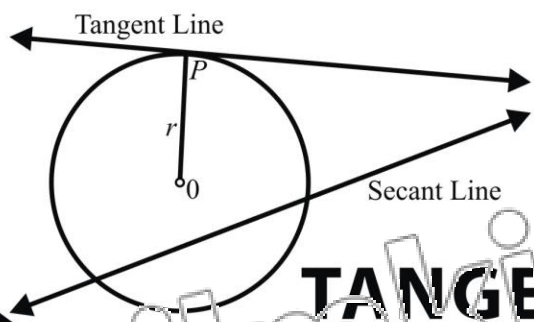
UNIT 10



TANGENT TO A CIRCLE

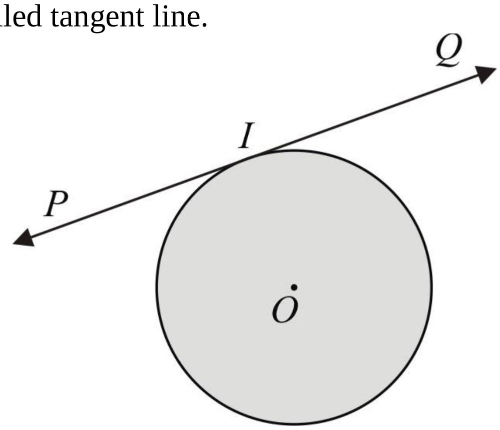
Tangent Line (K.B)

(LHR 2014, 15, GRW 2014, FSD 2014, BWP 2015, SWL 2014, SGD 2014)

A line which has one point common with a circle is called tangent line.

Or

A straight line which touches the circumference of circle at one point only is called tangent line.



In the figure $\overleftrightarrow{PIQ}$ is tangent line.

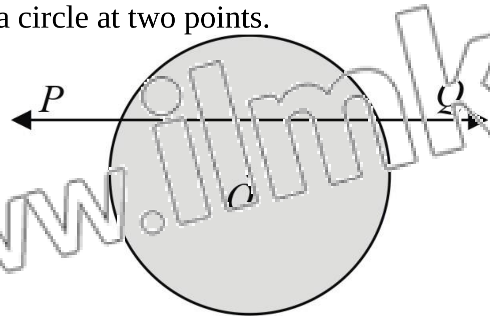
Secant Line (K.B)

(FSD 2015, SGD 2014, 15, RWP 2014, D.G.K 2014)

A line which has two points common with a circle is called secant line.

Or

A straight line which cuts the circumference of a circle at two points.

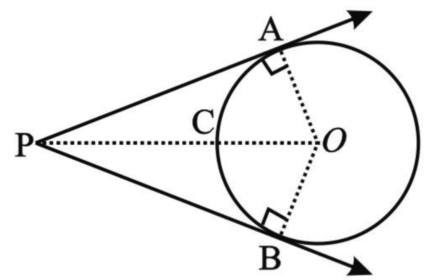


In the figure, $\overline{PQ}$ is secant line.

Length of Tangent (K.B)

(SWL 2014, RWP 2015, D.G.K 2014, 15)

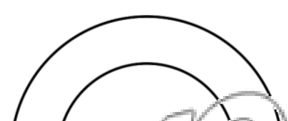
If a tangent is drawn from a point out side a circle, then distance between that point and the point of contact of the circle is called length of tangent.



In the figure $m\overline{PA} = m\overline{PB}$ is length of tangent.

Concentric Circles (K.B)

Circles having same centres and different radii are called concentric circles.



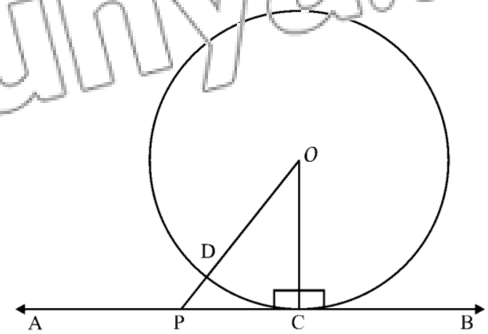
In the above figure, circles having centre O are concentric circles.

