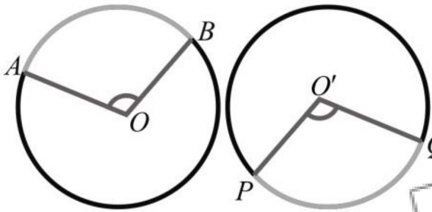
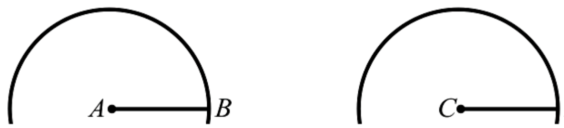


UNIT 11
CHORDS AND ARCS

Congruent Circles **(K.B)**

Circles having different centres but same radii are called congruent circles.



In the given figure, circles with centre A and C are congruent, if $m\overline{AB} = m\overline{CD}$.

Congruent Arcs **(K.B)**

Two arcs of same circle or of different circles are congruent, if their central angles are congruent.

In the given figure, arcs BC and QR are congruent, if $\angle CAB \cong \angle RPQ$.

Theorem 1 **(A.B)**

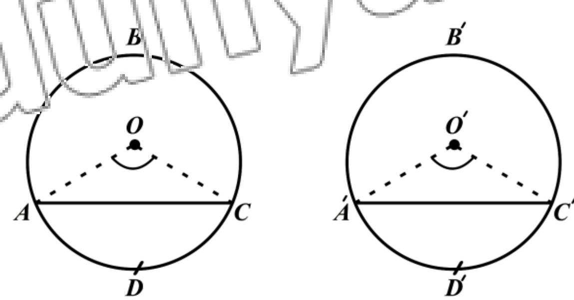
11.1(*i*)

Statement:

If two arcs of a circle (or of congruent circles) are congruent then the corresponding chords are equal.

Given:

$ABCD$ and $A'B'C'D'$ are two congruent circles. With centres O and O' respectively. So that $m\widehat{ADC} = m\widehat{A'D'C'}$



To Prove:

$m\overline{AC} = m\overline{A'C'}$

Construction:

Join O with A, O with C, O' with A' and O' with C'. So that we can form $\Delta^s OAC$ and $O'A'C'$

Proof:

Statements	Reasons
In two equal circles $ABCD$ and $A'B'C'D'$ with centres O and O' respectively.	Given
$mADC = mA'D'C'$	Given
$\therefore m\angle AOC = m\angle A'O'C'$	Central angles subtended by equal arcs of the equal circles.
Now in $\Delta AOC \leftrightarrow A'O'C'$	
$m\overline{OA} = m\overline{O'A'}$	Radii of equal circles
$m\angle AOC = m\angle A'O'C'$	Already Proved
$m\overline{OC} = m\overline{O'C'}$	Radii of equal circles
$\therefore \Delta AOC \cong \Delta A'O'C'$	$S.A.S \cong S.A.S$
and in particular $m\overline{AC} = m\overline{A'C'}$	
Similarly we can prove the theorem in the same circle.	

Theorem 2 **(A.B)**

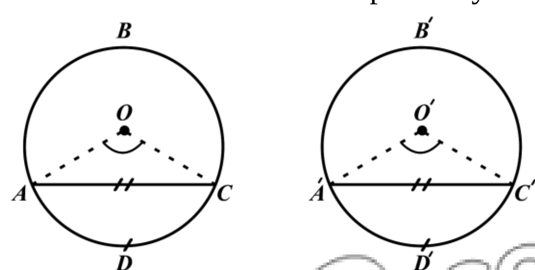
11.1(*ii*)

Statement:

If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent. In equal circles or in the same circle, if two chords are equal, they cut off equal arcs.

Given:

$ABCD$ and $A'B'C'D'$ are two congruent circles with centres O and O' respectively. So that $m\overline{AC} = m\overline{A'C'}$.



To Prove:

$mADC = mA'D'C'$

Construction:

Join O with A, O with C, O' with A' and O' with C'.

