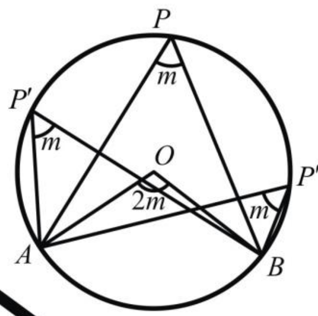
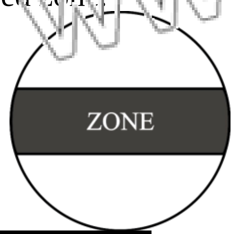


UNIT 12

ANGLE IN A SEGMENT OF A CIRCLE

Zone **(K.B)**

Circular region bounded by two parallel chords is called zone.



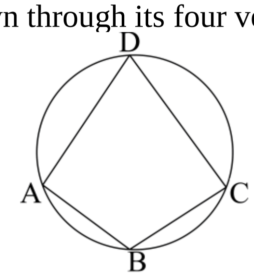
Cyclic Quadrilateral **(K.B)**

(LHR 2014, 15, GRW 2014, 17, BWP 2016, 17, RWP 2016, MTN 2016, 17, D.G.K 2017)

A quadrilateral whose vertices lie on the circumference of a circle is called cyclic quadrilateral.

Or

A quadrilateral is called cyclic when a circle can be drawn through its four vertices.



Circum Angle **(K.B)**

(LHR 2014, 15, GRW 2017, BWP 2016, 17, RWP 2016, MTN 2016, 17, D.G.K 2017)

The angle subtended by an arc of a circle at its circumference is called circum angle.

Or

An angle whose vertex lies on the circumference and whose arms pass through end points of the arc.

In the figure, $\angle ACB$ is the circum angle.

Central Angle **(K.B)**

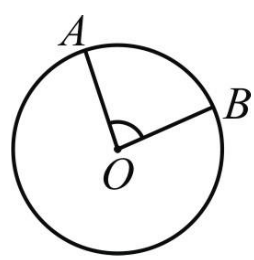
(LHR 2016, GRW 2016, FSD 2015, 16, 17, RWP 2016, 17, SWL 2017, SGD 2016, 17, MTN 2017, D.G.K 2016)

The angle subtended by an arc of a circle at its centre is called circum angle.

Or

"An angle formed by two radial segments at the centre of the circle is called central angle".

In the given figure, $\angle AOB$ is central angle of AB.



Note **(K.B + U.B)**

(i) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

(ii) Any two angles in the same segment of a circle are equal.

(iii) The angle

- in a semi-circle is a right angle,
- in a segment greater than a semi circle is less than a right angle,
- in a segment less than a semi-circle is greater than a right angle.

(iv) The opposite angles of any quadrilaterel inscribed in a circle are supplementary.

Theorem 1 **(A.B)**

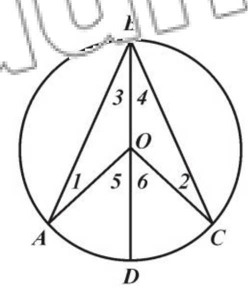
12.1(*i*)

Statement:

The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Given:

AC is an arc of a circle with centre O. Where as $\angle AOC$ is the central angle and $\angle ABC$ is circum angle.



To Prove:

$m\angle AOC = 2m\angle ABC$

Construction:

Join B with O and produce it to meet the circle at D. Write angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
As $m\angle 1 = m\angle 3 \rightarrow \text{(i)}$	Angles opposite to equal sides in ΔOAB
And $m\angle 2 = m\angle 4 \rightarrow \text{(ii)}$	Angles opposite to equal sides in ΔOBC
Now $m\angle 5 = m\angle 1 + m\angle 3 \rightarrow \text{(iii)}$	External angle is the sum of internal opposite angles.
Similarly $m\angle 6 = m\angle 2 + m\angle 4 \rightarrow \text{(iv)}$	
Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3 \rightarrow \text{(v)}$	Using (i) and (iii)
And $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4 \rightarrow \text{(vi)}$	Using (ii) and (iv)
Then from figure	
$\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (v) and (vi)
$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4) = 2m\angle ABC$	

Theorem 2 **(GRW 2016, SWL 2016, RWP 2016, MTN 2017, SGD 2017, BWP 2015)** **(A.B)**

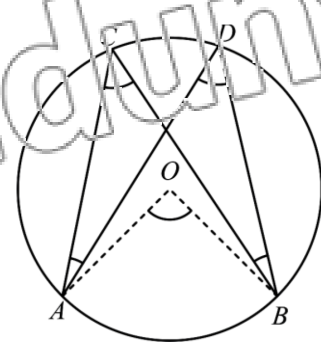
12.1(*ii*)

Statement:

Any two angles in the same segment of a circle are equal.