Theorem 1 **(A.B)**

10.1(*i*)

Statement:

If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.



Given:

A circle with centre *O* and $\overline{OC}$ is the radial segment. $\overleftrightarrow{AB}$ is perpendicular to $\overline{OC}$ at its outer end *C*.

To Prove:

$\overleftrightarrow{AB}$ is a tangent to the circle at *C*

Construction:

Take any point *P* other then *C* on $\overleftrightarrow{AB}$. Join *O* with *P*.

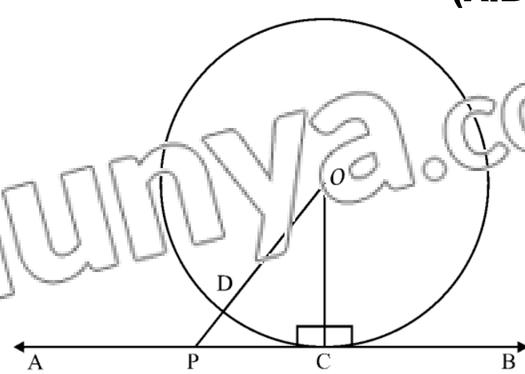
Proof:

Statements	Reasons
In ΔOCP,	
$m\angle OCP = 90°$	$\overleftrightarrow{AB} \perp \overline{OC}$ (given)
and $m\angle OPC < 90°$	Acute angle of right angled Triangle.
$m\overline{OP} > m\overline{OC}$	Greater angle has greater side opposite to it.
$\therefore P$ is a point outside the circle.	$\overline{OC}$ is the radial segment.
Similarly, every point on $\overleftrightarrow{AB}$ except *C* lies outside the circle.	
Hence $\overleftrightarrow{AB}$ intersects the circle at one point *C* only.	
i.e., $\overleftrightarrow{AB}$ is a tangent to the circle at one point only.	

Theorem 2 **(A.B)**

10.1(*ii*)

The tangent to a circle and the radial segment, joining the point of contact and the centre are perpendicular to each other.



Given:

In a circle with centre *O* and radius $\overline{OC}$.

Also $\overleftrightarrow{AB}$ is the tangent to the circle at point *C*.

To Prove:

$\overleftrightarrow{AB}$ and radial segment $\overline{OC}$ are perpendicular to each other.

Construction:

Take any point *P* other then *C* on the tangent line $\overleftrightarrow{AB}$. Join *O* with *P* so that $\overline{OP}$ meets the circle at *D*.

Proof:

Statements	Reasons
$\overleftrightarrow{AB}$ is the tangent to the circle at point C. Whereas $\overline{OP}$ cuts the circle at D.	Given Construction
$\therefore m\overline{OC} = m\overline{OD} \rightarrow (\text{i})$	Radii of the same circle
But $m\overline{OD} < m\overline{OP}$ (ii)	Point P is outside the circle.
$\therefore m\overline{OC} < m\overline{OP}$	Using (i) and (ii)
So radius $\overline{OC}$ is shortest of all lines that can be drawn from O to the tangent line $\overleftrightarrow{AB}$	
Also $\overline{OC} \perp \overleftrightarrow{AB}$	
Hence, radial segment $\overline{OC}$ is perpendicular to the tangent $\overleftrightarrow{AB}$.	

Corollary **(A.B + U.B)**

There can only be one perpendicular draw to the radial segment $\overline{OC}$ at the point C of the circle. It follows that one and only one tangent can be drawn to the circle at the given point C on its circumference.

Theorem 3 **(A.B)**

10.1(*iii*)

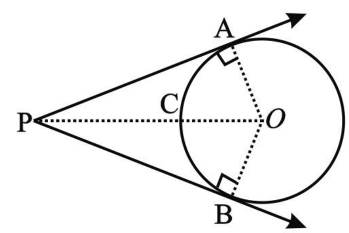
Two tangents drawn to a circle from a point outside it, are equal in length.