Proof:

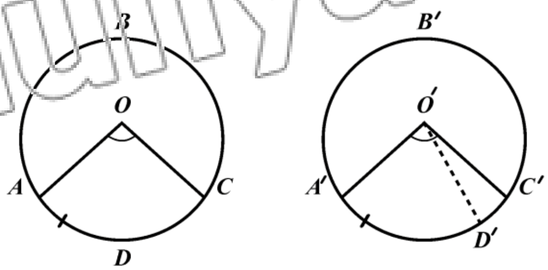
Statements	Reasons
In $\Delta AOC \leftrightarrow \Delta A'O'C'$	
$m\overline{OA} = m\overline{O'A'}$	Radii of equal circles
$m\overline{OC} = m\overline{O'C'}$	Radii of equal circles
$m\overline{AC} = m\overline{A'C'}$	Given
$\therefore \Delta\angle AOC \cong \Delta\angle A'O'C'$	$S.S.S \cong S.S.S$
$\Rightarrow m\angle AOC = m\angle A'O'C'$	Corresponding angles of congruent triangles
Hence $mADC = mA'D'C'$	Arcs corresponding to equal chords in a circle.

Theorem 3 **(A.B)**

11.1(*iii*)

Statement:

Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).



Given:

ABC and $A'B'C'$ are two congruent circles with centres O and O' respectively. So that $\overline{AC} = \overline{A'C'}$

To Prove:

$\angle AOC \cong \angle A'O'C'$

Construction:

Let if possible $m\angle AOC \neq m\angle A'O'C'$ *then consider* $\angle AOC \cong \angle A'O'D'$

Proof:

Statements	Reasons
$\angle AOC \cong \angle A'O'D'$	Construction
$\therefore AC \cong A'D' \rightarrow (\text{i})$	Arcs subtended by equal Central angles in congruent circles
$\overline{AC} \cong \overline{A'D'} \rightarrow (\text{ii})$	Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).
But $\overline{AC} \cong \overline{A'C'} \rightarrow (\text{iii})$	Given
$\therefore \overline{A'C'} = \overline{A'D'}$ Which is only possible, if C' coincides with D'. Hence $m\angle A'O'C' = m\angle A'O'D' \rightarrow (\text{iv})$	Using (i) and (ii)
But $m\angle AOC = m\angle A'O'D' \rightarrow (\text{v})$	Construction
$\Rightarrow m\angle AOC = m\angle A'O'C'$	Using (iv) and (v)

Corollary 1 **(K.B)**

In congruent circles or in the same circle, if central angles are equal then corresponding sectors are equal.

Corollary 2 **(K.B)**

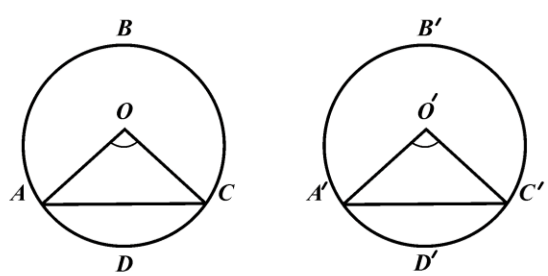
In congruent circles or in the same circle, unequal arcs will subtend unequal central angles.

Theorem 4 **(A.B)**

11.1(*iv*)

Statement:

If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, the chords are equal.



Given:

$ABCD$ and $A'B'C'D'$ are two congruent circles with centre O and O' respectively. $\overline{AC}$ and $\overline{A'C'}$ are chords of circles $ABCD$ and $A'B'C'D'$ respectively and $m\angle AOC = m\angle A'O'C'$

To Prove:

$m\overline{AC} = m\overline{A'C'}$

Proof:

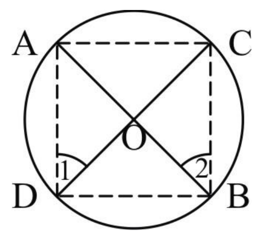
Statements	Reasons
In $\Delta OAC \leftrightarrow \Delta O'A'C'$	
$m\overline{OA} = m\overline{O'A'}$	Radii of congruent circles
$m\angle AOC = m\angle A'O'C'$	Given
$m\overline{OC} = m\overline{O'C'}$	Radii of congruent circles
$\therefore \Delta OAC \leftrightarrow \Delta O'A'C'$	$SAS \cong SAS$
Hence $m\overline{AC} = m\overline{A'C'}$	Corresponding sides of congruent triangles