Given:

$\angle ACB$ and $\angle ADB$ are the circum angles in the same segment of a circle with centre O.



To Prove:

$m\angle ACB = m\angle ADB$

Construction:

Join O with A and O with B. So that $m\angle AOB$ is the central angle.

Proof:

Statements	Reasons
Standing on the same arc AB of a circle.	
$\angle AOB$ is the central angle whereas	Construction
$\angle ACB$ and $\angle ADB$ are circum angles	Given
$\therefore m\angle AOB = 2m\angle ACB \rightarrow (\text{i})$	The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
and $m\angle AOB = 2m\angle ADB \rightarrow (\text{ii})$	Same as above
$\Rightarrow 2m\angle ACB = 2m\angle ADB$	Using (i) and (ii)
Hence $m\angle ACB = m\angle ADB$	

Theorem 3 (LHR 2016, MTN 2015) **(A.B)**

12.1(*iii*)

Statement:

The angle

- In a semi-circle is a right angle,
- In a segment greater than a semi circle is less than a right angle,
- In a segment less than a semi-circle is greater than a right angle.

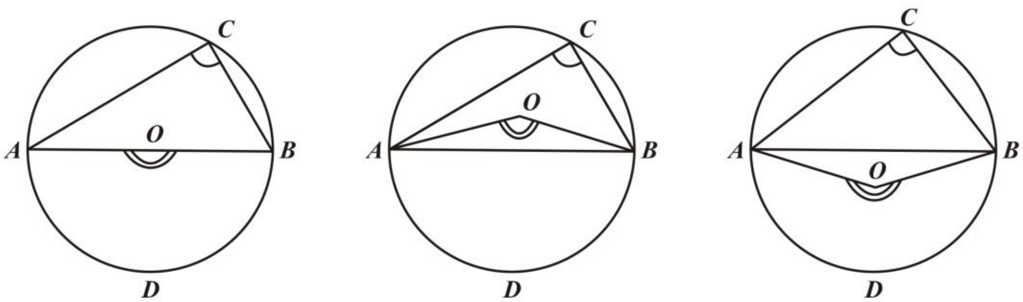


Fig I **Fig II** **Fig III**

Given:

$\overline{AB}$ is the chord corresponding to an arc ADB. Whereas $\angle AOB$ is a central angle and $\angle ACB$ is a circum angle of a circle with centre O.

To Prove:

In fig (I) if sector ACB is a semi circle then $m\angle ACB = 1\angle rt$

In fig (II) if sector ACB is greater than a semi circle then $m\angle ACB < 1\angle rt$

In fig (III) if sector ACB is less than s semi circle then $m\angle ACB > 1\angle rt$

Proof:

Statements	Reasons
In each figure, $\overline{AB}$ is the chord of a circle with centre O. $\angle AOB$ is the central angle standing on an arc ADB. Whereas $\angle ACB$ is the circum angle	Given
Such that $m\angle AOB = 2m\angle ACB \rightarrow$ (i)	The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
Now in fig (I) $m\angle AOB = 180°$	A straight angle
$\therefore m\angle AOB = 2\angle rt \rightarrow$ (ii)	
$\Rightarrow m\angle ACB = 1\angle rt$	Using (i) and (ii)
In fig (II) $m\angle AOB < 180°$	
$\therefore m\angle AOB = 2\angle rt \rightarrow (\text{iii})$	
$\Rightarrow m\angle ACB < 1\angle rt$	Using (i) and (iii)
In fig (III) $m\angle AOB > 180°$	
$\therefore m\angle AOB > 2\angle rt \rightarrow (\text{iv})$	
$\Rightarrow 2m\angle ACB > 2\angle rt$ $\Rightarrow m\angle ACB > 1\angle rt$	Using (i) and (iv)

Theorem 4 **(A.B)**

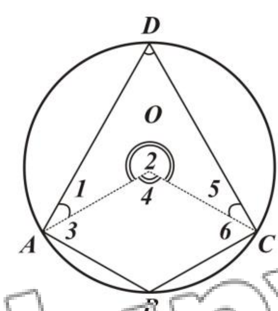
12.1(*iv*)

Statement:

The opposite angles of any quadrilaterel inscribed in a circle are supplementary.