Given:

Two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P to the circle with centre O.

To Prove:

$m\overline{PA} = m\overline{PB}$



Construction:

Join O with A, B and P, so that we form $\angle rt\Delta^s OAP$ and OBP.

Proof:

Statements	Reasons
In $\angle rt\Delta^s\ OAP \leftrightarrow OBP$	
$m\angle OAP = m\angle OBP = 90°$	Radii $\perp$ to the tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$
hyp.$\overline{OP}$ = hyp.$\overline{OP}$	Common
$m\overline{OA} = m\overline{OB}$	Radii of the same circle.
$\therefore \Delta OAP \cong \Delta OBP$	$\angle rt\Delta^s$ H.S≅H.S
Hence $m\overline{PA} = m\overline{PB}$	Corresponding sides of congruent triangles

Note

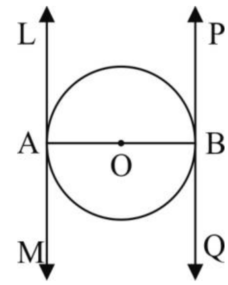
The length of a tangent to a circle is measured from the given point to the point of contact.

Corollary **(A.B + U.B)**

If O is the centre of a circle and two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from an external point P then $\overline{OP}$ is the right bisector of the chord of contact $\overline{AB}$.

Exercise 10.1

Q.1 Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel. **(A.B)**



Given

In a circle with centre 'O', $\overline{AB}$ is a diameter, $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ are two tangents passing through point A and B.

To prove

$\overleftrightarrow{LM} \parallel \overleftrightarrow{PQ}$

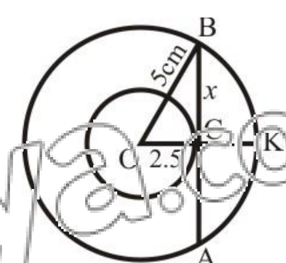
Proof

Statements	Reasons
$\overline{OA}$ and $\overline{OB}$ are radial segments	Given
$\therefore m\angle OAL = 90° \rightarrow (\text{i})$	Tangent is $\perp$ to a radial segment.
Similarly	
$\therefore m\angle OBQ = 90° \rightarrow (\text{ii})$	As in (i)
$\therefore m\angle OAL = m\angle OBQ$	From is (i) and (ii)
Or $\overleftrightarrow{LM} \parallel \overleftrightarrow{PQ}$	Alternate angles are congruent.

Q.2 The diameters of two concentric circles are 10 cm and 5 cm respectively. Look for the length of any chord of the outer circle which touches the inner one. **(A.B)**

Solution:

From ΔOCB,



$\left(m\overline{BC}\right)^2 = \left(m\overline{OB}\right)^2 - \left(m\overline{OC}\right)^2 \because (hyp)^2 = (perp)^2 + (base)^2$

$= (5)^2 - (2.5)^2$

$= 25 - 6.25$

$= 18.75$

Taking square root on both sides

$m\overline{BC} = \sqrt{18.75}$

$x = 4.33$

$m\overline{AB} = 2x$

$= 2(4.33)$

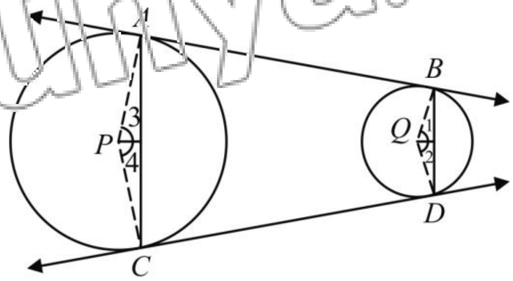
$= 8.66$ cm

$\therefore$ Length of chord = $m\overline{AB} = 8.66$ cm

Q.3 $\overleftrightarrow{AB}$ **and** $\overleftrightarrow{CD}$ **are the common tangents drawn to the pair of circles. If** A **and** C **are the points of tangency of 1st circle where** B **and** D **are the points of tangency of 2nd circle, then prove that** $\overline{AC}$ □ $\overline{BD}$. **(A.B)**

Given

Two circles with centre P and Q. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are common tangents of A is joined with C and B with D.



