5$.
(iii) Solve: $\sqrt{3x+18}=x$
(iv) Solve: $2x^4-11x^2+5=0$
(v) Solve: $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Q.3 Answer the following Questions. **(4+4=8)**

(a) Solve: $3^{2x+2}=12.3^x-3$
(b) Solve the equation by completing square method. $x^2-2x-195=0$.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.