Given:

ABCD is a quadrilateral inscribed in a circle with centre *O.*



To Prove:

$$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$. Write $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
Standing on the same arc ADC, $\angle 2$ is a central angle. Whereas $\angle B$ is the circum angle	Arc ADC of the circle with centre O.
$\therefore m\angle B = \frac{1}{2}(m\angle 2) \rightarrow$ (i)	The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$ is the circum angle	Arc ABC of the circle with centre O.

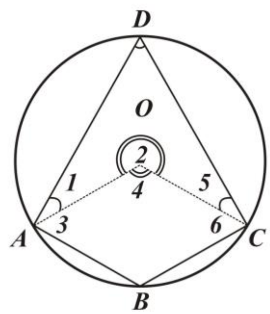
$\therefore m\angle D = \frac{1}{2}(m\angle 4) \rightarrow (\text{ii})$	The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
$\Rightarrow m\angle B + m\angle D = \frac{1}{2}m\angle 2 + \frac{1}{2}m\angle 4$	Adding (i) and (ii)
$= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$ (Total central angle)	
$i.e., m\angle B + m\angle D = \frac{1}{2}(4\angle rt) = 2\angle rt$	
Similarly $m\angle A + m\angle C = 2\angle rt$	

Exercise 12.1

Q.1 Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely. **(A.B)**

Given:

ABCD is a quadrilateral inscribed in a circle with centre *O*.



To Prove:

$$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$$

Construction:

Draw $\overline{OA}$ and $\overline{OC}$. Write $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.

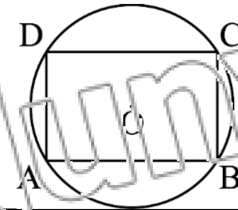
Proof:

Statements	Reasons
Standing on the same arc *ADC*, $\angle 2$ is a central angle Whereas $\angle B$ is the circum angle	Arc *ADC* of the circle with centre *O*.
$\therefore m\angle B = \frac{1}{2}(m\angle 2) \rightarrow (\text{i})$	The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
Standing on the same arc *ABC*, $\angle 4$ is a central angle whereas $\angle D$ is the circum angle	Arc *ABC* of the circle with centre *O*.
$\therefore m\angle D = \frac{1}{2}(m\angle 4) \rightarrow (\text{ii})$	The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
$\Rightarrow m\angle B + m\angle D = \frac{1}{2}m\angle 2 + \frac{1}{2}m\angle 4$	Adding (i) and (ii)
$= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}$ (Total central angle)	
i.e.,$m\angle B + m\angle D = \frac{1}{2}(4\angle rt) = 2\angle rt$	
Similarly $m\angle A + m\angle C = 2\angle rt$	

Q.2 Show that parallelogram inscribed in a circle will be a rectangle. **(A.B)**

Given

ABCD is a parallelogram inscribed in a circle with centre '*O*' .



To prove

ABCD is a rectangle.

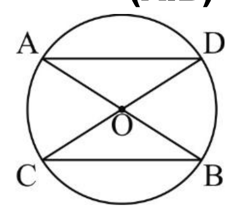
Proof

Statements	Reasons
$\angle A \cong \angle C \rightarrow (\text{i})$	Opposite angles of a parallelogram
$m\angle A + m\angle C = 180° \rightarrow (\text{ii})$	Th-12.4 (opposite angles are supplementary)
$2m\angle A = 180°$	From (i) and (ii)
$m\angle A = 90° \rightarrow (\text{iii})$	Div both sides by 2
$m\angle C = 90° \rightarrow (\text{iv})$	
Similarly	
$m\angle B = m\angle D = 90° \rightarrow (\text{v})$	
Hence ABCD is a rectangle.	From (iii),(iv) and (v)

Q.3 *AOB* and *COD* are two intersecting chords of a circle. Show that Δ^s *AOD* and *BOC* are equiangular. **(A.B)**

Given

In a circle with centre '*O*', AOB and COD are two intersecting chords. AD and BC are joined



To prove

Triangles *AOD* and *BOD* are equiangular

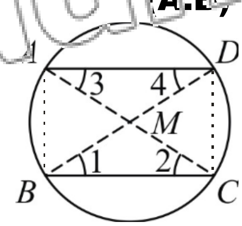
Proof

Statement	Reasons
In $\Delta AOD \leftrightarrow \Delta BOC$	
$\angle D \cong \angle B$	Th-12.2 Circum angles
Similarly	Subtended by are *AC*.
$\angle A \cong \angle C$	Circum angles subtended by arc *BD*.
$\angle AOD \cong \angle BOC$	Vertical angle
$\therefore \Delta AOD \sim \Delta BOC$	A.A.A postulate
$\therefore \Delta^s$ *AOD* and *BOD* are equiangular.	