To prove

$\overline{AC}$ □ $\overline{BD}$

Construction

Join P with A and C and Q with B and D. Name the angles $\angle 1, \angle 2, \angle 3, \angle 4$ as shown in the figure.

Proof

Statement	Reasons
$\overline{AP} \perp \overleftrightarrow{AB} \rightarrow$ (i)	Theorem 10.2
$\overline{BQ} \perp \overleftrightarrow{AB} \rightarrow$ (ii)	
$\overline{AP}$ □ $\overline{BQ}$	From (i),(ii)
$\angle 3 \cong \angle 1 \rightarrow$ (iii)	Corresponding angles
Similarly	
$\angle 4 \cong \angle 2 \rightarrow$ (iv)	
$m\angle 3 + m\angle 4 = m\angle 1 + m\angle 2$	Adding (iii) and (iv)
$m\angle APC = m\angle BQD$	Sum of angles postulate
$\dfrac{m\overline{AP}}{m\overline{BQ}} = \dfrac{m\overline{PC}}{m\overline{QD}}$	
$\therefore m\angle PCA = m\angle QDB$	
Hence $\overline{AC}$ □ $\overline{BD}$	

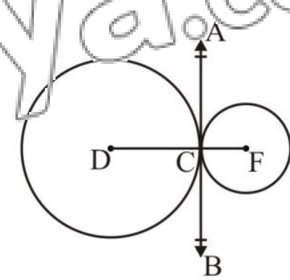
Theorem 4 (a) **(A.B)**

10.1(*iv*)

If two circles touch externally then the distance between their centers is equal to the sum of their radii.

Given:

Two circles with centres D and F respectively touch each other externally point C. So that $\overline{CD}$ and $\overline{CF}$ are respectively the radii of the two circles.



To Prove:

Point C lies on the join of centres D and F and $m\overline{DF} = m\overline{DC} + m\overline{CF}$

Construction:

Draw $\overleftrightarrow{ACB}$ as a common tangent to the pair of circles at C.

Proof:

Statements	Reasons
Both circles touch externally at C whereas $\overline{CD}$ is radial segment and $\overleftrightarrow{ACB}$ is the common tangent.	
$\therefore m\angle ACD = 90°$ (i)	Radial segment $\overline{CD}$ $\perp$ the Tangent line $\overline{AB}$
Similarly $\overline{CF}$ is radial segment and $\overleftrightarrow{ACB}$ is the common tangent	
$\therefore m\angle ACF = 90°$(ii)	Radial segment $\overline{CF} \perp$ the tangent line $\overline{AB}$
$m\angle ACD + m\angle ACF = 90° + 90°$	Adding (i) and (ii)
$m\angle DCF = 180°$(iii)	Sum of supplementary adjacent angles.
Hence DCF is a line segment with point C between D and F and $m\overline{DF} = m\overline{DC} + m\overline{CF}$	

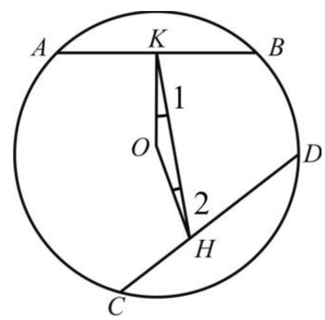
Exercise 10.2

Q.1 $\overline{AB}$ and $\overline{CD}$ are two equal chords in a circle with centre *O*. *H* and *K* are respectively the midpoints of the chords. Prove that $\overline{HK}$ makes equal angles with $\overline{AB}$ AND $\overline{CD}$.

(A.B)

Given

In a circle with centre 'O', H and K are midpoints of chord $\overline{AB}$ and $\overline{CD}$ respectively.