Exercise 11.1

Q.1 In a circle two equal diameters $\overline{AB}$ and $\overline{CD}$ intersect each other. Prove that $m\overline{AD} = m\overline{BC}$. **(A.B)**

Given

In a circle with centre 'O', two chords $\overline{AB}$ and $\overline{CD}$ intersect each other, such that $\overline{AB} \cong \overline{CD}$.



To prove

$m\overline{AD} = m\overline{BC}$

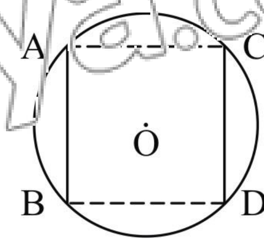
Proof

Statement	Reasons
In $\Delta^s CBD$ and BCA	
$\overline{BC} \cong \overline{BC} \rightarrow (\text{i})$	Common
$\overline{AB} \cong \overline{CD} \rightarrow (\text{ii})$	Given
$\angle 2 \cong \angle 1 \rightarrow (\text{iii})$	Circum angles of same segment
$\angle CBD \cong \angle BCA \rightarrow (\text{iv})$	Opposite angles in the same segment
$m\angle CBD - m\angle 2 = m\angle BCA - m\angle 1$	From (iii) and (iv)
$\therefore m\angle CBA = m\angle BCD \rightarrow (\text{v})$	
From (i), (ii) and (v)	
$\Delta CBD \cong \Delta BCA$	S.A.S postulate
$\overline{AC} \cong \overline{BD}$	Corresponding sides of $\cong$ triangles

Q.2 In a circle prove that the arcs between two parallel and equal chords are equal. **(A.B)**

Given

In a circle with centre 'O', $\overline{AB}$ and $\overline{CD}$ are two chords, such that $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$



To prove

$mAC = mBD$

Construction

Join A to C and B to D.

Proof

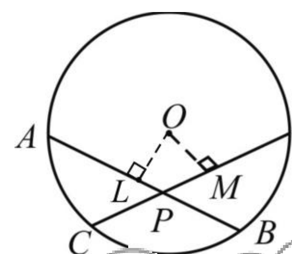
Statements	Reasons
$m\overline{AB} = m\overline{CD}$ Also $\overline{AB} \parallel \overline{CD}$	Given
$\therefore ABCD$ is a $\parallel^{gm}$	A quadrilateral having two sidesparallel and congruent is a parallelogram.
$m\overline{AC} = m\overline{BD}$	Opposite sides of a parallelogram
$\therefore mAC = mBD$	Th-11.2 (chords area equal)

Q.3 Give a geometric proof that a pair of bisecting chords are the diameters of a circle. **(A.B)**

Given

In a circle with centre 'O', two chords $\overline{AB}$ and $\overline{CD}$ bisect each other at point 'P'.

i.e $m\overline{PC} = m\overline{PD}$ and $m\overline{PB} = m\overline{PA}$.



To prove

Chords $\overline{AB}$ and $\overline{CD}$ pass through point 'O'.

Construction

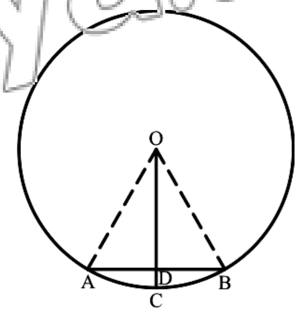
Draw $\overline{OM} \perp \overline{CD}$ and $\overline{OL} \perp \overline{AB}$