Q.4 $\overline{AD}$ and $\overline{BC}$ are two parallel chords of a circle. Prove that arc $\widehat{AB} \cong$ arc $\widehat{CD}$ and arc $\overline{AC} \cong$ are $\overline{BD}$ **(A.B)**

Given:

In a circle with centre '*M*' chord $\overline{AD} \parallel$ chord $\overline{BC}$



To Prove:

$AC \cong BD$

Construction:

Join *A* to *C* and *B* to *D*. $\overline{AC}$ and $\overline{BD}$ intersect at point *M*. Name of the angles as shown in the figure.

Proof:

Statement	Reasons
$\angle 1 \cong \angle 4 \rightarrow$ (i)	Alternate angles
$\angle 2 \cong \angle 3 \rightarrow$ (ii)	Alternate angles
$\angle 1 \cong \angle 3 \rightarrow$ (iii)	Angles in the same segments
$\angle 3 = \angle 4$	From (i) and (iii)
In ΔAMD, $\overline{AM} \cong \overline{DM} \rightarrow (iv)$	Opposite sides of $\cong$ angles
Similarly	
$\overline{MC} \cong \overline{BM} \rightarrow$ (v)	
$m\overline{AM} + m\overline{CM} = m\overline{DM} + m\overline{BM}$	Adding (iv) &(v)
$\overline{AC} \cong \overline{BD}$	
In $\Delta BCA \leftrightarrow \Delta BCD$	
$\overline{BC} \cong \overline{BC}$	Common
$\angle 1 \cong \angle 2$	From (i) and (ii)
$\overline{AC} \cong \overline{BD}$	Already proved
$\therefore \Delta BCD \cong \Delta BCA$	SAS postulate
$\overline{AB} \cong \overline{CD}$	Corresponding sides of $\cong$ Δ^S.
Hence $AB \cong CD$	

Miscellaneous Exercise 12

Q.1 Multiple choice questions

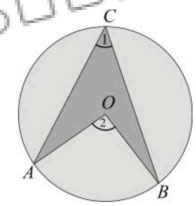
Four possible answers are given for the following question. Tick (✓) the correct answer.

(i) **A circle passes through the vertices of a right angled $\triangle ABC$ with $m\overline{AC} = 3cm$ and $m\overline{BC} = 4cm, m\angle C = 90°$. Radius of the circle is:** **(K.B + U.B)**

(a) $1.5cm$ **(b)** $2.0cm$

(c) $2.5cm$ **(d)** $3.5cm$

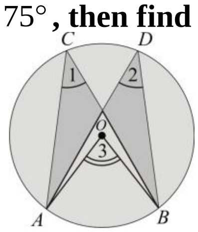
(ii) **In the adjacent circular figure, central and inscribed angles stand on the same arc AB. Then** **(FSD 2017)** **(K.B + U.B)**



(a) $m\angle 1 = m\angle 2$ **(b)** $m\angle 1 = 2m\angle 2$

(c) $m\angle 2 = 3m\angle 1$ **(d)** $m\angle 2 = 2m\angle 1$

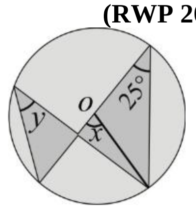
(iii) **In the adjacent figure if $m\angle 3 = 75°$, then find $m\angle 1$ and $m\angle 2$.** **(GWR 2014)** **(K.B + U.B)**



(a) $37\frac{1°}{2}, 37\frac{1°}{2}$ **(b)** $37\frac{1°}{2}, 75°$

(c) $75°, 37\frac{1°}{2}$ **(d)** $75°, 75°$

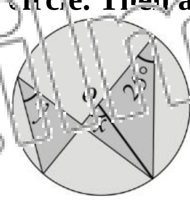
(iv) **Given that O is the centre of the circle. Then angle marked x will be:**
(RWP 2014, SGD 2014, D.G.K 2015) **(K.B + U.B)**



(a) $12\frac{1°}{2}$ **(b)** $25°$

(c) $50°$ **(d)** $75°$

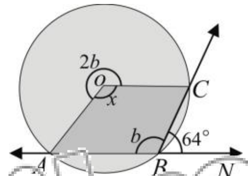
(v) **Given that O is the centre of the circle. Then angle marked y will be:** **(K.B + U.B)**