Solution

$m\overline{AB} = m\overline{CD}$

To prove

$m\angle BKH = m\angle DHK$

Construction

Join O to H and K and name the angles as shown in the figure.

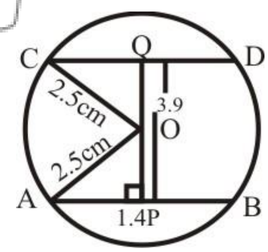
Proof

Statements	Reasons
$m\overline{OH} = m\overline{OK}$	(Given)$\left(\overline{AB} \cong \overline{CD}\right)$
Also $m\angle OHD = m\angle OKB = 90° \rightarrow$(i)	
$\angle 1 \cong \angle 2 \rightarrow$(ii)	Angles opposite to congruent sides
$m\angle BKH = m\angle BKO - m\angle 1$ $= 90° - m\angle 1 \rightarrow$(iii)	
$m\angle DHK = m\angle DHO - m\angle 2$ $= 90° - m\angle 2$	
$= 90° - m\angle 1 \rightarrow$(iv)	From (ii)
$\therefore m\angle BKH = m\angle DHK$	From (iii) and (iv)

Q.2 The radius of a circle is 2.5m. $\overline{AB}$ and $\overline{CD}$ are two chords 3.9 cm apart. If $m\overline{AB} = 1.4cm$, then find the measurement of other chord. **(A.B)**

Solution:

In a circle with centre 'O', $\overline{AB}$ and $\overline{CD}$ are two chords and distance between then $\overline{PQ}$ is 3.9cm.



From the fig, $\overline{AP} = \frac{1.4}{2} = 0.7$ $\quad \because \overline{OP} \perp \overline{AB}$

From Δ OAP,

$(m\overline{OP})^2 = (m\overline{OA})^2 - (m\overline{AP})^2$

$= (2.5)^2 - (0.7)^2$

$= 6.25 - 0.49$

$= 5.76$

Taking square root

$m\overline{OP} = \sqrt{5.76}$

$\Rightarrow m\overline{OP} = 2.4$

$m\overline{OQ} = m\overline{PQ} - m\overline{OP}$

$= 3.9 - 2.4$

$= 1.5$ cm

From ΔOCQ,

$(m\overline{CQ})^2 = (m\overline{OC})^2 - (m\overline{OQ})^2$

$(m\overline{CQ})^2 = (2.5)^2 - (1.5)^2$

$= 6.25 - 2.25$

$= 4$

Taking square root

$m\overline{CQ} = 2$

Since chord $\overline{CD} = 2m\overline{CQ}$

$= 2(2)$

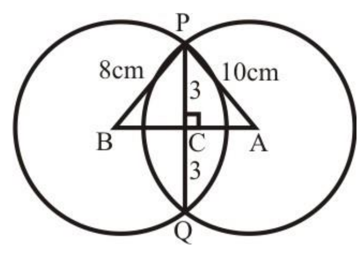
$= 4$

Result:

Length of other chord $m\overline{CD} = 4cm$

Q.3 The radii of two intersecting circles are 10 cm and 8 cm. If the length of their common chord is 6cm then find the distance between the centers. **(A.B)**

Solution:



In ΔACP

$(m\overline{AC})^2 = (m\overline{AP})^2 - (m\overline{PC})^2$

$(m\overline{AC})^2 = (10)^2 - (3)^2$

$(m\overline{AC})^2 = 100 - 9$

$\left(m\overline{AC}\right)^2 = 91$

$\left(m\overline{AC}\right)^2 = \sqrt{91}$

In ΔBCP

$\left(m\overline{BC}\right)^2 = \left(m\overline{BP}\right)^2 - \left(m\overline{PC}\right)^2 \quad \because (hypotenuse)^2 = (perpendicular)^2 + (base)^2$