Proof

Statements	Reasons
L is mid point of $\overline{AB}$	$\overline{OL} \perp \overline{AB}$
P is mid point of $\overline{AB}$	Given
$\therefore P$ and L coincide	
M is the mid point $\overline{CD}$	$\overline{OM} \perp \overline{CD}$
P is mid point of $\overline{CD}$	Given
$\therefore M$ and P coincide	
So L, M and P coincide which is possible only if L, M and P coincide with O	
$\therefore \overline{AB}$ and $\overline{CD}$ pass through O	
Hence $\overline{AB}$ and $\overline{CD}$ are diameters	

Q.4 If C is the mid point of an arc ACB in a circle with centre O. Show that line segment $\overline{OC}$ bisects the chord AB. **(A.B)**

Given



In a circle with centre 'O', C is mid point of ABC, $\overline{OC}$ intersect $\overline{AB}$ at point D

To prove

$\overline{AD} \cong \overline{BD}$

Construction

Join O to A and B

Proof

Statements	Reasons
C is mid point of ACB	Given
Then $AC \cong BC$	
$\therefore \angle AOC \cong \angle BOC$	{Central angles of congruent arcs.
Or $\angle AOD \cong \angle BOD$	
In $\Delta OAD \leftrightarrow \Delta OBD$	
$\overline{OA} \cong \overline{OB}$	Radii of same circle
$\angle AOD \cong \angle BOD$	Proved
$\overline{OD} \cong \overline{OD}$	Common
$\therefore \Delta OAD \cong \Delta OBD$	S.A.S postulate
$\overline{AD} \cong \overline{BD}$	Corresponding sides of congruent triangles

Miscellaneous Exercise 11

Q.1 Multiple choice questions
Four possible answers are given for the following question. Tick (✓) the correct answer.

(1) **A** $4cm$ **long chord subtends a central angle of** $60°$**. The radial segment of this circle is:** **(SWL 2015)** **(K.B)**
(a) 1 **(b)** 2
(c) 3 **(d)** 4

(2) **The length of a chord and the radial segment of a circle are congruent, the central angle made by the chord will be:** **(FSD 2014, RWP 2015)** **(K.B)**
(a) 30° **(b)** 45°
(c) 60° **(d)** 75°

(3) **Out of two congruent arcs of a circle, if one arc makes a central angle of** $30°$ **then the other arc will subtend the central angle of:** **(LHR 2015, SWL 2014)** **(K.B)**
(a) 15° **(b)** 30°
(c) 45° **(d)** 60°

(4) **An arc subtends a central angle of** $40°$ **then the corresponding chord will subtend a central angle of:** **(LHR 2015, GRW 2014, FSD 2014, D.G.K 2014, 15)** **(K.B)**
(a) 20° **(b)** 40°
(c) 60° **(d)** 80°

(5) **A pair of chords of a circle subtending two congruent central angles is:** **(K.B)**
(a) Congruent **(b)** Incongruent
(c) Over lapping **(d)** Parallel

(6) **If an arc of circle subtend a central angle of** $60°$**, then the corresponding chord of the arc will make the central angle of:** **(K.B)**
(a) 20° **(b)** 40°
(c) 60° **(d)** 80°

(7) **The semi circumference and the diameter of a circle both subtend a central angle of:** **(K.B)**
(a) 90° **(b)** 180°
(c) 270° **(d)** 360°

(8) **The chord length of a circle subtending a central angle of** $180°$ **is always:** **(K.B)** **(D.G.K 2014)**
(a) Less than radial segment **(b)** Equal to the radial segment
(c) Double of the radial segment **(d)** None of these

(9) **If a chord of a circle subtend a central angle of** $60°$**, then the length of the chord and the radial segment are:** **(MTN 2015)** **(K.B)**
(a) Congruent **(b)** Incongruent
(c) Parallel **(d)** Perpendicular

(10) **The arcs opposite to incongruent central angles of a circle arc always:** **(K.B)** **(D.G.K 2014)**
(a) Congruent **(b)** Incongruent
(c) Parallel **(d)** Perpendicular

ANSWER KEY

1	2	3	4	5	6	7	8	9	10
d	c	b	b	a	c	b	c	a	b