(a) $12\frac{1°}{2}$ **(b)** $25°$

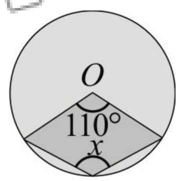
(c) $50°$ **(d)** $75°$

(vi) **In the figure, O is the centre of the circle and $\overleftrightarrow{ABN}$ is a straight line. The obtuse angle $AOC = x$ is:** **(K.B + U.B)**



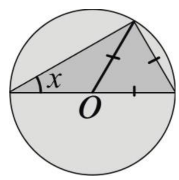
(a) 32° **(b)** 64°
(c) 96° **(d)** 128°

(vii) **In the figure, O is the centre of the circle, then the angle x is:** **(K.B + U.B)**
(SWL 2014, SGD 2014, D.G.K 2015)



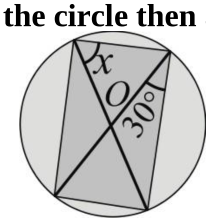
(a) 55° **(b)** 110°
(c) 220° **(d)** 125°

(viii) **In the figure, O is the centre of the circle then angle x is:** **(K.B + U.B)**
(GWR 2014, D.G.K 2014)



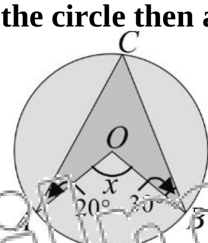
(a) 15° **(b)** 30°
(c) 45° **(d)** 60°

(ix) **In the figure, O is the centre of the circle then angle x is:** **(K.B + U.B)**



(a) 15° **(b)** 30°
(c) 45° **(d)** 60°

(x) **In the figure, O is the centre of the circle then angle x is:** **(RWP 2015)** **(K.B + U.B)**



(a) 50° **(b)** 75°
(c) 100° **(d)** 125°

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii	ix	x
c	d	a	c	b	d	d	b	d	c

CUT HERE

SELF TEST

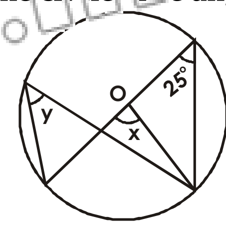
Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 A circle passes through the vertices of a right angled triangle ABC with $m\overline{AC} = 3cm,\ m\overline{BC} = 4cm,\ m\angle C = 90^o$. **Radius of the circle is:**

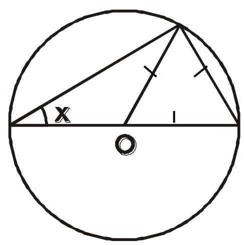
(A) $1.5cm$ **(A)** $2.0cm$
(C) $2.5cm$ **(D)** $3.5cm$

2 Given that O is the centre of the circle. The angle marked *x* will be:



(A) $12\frac{1}{2}^o$ **(A)** 25^o
(C) 50^o **(D)** 75^o

3 In the figure, O is the centre of the circle then angle *x* is:



(A) 15^o **(A)** 30^o
(C) 45^o **(D)** 60^o

4 Inscribed angle of minor segment is:

(A) Acute **(A)** Obtuse
(C) Right **(D)** Complete

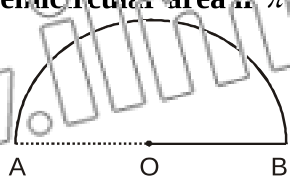
5 A 4 cm long chord subtends a central angle of 60º. The radial segment of this circle is:

(A) 1 **(A)** 2
(C) 3 **(D)** 4

6 Opposite angles in cyclic quadrilateral are:

(A) Complementary **(A)** Supplementary
(C) Equal **(D)** Alternate

7 In the adjacent figure find semicircular area if $\pi = 3.1416$ **and** $m\overline{OA} = 20cm$ **:**



(A) 62.83 sq cm **(A)** 314.16 sq cm
(C) 436.20 sq cm **(D)** 628.32 sq cm

CUT HERE

Give Short Answers to following Questions. **(5×2=10)**

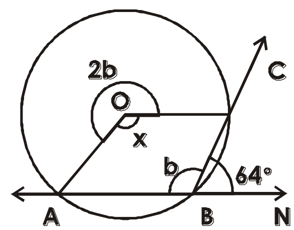
(i) Define cyclic quadrilateral.

(ii) Define circum angle or inscribed angle.

(iii) Define congruent arcs.

(iv) Define length of tangent.

(v) In the figure, O is the centre of the circle and $\overleftrightarrow{ABN}$ is a straight line. Then find obtuse angle $AOC = x$.



Q.2 Prove that, **(8)**

Prove that the measure of a central angle of a minor arc of a circle, is doubled that of the angle subtended by the corresponding major arc.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.