$\left(m\overline{BC}\right)^2 = (8)^2 - (3)^2$

$\left(m\overline{BC}\right)^2 = 64 - 9$

$\left(m\overline{BC}\right)^2 = 55$

$\left(m\overline{BC}\right)^2 = \sqrt{55}$

Distance between centers = $\mathrm{m}\overline{\mathrm{AB}} = \mathrm{m}\overline{\mathrm{AC}} + \mathrm{m}\overline{\mathrm{BC}}$

$= \sqrt{91} + \sqrt{55}$

= 16.96 cm

Q.4 Show that greatest chord in a circle is its diameter. **(A.B)**

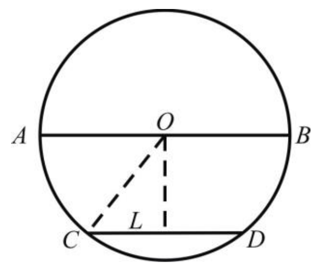
In a circle with centre. '*O*'

Given

$\overline{AB}$ is a diameter and $\overline{CD}$ is a chord.

To prove

$m\overline{AB} > m\overline{CD}$



Construction

Draw $\overline{OL} \perp \overline{CD}$ and join O to C.

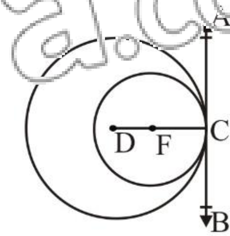
Proof

Statements	Reasons
$\overline{OL} \perp \overline{CD}$	Construction
$\therefore$ L is midpoint of $\overline{CD}$.	Perpendicular drawn from centre bisect the chord.
In ΔOLC	
$m\angle OLC = 90$	$\overline{OL} \perp \overline{CD}$
$m\angle COL \angle 90$	Acute angle of right triangle
$\therefore m\angle OLC > m\angle COL$	From (i) and (ii)
$\therefore m\overline{OC} > m\overline{CL}$	Side opposite to greater angle
$2m\overline{OC} > 2m\overline{CL}$	Multiply by '2'
Diameter $> m\overline{CD}$	L is midpoint of $\overline{CD}$
$\therefore m\overline{AB} > m\overline{CD}$	(Proved)

Theorem 4 (b) **(A.B)**

10.1(v)

If two circles touch each other internally, then the point of contact lies on the line segment through their centres and distance between their centres is equal to the difference of their radii.



Given:

Two circles with centres D and F touch each other

Internally at point C. So that $\overline{CD}$ and $\overline{CF}$ are the radii of two circles.

To Prove:

Point C lies on the join of centres D and F extended and $m\overline{DF} = m\overline{DC} - m\overline{CF}$

Construction:

Draw $\overleftrightarrow{ACB}$ as the common tangent to the pair of circles at C.

Proof:

Statements	Reasons
Both circles touch internally at C whereas $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CD}$ is the radial segment Of the first circle.	
$\therefore m\angle ACD = 90^\circ$ (i)	Radial segment $\overline{CD}$ $\perp$ the tangent line $\overline{AB}$
Similarly $\overleftrightarrow{ACB}$ is the common tangent and $\overline{CF}$ is the radial segment of the second circle.	
$\therefore m\angle ACF = 90^\circ$ (ii)	Radial segment $\overline{CF}$ $\perp$ the tangent line $\overline{AB}$
$\Rightarrow m\angle ACD = m\angle ACF = 90^\circ$ Where $\angle ACD$ and $\angle ACF$ coincide each other with point F between D and C. Hence $m\overline{DC} = m\overline{DF} + m\overline{FC}$ i.e., $m\overline{DC} - m\overline{FC} = m\overline{DF}$ Or $m\overline{DF} = m\overline{DC} - m\overline{FC}$	Using (i) and (ii)

Exercise 10.3

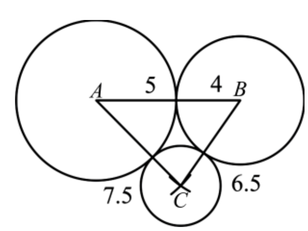
Q.1 **Two circles with radii $5cm$ and $4cm$ touch each other externally. Draw another circle with radius $2.5cm$ touching the first pair externally.** **(A.B)**

To construct

A circle of radius $2.5cm$ touching given two circles externally.

Construction

Steps of construction



(i) With centre A, draw an arc of radius $7.5cm\,(5+2.5=7.5)$

(ii) With centre B, draw an arc of radius

(iii) Both arcs cut each other at point C.

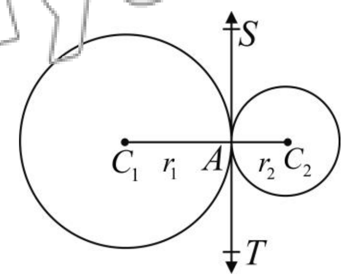
(iv) With centre C, draw a circle of radius $2.5cm$.

Q.2 If the distance between the centres $6.5cm\left(4+2.5=6.5\right)$ **of two circles is the sum or the difference of their radii they will touch each other.** **(A.B)**

Solution:

(i) Given

Two circles with centres C_1 and C_2, radii of measure r_1 and r_2 such that $m\overline{C_1C_2}=r_1+r_2$



To prove

Circles touch each other externally

Construction

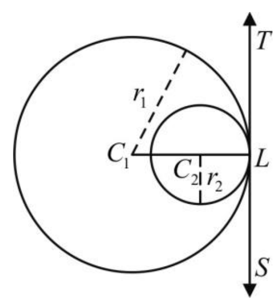
Draw TS tangent to the circle with centre C_1 at A

Proof

Statements	Reasons
TA is tangent to circle 3with centre C_1	
$\therefore m\angle C_1AT=90^\circ$	
$m\angle C_2AT=180^\circ-90^\circ$	$\because C_1AC_2$ is a straight line
$m\angle C_2AT=90^\circ$	
TA is perpendicular to radial segment AC_2	
So TA is tangent to the circle with centre C_2	
$\therefore TA$ is common tangent at A	
Hence circles touch each other externally	

(ii) Given **(A.B)**

Two circles with centres C_1 and C_2, radii r_1 and r_2 such that $m\overline{C_1C_2}=r_1-r_2$



To prove

Circles touch each other internally

Construction

Produce $\overline{C_1C_2}$ to meet the circle with centre C_1 at L

Draw TS tangent to the circle with centre C_1 at L

Proof

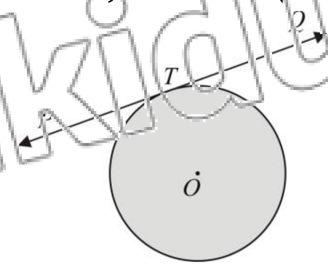
Statements	Reasons
As TLS is tangent to the circle with centre C_1	Construction
$C_1L\perp TL$	Tangent $\perp$ radial segment
$\therefore m\angle C_1LT=90^\circ$	
But C_1C_2L is a straight line	
$\therefore m\angle C_2LT=90^\circ$	
i.e $C_2L\perp TL$	
$\therefore TS$ is tangent to the circle with centre C_2	
$\therefore TL$ is common tangent	
Hence circles touch each other internally	

Miscellaneous Exerciser 10

Q.1 Multiple choice questions

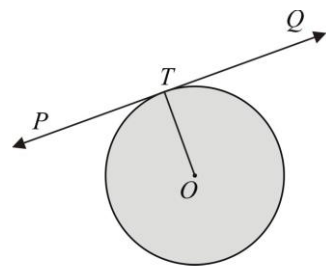
Four possible answers are given for the following question. Tick (✓) the correct answer.

(i) In the adjacent figure of the circle, the line $\overleftrightarrow{PTQ}$ is named as **(K.B)**



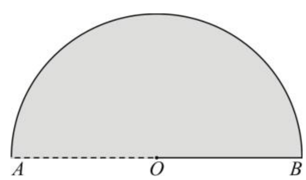
(a) An arc **(b)** A chord
(c) A tangent **(d)** A secant

(ii) In a circle with centre O, if $\overline{OT}$ is the radial segment and $\overleftrightarrow{PTQ}$ is the segment line, then **(K.B)**



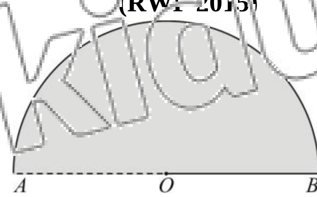
(a) $\overline{OT} \perp \overleftrightarrow{PQ}$ **(b)** $\overline{OT} \not\perp \overleftrightarrow{PQ}$
(c) $\overline{OT} \parallel \overleftrightarrow{PQ}$ **(d)** $\overline{OT}$ is right bisector of $\overleftrightarrow{PQ}$

(iii) In the adjacent figure, find semicircular area if $\pi \approx 3.1416$ and $m\overline{OA} = 20cm$. **(K.B)**
(GRW 2014)



(a) $62.83 sq\,cm$ **(b)** $314.16 sq\,cm$
(c) $436.20 sq\,cm$ **(d)** $628.32 sq\,cm$

(iv) In the adjacent figure, find half the perimeter of circle with centre O if $\pi \approx 3.1416$ and $m\overline{OA} = 20cm$. **(RWP 2015)** **(K.B)**



(a) $31.42cm$ **(b)** $62.832cm$
(c) $125.65cm$ **(d)** $188.50cm$

(v) A line which has two points in common with a circle is called: **(K.B)**
(a) Sine of a circle **(b)** Cosine of a circle
(c) Tangent of a circle **(d)** Secant of a circle

(vi) **A line which has only one point in common with a circle is called:** **(K.B)** **(D.G.K 2014)**

(a) Sine of a circle **(b)** Cosine of circle
(c) Tangent of a circle **(d)** Secant of a circle

(vii) **Two tangents drawn to a circle from a point outside it are of … in length.** **(K.B)**

(a) Half **(b)** Equal
(c) Double **(d)** Triple

(viii) **A circle has only one:** **(K.B)**

(a) Secant **(b)** Chord
(c) Diameter **(d)** Centre

(ix) **A tangent line intersects the circle at:** **(K.B)**

(a) Three points **(b)** Two points
(c) Single point **(d)** No point at all

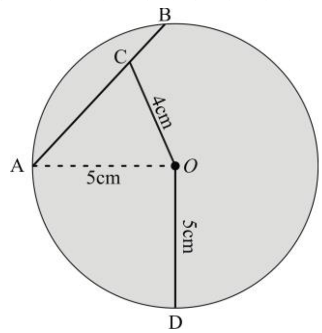
(x) **Tangents drawn at the ends of diameter of a circle are … to each other.** **(K.B)** **(LHR 2015)**

(a) Parallel **(b)** Non-parallel
(c) Collinear **(d)** Perpendicular

(xi) **The distance between the centres of two congruent touching circles externally is:** **(K.B)**

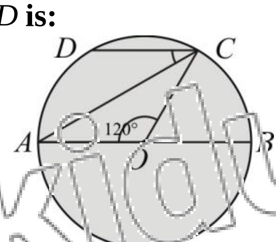
(a) Of zero length **(b)** The Radius of each circle
(c) The diameter of each circle **(d)** Twice the diameter of each circle

(xii) **In the adjacent circular figure with centre O and radius $5cm$, the length of the chord intercepted at $4cm$ away from the centre of this circle is:** **(K.B)**



(a) $4cm$ **(b)** $6cm$
(c) $7cm$ **(d)** $9cm$

(xiii) **In the adjoining figure, there is a circle with centre O. If $\overline{DC}$ ∥ diameter $\overline{AB}$ and $m\angle AOC = 120°$, then $m\angle ACD$ is:** **(K.B)**



(a) $40°$ **(b)** $30°$
(c) $50°$ **(d)** $60°$

ANSWER KEY

i	c	iv	b	vii	b	x	a	xiii	b
ii	a	v	d	viii	d	xi	c		
iii	d	vi	c	ix	c	xii	b		